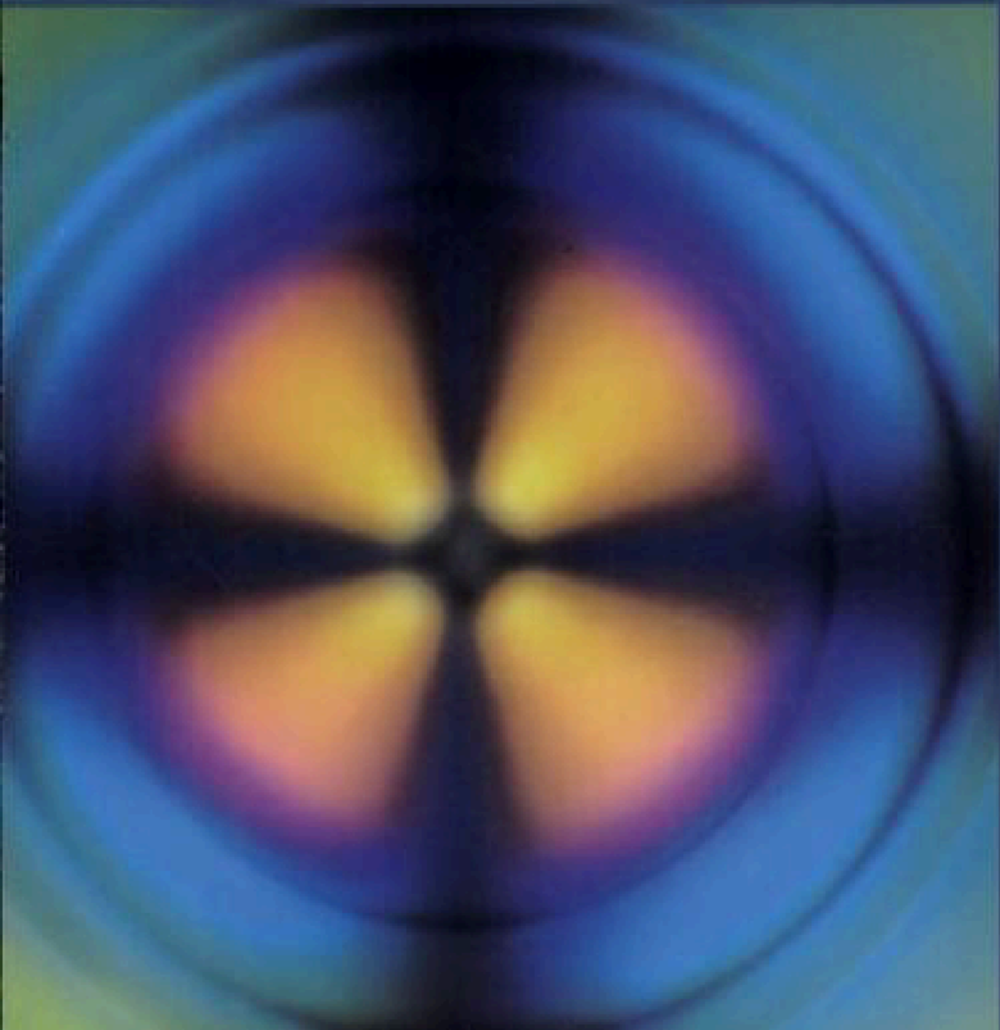
A-Level PHYSICS

FOURTH EDITION



Roger Muncaster

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A-Level PHYSICS

FOURTH EDITION

Roger Muncaster

BSc PhD

Formerly Head of Physics
Bury Metropolitan College of Further Education

First published in 1981
Second edition 1985
Third edition 1989
Fourth edition 1993

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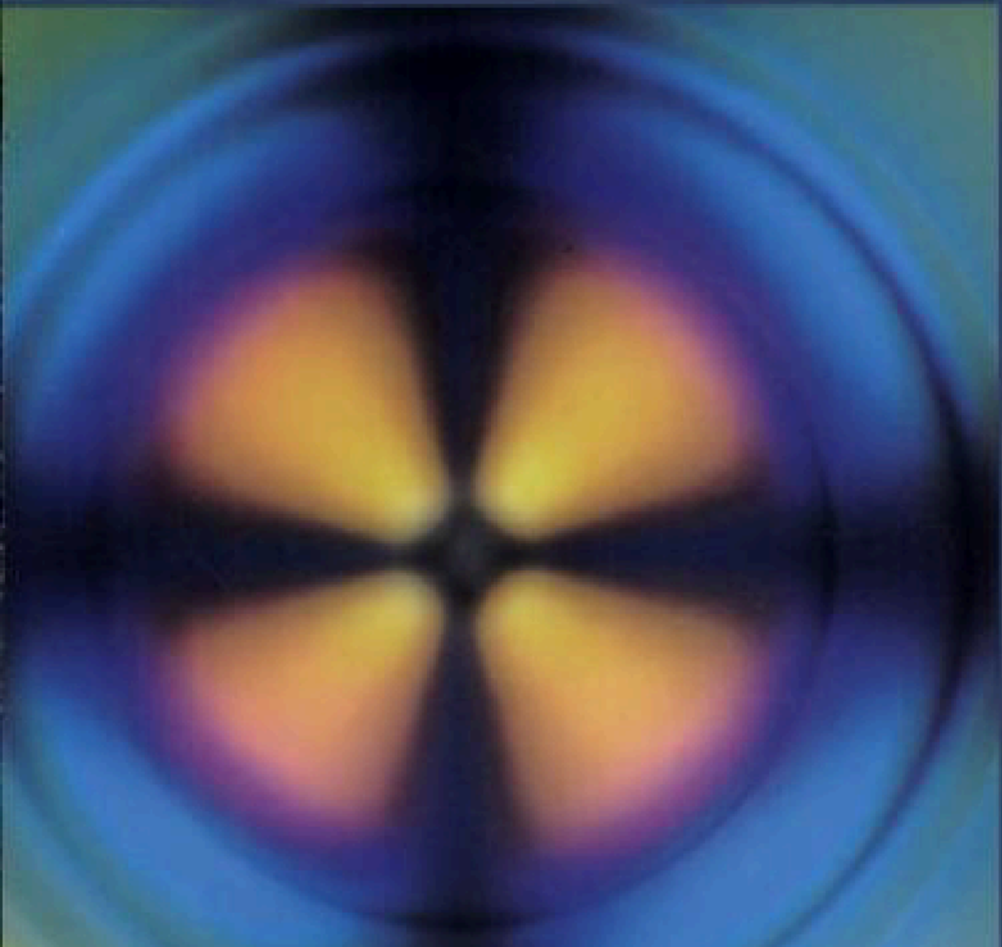

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Preface

This book is intended to cover the NEAB, London, and AEB A-level syllabuses in Physics. It will also be found to cover the bulk of all the other syllabuses for A-level Physics, including those used overseas. Students following BTEC National courses involving Physics should also find the book useful, as should those university students who are studying Physics as a subsidiary subject. SI units are used throughout.

The aim has been to produce a book which is not so long that students are unlikely to read it. On the other hand, the book is not a set of 'revision notes' and it has been my intention to explain every topic thoroughly. It is hoped that the explanations are such that all students will understand them; at the same time, the content is intended to be such that the book will provide a proper basis for those students who are going on to study Physics at degree level.

The book has been arranged in seven main sections (A to G). Though there is no need to read the sections in the order in which they are presented, on the whole it is advisable to keep to the chapter sequence within any one section.

Practical details are given of those experiments which students are required to describe at length in examinations. The book contains many worked examples.

Chapters 9, 11 and 55 were extended for the second edition of the book; a chapter on thermodynamics was added at the same time. Sections on pressure, density, Archimedes' principle, reflection at plane surfaces, defects of vision, magnetic domains, U-values and impulse were added for the third edition. The treatments of various other topics were also revised and the number of experimental investigations was increased.

Since the advent of the GCSE examination and double science, students starting A-level courses tend to have less knowledge of Physics than they did previously. In the light of this, I have needed to make further additions to the book.

The number of worked examples has been greatly increased. Many of these are easier than was previously considered necessary. Questions have been added at relevant points in the text so that students can obtain an immediate test of their understanding of a topic. 'Consolidation' sections have been added at the ends of selected Chapters. These are designed to stress key points and, in some cases, to present an overview of a topic in a manner which would not be possible in the main text. Definitions and fundamental points are now highlighted – either by the use of screening or bold type.

At the end of each of the seven sections of the book there are questions, most of which are taken from past A-level papers. Over two hundred of these have been added for the fourth edition of the book.

A new edition gives me the opportunity to thank all those people who have suggested ways in which the book might be improved. I am particularly grateful to Jeni Davies for undertaking the laborious task of assisting with proof-reading, and for the invaluable suggestions she has made throughout the preparation of this edition.

Finally, I express my gratitude to the following examinations boards for permission to use questions from their past examination papers:

Associated Examining Board [AEB]
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Where only part of the original question has been used, this is indicated by an asterisk in the acknowledgement to the board concerned thus [L*].

R. MUNCASTER
Helmshore

SECTION A

MECHANICS

1

VECTORS

1.1 VECTORS AND SCALARS

Vector quantities have both magnitude and direction; **scalar quantities** have magnitude only.

Examples of each type of quantity are given in Table 1.1.

Table 1.1
Examples of vectors and scalars

Scalars	*Vectors*
Distance	Displacement
Speed	Velocity
Mass	Force (weight)
Energy (work)	Acceleration
Volume	Momentum
Charge	Torque

Vectors can be represented by a line drawn in a particular direction. The length of the line represents the magnitude of the vector; the direction of the line represents the direction of the vector. In print, vector quantities are indicated by using bold type (e.g. $\boldsymbol{F}$) or by using an arrow (e.g. $\overrightarrow{F}$). The same symbol without the use of either bold type or an arrow (e.g. F) represents the magnitude of the vector.

Two vector quantities are equal only if they have the same magnitude and direction.

1.2 DISPLACEMENT

The displacement of a body may be defined as being the length and direction of the imaginary line joining it to some reference point.

Displacement is therefore a vector; the magnitude of the displacement is equal to the distance from the reference point.

Suppose a body moves from O to Y along the path OXY (Fig. 1.1). When the body is at Y its displacement from O is the vector, **OY**. The magnitude of the displacement is simply the length of OY. This is quite clearly less than the path length OXY, illustrating that the magnitude of the displacement of a body is not necessarily equal to the distance the body has actually moved.

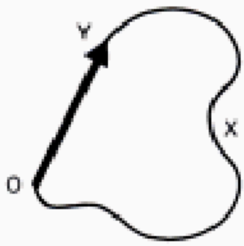


Fig. 1.1
To illustrate the difference between displacement and distance

1.3 SOME DEFINITIONS

Velocity is the rate of change of displacement, i.e. the rate of change of distance in a given direction.

Speed is the rate of change of distance.

Momentum is the product of mass and velocity.

Acceleration is the rate of change of velocity.

Velocity, momentum and acceleration are vectors

Note A body moving along a circular path may have constant speed but, because its direction is changing, it cannot have a constant velocity. It follows that if a body is moving around a circle, even if it has constant speed, it is being accelerated because its velocity is changing.

1.4 RELATIONSHIP BETWEEN SPEED AND VELOCITY

If a body moves along a straight line (without ever reversing its direction of motion), the distance it moves is equal to the magnitude of its displacement from the starting point. It follows, therefore, that since

$$\text{Speed} = \frac{\text{Distance moved}}{\text{Time taken}}$$

and

$$\text{Magnitude of velocity} = \frac{\text{Magnitude of displacement}}{\text{Time taken}}$$

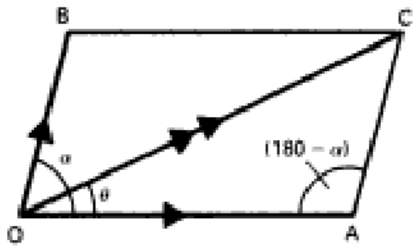
then

$$\text{Speed} = \text{Magnitude of velocity}$$

It should be noted that this relationship is not necessarily true if the motion is not along a straight line, for then the magnitude of the displacement is less than the distance moved. The relationship does hold, though, if the time interval under consideration is infinitesimally short, for then the path length will also be infinitesimally short and therefore can be considered linear. Thus, for all types of motion

$$\text{Instantaneous speed} = \text{Magnitude of instantaneous velocity}$$

Fig. 1.5
The parallelogram rule



Thus, in Fig. 1.5,

$$\mathbf{OA} + \mathbf{OB} = \mathbf{OC}$$

Subtraction

This can be achieved by adding a vector of the same magnitude as that being subtracted but which acts in the opposite direction. For example

$$\mathbf{OA} - \mathbf{OB} = \mathbf{OA} + \mathbf{BO}$$

$$\therefore \quad \mathbf{OA} - \mathbf{OB} = \mathbf{BC} + \mathbf{BO}$$

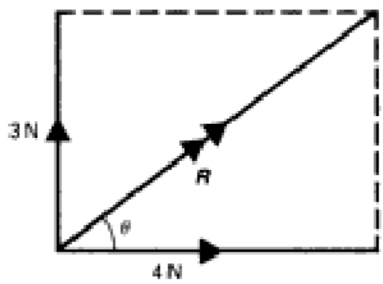
$$\text{i.e.} \quad \mathbf{OA} - \mathbf{OB} = \mathbf{BA}$$

EXAMPLE 1.1

A force of 3 N acts at 90° to a force of 4 N. Find the magnitude and direction of their resultant, $\boldsymbol{R}$.

Solution

Fig. 1.6
Diagram for Example 1.1



Refer to Fig. 1.6. By Pythagoras

$$R^2 = 3^2 + 4^2 = 25 \qquad \therefore R = 5\,\text{N}$$

Also

$$\tan\theta = \tfrac{3}{4} \qquad \therefore \theta = \tan^{-1}\left(\tfrac{3}{4}\right) = 37^\circ$$

The resultant is therefore a force of 5 N acting at 37° to the 4 N force and at 53° to the 3 N force.

EXAMPLE 1.2

A force of 3 N acts at 60° to a force of 5 N. Find the magnitude and direction of their resultant, $\boldsymbol{R}$.

Solution

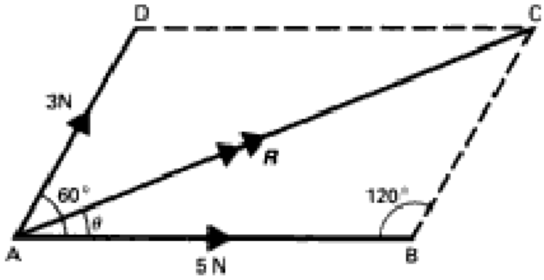


Fig. 1.7
Diagram for Example 1.2

Refer to Fig. 1.7. Applying the cosine rule (Appendix 3.7) to △ ABC gives

$$R^2 = 5^2 + 3^2 - 2\cdot5\cdot3\cos 120^\circ$$
$$= 25 + 9 + 15 = 49 \qquad \therefore \quad R = 7\,\text{N}$$

Applying the sine rule (Appendix 3.7) to △ ABC gives

$$\frac{R}{\sin 120^\circ} = \frac{3}{\sin\theta}$$

$$\therefore \qquad \sin\theta = \frac{3\sin 120^\circ}{R} = \frac{3\sin 120^\circ}{7} = 0.3712$$

$$\therefore \qquad \theta = \sin^{-1} 0.3712 = 21.8^\circ \text{ or } 180^\circ - 21.8^\circ = 158.2^\circ$$

Your calculator will give you $\sin^{-1}$ 0.3712 as 21.8° but in general $\sin\alpha = \sin(180^\circ - \alpha)$ and therefore 158.2° is also a possibility.

It is obvious from the diagram that θ must be acute and therefore the required value is 21.8°. The resultant is therefore a force of 7 N acting at 21.8° to the 5 N force and at 38.2° to the 3 N force.

EXAMPLE 1.3

A particle which is moving due east at 4 m s^{-1} changes direction and starts to move due south at 3 m s^{-1}. Find the change in velocity.

Solution

The change in velocity is the 'new' velocity minus the 'old' velocity, just as a change in temperature, for example, would be the 'new' temperature minus the 'old' temperature. Therefore

$$\text{Change in velocity} = 3\text{ m s}^{-1}\text{ (S)} - 4\text{ m s}^{-1}\text{ (E)}$$
$$= 3\text{ m s}^{-1}\text{ (S)} + 4\text{ m s}^{-1}\text{ (W)}$$

The change in velocity is therefore the vector $\boldsymbol{R}$ of Fig. 1.8.

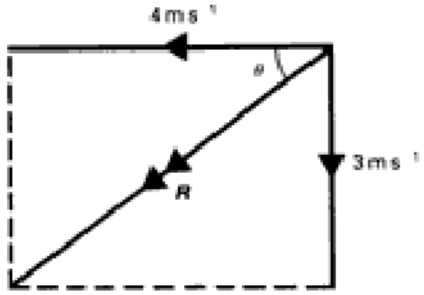


Fig. 1.8
Diagram for Example 1.3

$$R^2 = 3^2 + 4^2 = 25 \qquad \therefore \quad R = 5\ \text{m s}^{-1}$$

$$\tan\theta = \tfrac{3}{4} \qquad \therefore \quad \theta = \tan^{-1}\left(\tfrac{3}{4}\right) = 37^\circ$$

i.e. Change in velocity = $5\ \text{m s}^{-1}$ at 37° S of W.

Alternatively, we can say that the velocity has increased by $5\ \text{m s}^{-1}$ in the direction 37° S of W.

Note The parallelogram rule can also be used to obtain the resultant of more than two vectors. For example, suppose that the resultant of three vectors is required. The procedure is to use the rule to find the resultant of any two of them, and then to use it again to add this to the remaining vector.

QUESTIONS 1A

1. Find the magnitude and direction of the resultant of each of the following pairs of forces.
(a) 7 N at 90° to 24 N,
(b) 20 N at 60° to 30 N,
(c) 40 N at 110° to 50 N,
(d) 60 N at 150° to 20 N.

2. Find the resultant of a displacement of 30 m due east followed by a displacement of 70 m due south.

3. Find: **(a)** the increase in speed, **(b)** the increase in velocity when a body moving south at $20\ \text{m s}^{-1}$ changes direction and moves north at $30\ \text{m s}^{-1}$.

4. Find the magnitude and direction of the increase in velocity when a body which has been moving due S at $6.0\ \text{m s}^{-1}$ changes direction and moves NW at $8.0\ \text{m s}^{-1}$.

1.6 COMPONENTS OF VECTORS

It follows from the parallelogram rule that any vector can be treated as if it is the sum of a pair of vectors. There is an infinite number of these pairs and three are shown in Fig. 1.9. A perpendicular pair such as ***P*** and ***Q*** is the most useful.

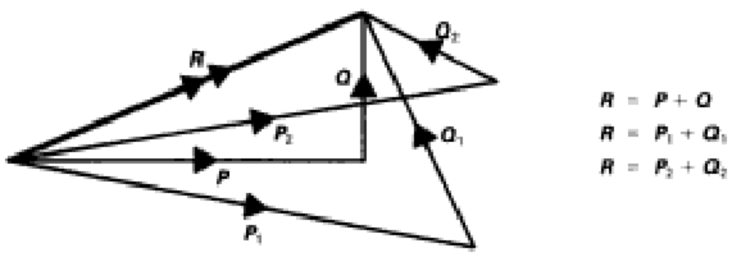


Fig. 1.9
Components of a vector

Consider a vector, $\boldsymbol{F}$, resolved into two perpendicular vectors of magnitudes AB and AD (Fig. 1.10). From simple trigonometry, $AB = F\cos\theta$ and $AD = F\sin\theta$, and therefore $\boldsymbol{F}$ can be resolved into two perpendicular vectors (called **the perpendicular components of $\boldsymbol{F}$**) of magnitudes $F\sin\theta$ and $F\cos\theta$ (Fig. 1.11).

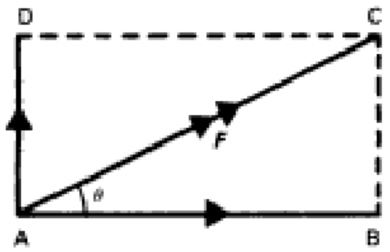


Fig. 1.10
Resolving a vector into two perpendicular components

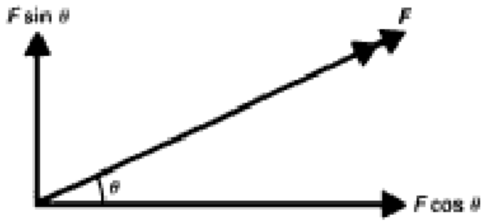


Fig. 1.11
The perpendicular components of a vector

EXAMPLE 1.4

Calculate the horizontal and vertical components of a force of 50 N which is acting at 40° to the horizontal.

Solution

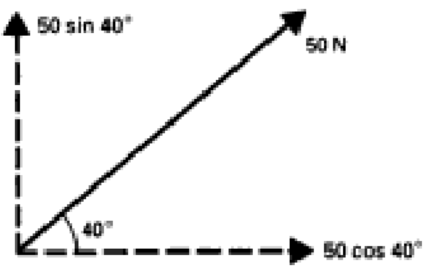


Fig. 1.12
Diagram for Example 1.4

Refer to Fig. 1.12.

Horizontal component $= 50\cos 40° = 38\,\text{N}$

Vertical component $= 50\sin 40° = 32\,\text{N}$

EXAMPLE 1.5

A body of weight 100 N rests on a plane which is inclined at 30° to the horizontal. Calculate the components of the weight parallel and perpendicular to the plane.

Solution

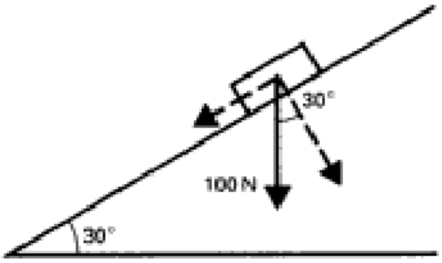


Fig. 1.13
Diagram for Example 1.5

Refer to Fig. 1.13.

Component parallel to plane = 100 sin 30° = 50.0 N

Component perpendicular to plane = 100 cos 30° = 86.6 N

EXAMPLE 1.6

Find the resultant of the system of forces shown in Fig. 1.14.

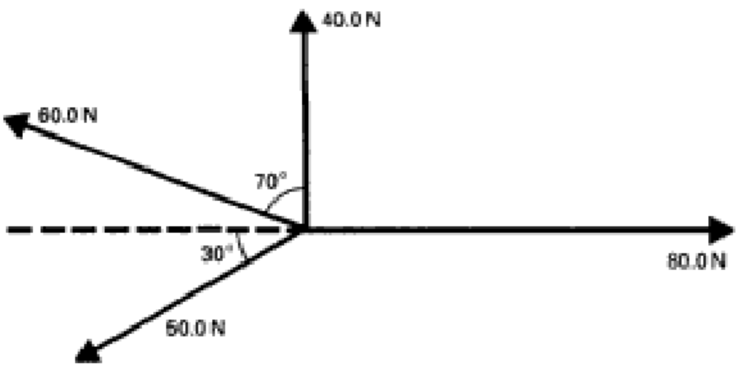


Fig. 1.14
Diagram for Example 1.6

Solution

Total upward force = 40.0 + 60.0 cos 70° − 50.0 sin 30° = 35.52 N

Total force to right = 80.0 − 60.0 sin 70° − 50.0 cos 30° = −19.68 N

The minus sign implies that the horizontal force is to the left. The resultant, $\boldsymbol{R}$, is as shown in Fig. 1.15.

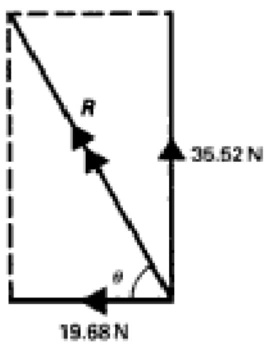


Fig. 1.15
Diagram for Example 1.6

$$R^2 = 35.52^2 + 19.68^2 \qquad \therefore \quad R = 40.6\ \text{N}$$

$$\tan\theta = \frac{35.52}{19.68} \qquad \therefore \quad \theta = 61.0^\circ$$

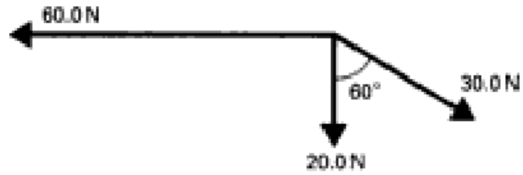
The resultant is therefore a force of 40.6 N acting at 61.0° to the horizontal.

QUESTIONS 1B

1. Find the horizontal and vertical components of: **(a)** a force of 30.0 N acting at 30° to the horizontal, **(b)** a velocity of 50.0 m s^{-1} at 60° to the horizontal.

2. A particle at weight 200 N rests on a plane inclined at 50° to the horizontal. What are the components of the weight: **(a)** parallel, **(b)** perpendicular to the plane?

3. Find the resultant of the system of forces shown below.



CONSOLIDATION

Vectors have both magnitude and direction; **scalars** have magnitude only.

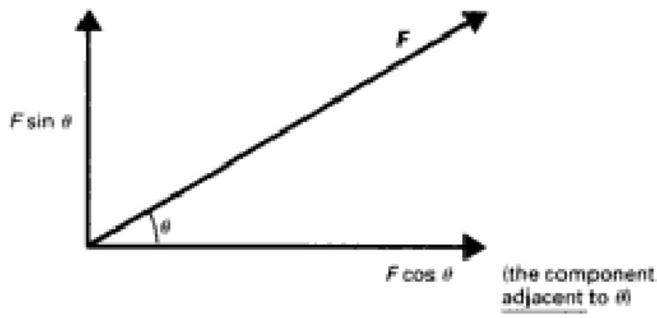
Displacement and **velocity** are vectors; **distance** and **speed** are scalars.

Distance from reference point = magnitude of displacement.

Instantaneous speed = magnitude of instantaneous velocity

Vectors can be added and subtracted by using the **parallelogram rule**.

Components of Vectors



2

MOTION

2.1 NEWTON'S LAWS OF MOTION

In 1687 Sir Isaac Newton published his *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Science*), in which he stated the three laws on which the science of mechanics is based.

Newton's First Law

Every body continues in its state of rest or of uniform (unaccelerated) motion in a straight line unless acted on by some external force.

This law expresses the concept of inertia. The **inertia** of a body can be described as being its reluctance to start moving, or to stop moving once it has started.

Events often seem to contradict the first law, for it is our natural experience that there are many familiar examples of motion in which moving objects come to rest when (apparently) left to their own devices. Closer examination of the circumstances, however, reveals that in every case there is some sort of retarding force acting. Such forces are often due to friction between solid surfaces or to air resistance.

A body of large mass requires a large force to change its speed or its direction by a noticeable amount, i.e. the body has a large inertia. Thus, **the mass of a body is a measure of its inertia.**

Newton's Second Law

The rate of change of momentum of a body is directly proportional to the external force acting on the body and takes place in the direction of the force.

In mathematical terms the second law may be written as

$$F \propto \frac{\mathrm{d}}{\mathrm{d}t}(mv)$$

where F = the applied force, and

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = \text{the rate of change of momentum.}$$

Introducing a constant of proportionality, k, this becomes

$$F = k\frac{\mathrm{d}}{\mathrm{d}t}(mv)$$

The SI unit of force (the newton) is defined in such a way that $k = 1$ provided that the rate of change of momentum is also expressed in the relevant SI unit ($kg\ m\ s^{-2}$), in which case

$$F = \frac{d}{dt}(mv) \tag{2.1}$$

If the mass is constant, equation [2.1] becomes

$$F = m\frac{dv}{dt}$$

i.e. $$\boldsymbol{F = ma} \tag{2.2}$$

where $a =$ the acceleration that results from the application of the force.

Equations [2.1] and [2.2] are the forms in which Newton's second law is normally used, but it should be remembered that they are valid only if a consistent set of units is used, and that equation [2.2] applies only in the case of a constant mass.

Equation [2.2] is used to define the newton. Thus:

The newton (N) is that force which produces an acceleration of $1\ m\ s^{-2}$ when it acts on a mass of 1 kg.

The experimental investigation of $F = ma$ is dealt with in section 2.15.

Newton's Third Law

If a body A exerts a force on a body B, then B exerts an equal and oppositely directed force on A.

This law is often misinterpreted as meaning that the two forces cancel each other out because they are of equal strength and act in opposite directions. There is, in fact, no possibility of this, because the two forces each act on different bodies.

Thus, if a man pushes on a large stationary crate, the crate pushes back on the man with a force of exactly the same size. Whether or not the crate starts to move, has nothing to do with the force that it exerts on the man. In accordance with Newton's second law, the crate will start to move if the force exerted by the man is greater than any forces which are acting on the crate in such a way as to resist its motion (e.g. friction between the crate and the ground).

The third law implies that forces always occur in pairs – some examples are given below.

(i) The Earth exerts a gravitational force of attraction on the Moon; the Moon exerts a force of the same size on the Earth.

(ii) A rocket moves forward as a result of the push exerted on it by the exhaust gases which the rocket has pushed out.

(iii) When a man jumps off the ground it is because he has pushed down on the Earth and the Earth, in accordance with Newton's third law, has pushed up on him. It should not be overlooked that the other result of this is that the Earth moves down.

(iv) If a car is accelerating forward, it is because its tyres are pushing backward on the road and the road is pushing forward on the tyres. Note that if the car is moving forward and slowing down, the tyres push forward and the road pushes backward.

2.2 MASS AND WEIGHT

The weight of a body is the force acting on its mass due to the gravitational attraction of the Earth.*

In accordance with Newton's second law, a body acquires an acceleration whenever there is a net force acting on it. The acceleration that results from the effect of gravity (i.e. that results from its weight) is known as the acceleration due to gravity, g. By equation [2.2], the weight of a body of mass m is given by

$$\textbf{Weight} = mg \qquad [2.3]$$

The force exerted by gravity is such that, at any given point in a gravitational field (and therefore at any given point on the Earth's surface), the acceleration due to gravity is the same for all bodies, no matter what their masses (see Chapter 8).

It follows that two bodies dropped from the same point above the surface of the Earth reach the ground at the same time even if their masses are different. (Note that this statement ignores the effect of air resistance; when the viscous drag of the air is significant, for example if one of the bodies is a feather or is falling by parachute, it is not even approximately true.) The acceleration due to gravity varies slightly from place to place on the Earth's surface, but it is normally sufficiently accurate to use a value of $9.8\ \mathrm{m\,s^{-2}}$ everywhere. Thus, from equation [2.3] the weight (in newtons) of a body which has a mass m (in kg) is given by

$$\text{Weight} = m \times 9.8$$

Another unit, the **kilogram force** (kgf), is often used as a measure of weight. It is defined such that a mass of 1 kg has a weight of 1 kgf. This is not an SI unit and must not be used in any equation where it is not possible to use it on both sides of the equation. If in doubt, it is best to convert kilograms force to newtons by making use of

$$1\ \text{kgf} = 9.8\ \text{N}$$

Summary of Differences between Mass and Weight

(i) The mass of a body is a measure of its resistance to acceleration (i.e. it is a measure of the inertia of the body). The weight of a body is the force exerted on its mass by gravity.

(ii) In SI units mass is measured in kilograms, weight is measured in newtons.

(iii) The mass of a body is the same everywhere. The weight of a body on the surface of the Earth has a slight dependence on where it is, and would have considerably different values at other places in the Universe.

*The weight of a body on the Moon is the force exerted on its mass by the gravitational attraction of the Moon.

EXAMPLE 2.1

A body of mass 7.0 kg rests on the floor of a lift. Calculate the force, R, exerted on the body by the floor when the lift: (a) has an upward acceleration of $2.0\ \mathrm{m\,s^{-2}}$, (b) has a downward acceleration of $3.0\ \mathrm{m\,s^{-2}}$, (c) is moving down with a constant velocity. (Assume $g = 10\ \mathrm{m\,s^{-2}}$.)

Solution

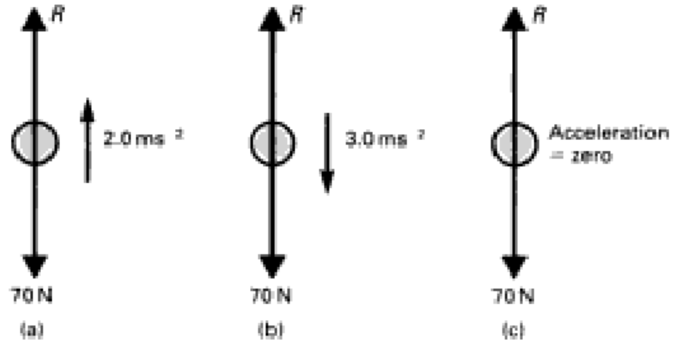


Fig. 2.1
Diagram for Example 2.1

(a) Refer to Fig. 2.1(a) (When using $F = ma$ the direction of F must be the same as that of a. The body has an upward acceleration and therefore we require the resultant upward force.)

By Newton's second law (equation [2.2])

$$\underbrace{R - 70}_{\text{upward force}} = 7.0 \times \underset{\text{upward acceleration}}{2.0}$$

$$\therefore \quad R - 70 = 14 \qquad \text{i.e.} \quad R = 84\ \mathrm{N}$$

(b) Refer to Fig. 2.1(b) (The acceleration is downward and therefore we require the resultant downward force.)

By equation [2.2].

$$\underbrace{70 - R}_{\text{downward force}} = 7.0 \times \underset{\text{downward acceleration}}{3.0}$$

$$\therefore \quad 70 - R = 21 \qquad \text{i.e.} \quad R = 49\ \mathrm{N}$$

(c) Refer to Fig. 2.1(c). There is no acceleration and therefore, by equation [2.2], no resultant force, in which case

$$R = 70\ \mathrm{N}$$

Notes (i) By Newton's third law, R is equal and opposite to the force exerted by the body on the floor of the lift. It follows that if the body were resting on a bathroom scale rather than directly on the floor of the lift, the scale would register its weight as 84 N, 49 N and 70 N in situations (a), (b) and (c) respectively. Thus the body appears heavier than it actually is when it has an upward acceleration and lighter when it has a downward acceleration. In case (c), where it has no acceleration, it appears neither heavy nor light.

(ii) These results do not depend on the direction in which the lift is moving. For example in (a) the lift has an upward directed acceleration and therefore may be moving up with increasing speed or moving down with decreasing speed.

EXAMPLE 2.2

A body of mass 5.0 kg is pulled up a smooth plane inclined at 30° to the horizontal by a force of 40 N acting parallel to the plane. Calculate the acceleration of the body and the force exerted on it by the plane. (Assume $g = 10\,\mathrm{m\,s^{-2}}$.)

Solution

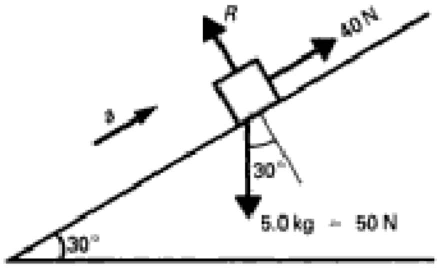


Fig. 2.2
Diagram for Example 2.2

The diagram shows the acceleration of the body and all the forces acting on it

Refer to Fig. 2.2. The plane is smooth and therefore the only force it exerts on the body is the normal reaction R. Let the acceleration of the body be a.

Consider the motion parallel to the plane. The weight has a component of $50 \sin 30°$ acting parallel to the plane (downwards) and therefore by Newton's second law (equation [2.2])

$$\underbrace{40 - 50 \sin 30°}_{\text{resultant force up the plane}} = 5.0 \times \underset{\text{acceleration up the plane}}{a}$$

$$\therefore \quad 40 - 25 = 5.0a \quad \text{i.e.} \quad a = 3.0\,\mathrm{m\,s^{-2}}$$

Note that the resultant force and the acceleration are in the same direction – up the plane.

Consider the motion perpendicular to the plane. The weight has a component of $50 \cos 30°$ perpendicular to the plane and in the opposite direction to R. There is no acceleration perpendicular to the plane and therefore no resultant force, in which case

$$R = 50 \cos 30° \quad \text{i.e.}\ R = 43\,\mathrm{N}$$

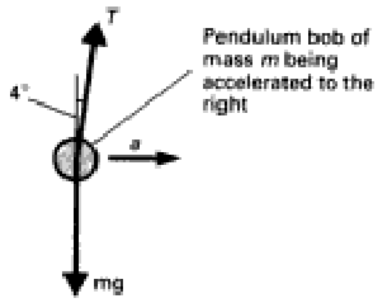
EXAMPLE 2.3

A train is moving along a straight horizontal track. A pendulum suspended from the roof of one of the carriages of the train is inclined at 4° to the vertical. Calculate the acceleration of the train. (Assume $g = 10\,\mathrm{m\,s^{-2}}$.)

Solution

Suppose that the mass of the pendulum bob is m. The forces acting on the bob are its weight, mg, which acts vertically downwards and the tension, T, in the string. (Fig. 2.3.)

Fig. 2.3
Diagram for Example 2.3



Consider the horizontal motion. The pendulum bob is at rest with respect to the train and therefore it too has a horizontal acceleration, a (to the right). The horizontal component of the tension is $T\sin 4^\circ$ and therefore by equation [2.2]

$$T\sin 4^\circ = ma \qquad [2.4]$$

Consider the vertical motion. There is no vertical component of acceleration and therefore

$$T\cos 4^\circ = mg \qquad [2.5]$$

Dividing equation [2.4] by equation [2.5] gives

$$\tan 4^\circ = a/g$$

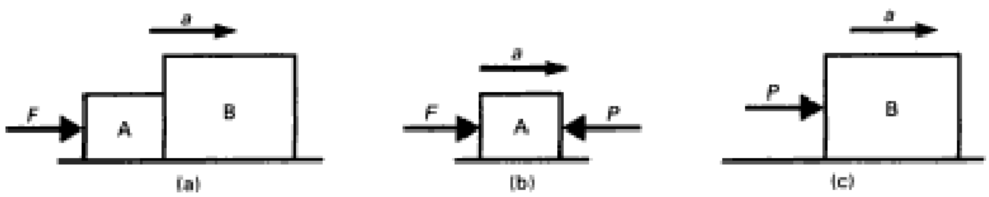
$$\therefore \quad a = g\tan 4^\circ \qquad \text{i.e.} \quad a = 0.70\ \text{m s}^{-2}$$

EXAMPLE 2.4

Two blocks, A of mass m and B of mass $3m$, are side by side and in contact with each other. They are pushed along a smooth floor under the action of a constant force F applied to A. Find: (a) the acceleration of the blocks, (b) the force exerted on B by A.

Solution

Fig. 2.4
The horizontal force(s): (a) on the whole system (b) on A, (c) on B



(a) Let the acceleration of the blocks be a. Consider the motion of the whole system (Fig. 2.4(a)). By Newton's second law (equation [2.2])

$$F = (m + 3m)a \qquad \text{i.e.} \quad a = \frac{F}{4m}$$

(b) Let the force on B due to A be P. By Newton's third law there will be an equal and opposite force on A (Fig. 2.4(b) and (c)). Applying Newton's second law to the motion of B gives

$$P = 3m \times a$$

$$\therefore \quad P = 3m \times \frac{F}{4m} \quad \text{i.e.} \quad P = \frac{3F}{4}$$

The reader should confirm that considering the motion of A gives the same result, though a little less easily.

QUESTIONS 2A

1. The resultant force on a body of mass 4.0 kg is 20 N. What is the acceleration of the body?

2. A body of mass 6.0 kg moves under the influence of two oppositely directed forces whose magnitudes are 60 N and 18 N. Find the magnitude and direction of the acceleration of the body.

3. Two forces, of magnitudes 30 N and 40 N and which are perpendicular to each other, act on a body of mass 25 kg. Find the magnitude and direction of the acceleration of the body.

4. Is the motion of the train in Example 2.3: (A) to the right, (B) to the left, or (C) is it impossible to tell?

5. A body of mass 3.0 kg slides down a plane which is inclined at 30° to the horizontal. Find the acceleration of the body: **(a)** if the plane is smooth, **(b)** if there is a frictional resistance of 9.0 N.
($g = 10\,\text{m}\,\text{s}^{-2}$.)

6. A railway truck of mass 6.0 tonnes moves with an acceleration of $0.050\,\text{m}\,\text{s}^{-2}$ down a track which is inclined to the horizontal at an angle α where $\sin\alpha = 1/120$. Find the resistance to motion, assuming that it is constant.
($g = 10\,\text{m}\,\text{s}^{-2}$, 1 tonne $= 1.0 \times 10^3$ kg.)

7. A body hangs from a spring-balance which is suspended from the ceiling of a lift. What is the mass of the body if the balance registers a reading of 70 N when the lift has an upward acceleration of $4.0\,\text{m}\,\text{s}^{-2}$?
($g = 10\,\text{m}\,\text{s}^{-2}$.)

8. What is the apparent weight during take-off of an astronaut whose actual weight is 750 N if the resultant upward acceleration is $5g$?

9. A body of mass 5.0 kg is pulled along smooth horizontal ground by means of a force of 40 N acting at 60° above the horizontal. Find: **(a)** the acceleration of the body, **(b)** the force the body exerts on the ground.
($g = 10\,\text{m}\,\text{s}^{-2}$.)

10. A railway engine of mass 100 tonnes is attached to a line of trucks of total mass 80 tonnes. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train: **(a)** has an acceleration of $0.020\,\text{m}\,\text{s}^{-2}$, **(b)** is moving at constant velocity.
(1 tonne $= 1.0 \times 10^3$ kg.)

11. A car of mass 1000 kg tows a caravan of mass 600 kg up a road which rises 1 m vertically for every 20 m of its length. There are constant frictional resistances of 200 N and 100 N to the motion of the car and to the motion of the caravan respectively. The combination has an acceleration of $1.2\,\text{m}\,\text{s}^{-2}$ with the engine exerting a constant driving force. Find: **(a)** the driving force, **(b)** the tension in the tow-bar.
($g = 10\,\text{m}\,\text{s}^{-2}$.)

Examples 2.1 to 2.4 are concerned with Newton's second law in the form '$F = ma$' (equation [2.2]). The examples that follow use the law in the form, 'Force = Rate of change of momentum' (equation [2.1]).

EXAMPLE 2.5

Water emerges at $2\,\text{m}\,\text{s}^{-1}$ from a hose pipe and hits a wall at right angles. The pipe has a cross-sectional area of $0.03\,\text{m}^2$. Calculate the force on the wall assuming that the water does not rebound. (Density of water $= 1000\,\text{kg}\,\text{m}^{-3}$.)

Note In solving problems of this type we determine the mass of substance that has its momentum changed in one second. We then find the change in momentum of this mass and so obtain the change in momentum per second, i.e. the rate of change of momentum.

Solution

In one second the volume of water that hits the wall is that which has left the pipe in one second, i.e. that which was contained in a cylinder of length 2 m and cross-sectional area $0.03\,\text{m}^2$, namely $2 \times 0.03 = 0.06\,\text{m}^3$. The mass of water hitting the wall in one second is therefore $0.06 \times 1000 = 60\,\text{kg}$. When the water hits the wall its speed changes from $2\,\text{m}\,\text{s}^{-1}$ to zero, and therefore the rate of change of momentum is $60 \times 2 = 120\,\text{N}$.

By Newton's second law, force = rate of change of momentum, and therefore the force exerted by the wall = 120 N. Therefore, by Newton's third law, the force exerted by the water = 120 N.

EXAMPLE 2.6

A helicopter of mass $1.0 \times 10^3\,\text{kg}$ hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept out by the blades is $80\,\text{m}^2$. Calculate the value of v. (Density of air $= 1.3\,\text{kg}\,\text{m}^{-3}$, $g = 10\,\text{m}\,\text{s}^{-2}$.)

Solution

The volume of air displaced in one second $= 80\,v$, and therefore the mass of air displaced in one second $= 1.3 \times 80\,v = 104\,v$. It follows that in one second the momentum of the air increases by $104\,v^2$. By Newton's second law, rate of change of momentum = force, and therefore the force exerted on the air by the blades $= 104\,v^2$. By Newton's third law, the upward force on the helicopter is also $104\,v^2$. Since the helicopter is hovering, the upward force is equal to the weight of the helicopter, and therefore

$$104\,v^2 = 1.0 \times 10^3\,g$$

i.e. $$v = 9.8\,\text{m}\,\text{s}^{-1}$$

EXAMPLE 2.7

Sand falls onto a conveyor belt at a constant rate of $2\,\text{kg}\,\text{s}^{-1}$. The belt is moving horizontally at $3\,\text{m}\,\text{s}^{-1}$. Calculate: (a) the extra force required to maintain the speed of the belt, (b) the rate at which this force is doing work, (c) the rate at which the kinetic energy of the sand increases.

Account for the fact that the answers to (b) and (c) are different.

Solution

(a) Every second 2 kg of sand acquire a horizontal velocity of $3\,\text{m s}^{-1}$, and therefore the rate of increase of horizontal momentum $= 2 \times 3 = 6\,\text{N}$. By Newton's second law, force = rate of change of momentum, and therefore the extra force required to maintain the speed of the belt = 6 N.

(b) In one second the force moves 3 m, and therefore (by equation [5.1] or [5.7]) the rate at which the force is working $= 6 \times 3 = 18\,\text{W}$.

(c) Kinetic energy $= \frac{1}{2}mv^2$ (see section 5.3), and therefore the rate at which the kinetic energy of the sand is increasing $= \frac{1}{2} \times 2 \times 3^2 = 9\,\text{W}$.

A finite time elapses before the sand acquires the speed of the belt. During this period the belt is slipping past the sand and therefore work has to be done to overcome friction between the sand and the belt. The rate at which work is done by the force is equal to the rate at which it is doing work against friction plus the rate at which it is doing work to increase the kinetic energy of the sand – hence the difference between (b) and (c). (**Note**. The rate at which work is done against friction is equal to the rate at which work is done to increase the kinetic energy of the sand no matter what the speed of the belt and no matter what the rate at which sand is falling onto the belt.)

QUESTIONS 2B

1. Water is squirting horizontally at $4.0\,\text{m s}^{-1}$ from a burst pipe at a rate of $3.0\,\text{kg s}^{-1}$. The water strikes a vertical wall at right angles and runs down it without rebounding. Calculate the force the water exerts on the wall.

2. A machine gun fires 300 bullets per minute horizontally with a velocity of $500\,\text{m s}^{-1}$. Find the force needed to prevent the gun moving backwards if the mass of each bullet is $8.0 \times 10^{-3}\,\text{kg}$.

3. Coal is falling onto a conveyor belt at a rate of 540 tonnes every hour. The belt is moving horizontally at $2.0\,\text{m s}^{-1}$. Find the extra force required to maintain the speed of the belt. (1 tonne = 1000 kg.)

4. The rotating blades of a hovering helicopter sweep out an area of radius 4.0 m imparting a downward velocity of $12\,\text{m s}^{-1}$ to the air displaced. Find the mass of the helicopter. ($g = 10\,\text{m s}^{-2}$, density of air $= 1.3\,\text{kg m}^{-3}$.)

5. Find the force exerted on each square metre of a wall which is at right angles to a wind blowing at $20\,\text{m s}^{-1}$. Assume that the air does not rebound. (Density of air $= 1.3\,\text{kg m}^{-3}$.)

6. Hailstones with an average mass of 4.0 g fall vertically and strike a flat roof at $12\,\text{m s}^{-1}$. In a period of 5.0 minutes six thousand hailstones fall on each square metre of roof and rebound vertically at $3.0\,\text{m s}^{-1}$. Calculate the force on the roof if it has an area of $30\,\text{m}^2$.

7. The speed of rotation of the blades of the helicopter in question **4** is increased so that the air now has a downward velocity of $13\,\text{m s}^{-1}$. Find the (upward) acceleration of the helicopter.

2.3 THE EQUATIONS OF MOTION FOR UNIFORM ACCELERATION

Equations [2.6]–[2.9] describe the motion of bodies which are moving with constant (uniform) acceleration.

$$v = u + at \quad [2.6]$$

$$v^2 = u^2 + 2as \quad [2.7]$$

$$s = ut + \tfrac{1}{2}at^2 \quad [2.8]$$

$$s = \tfrac{1}{2}(u+v)t \quad [2.9]$$

where u = the velocity when $t = 0$,

v = the velocity at time t,

a = the constant acceleration,

s = the distance from the starting point at time t, (this is not necessarily the distance moved).

When using these equations it is necessary to bear in mind that u, v, a and s are vectors. If, say, the positive direction is taken to be up, then:

(i) the velocity of a body which is moving down is negative,

(ii) points below the starting point have negative values of s,

(iii) downward directed accelerations are negative.

Notes (i) An acceleration produces retardation whenever it acts in the opposite direction to the velocity, irrespective of whether the acceleration itself is being taken to be positive or negative.

(ii) The equations of motion can be deduced from the definitions of velocity and acceleration and therefore do not introduce any new ideas; equation [2.8], however, highlights the important result that when a body moves from rest $s \propto t^2$.

(iii) For a body moving at constant velocity $a = 0$ and equations [2.6] and [2.7] reduce to $v = u$. Substituting for u in equation [2.8] (with $a = 0$) or in equation [2.9] gives

$$s = vt \qquad \text{at constant velocity}$$

Derivation of the Equations of Motion for Uniform Acceleration

Suppose that a body is moving with constant acceleration a and that in a time interval t its velocity increases from u to v and its displacement increases from 0 to s. Then, since

Acceleration = Rate of change of velocity

$$a = \frac{v-u}{t}$$

i.e. $$v = u + at \quad [2.6]$$

The average velocity is $\frac{1}{2}(u+v)$ and therefore, since

Displacement = Average velocity × time

$$s = \tfrac{1}{2}(u+v)t \quad [2.9]$$

Eliminating t between equations [2.6] and [2.9] leads to equation [2.7], and eliminating v between any two of the three equations that have now been derived leads to equation [2.8].

EXAMPLE 2.8

A ball is thrown vertically upwards with a velocity of $20\ \mathrm{m\,s^{-1}}$. Calculate:

(a) the maximum height reached,

(b) the total time for which the ball is in the air.
(Assume $g = 10\ \mathrm{m\,s^{-2}}$.)

Solution

(a) We shall take the upward direction to be positive. In the notation of this section:

$u = 20\ \mathrm{m\,s^{-1}}$ (the velocity with which the ball leaves the thrower's hand)

$v = 0$ (at the maximum height)

$a = -10\ \mathrm{m\,s^{-2}}$ (the minus sign is necessary because 'up' has been taken to be positive)

$s = h$ (where h is the maximum height)

From equation [2.7]

$$0^2 = 20^2 + 2(-10)h$$

i.e. $h = 20\ \mathrm{m}$

(b) $u = 20\ \mathrm{m\,s^{-1}}$

$a = -10\ \mathrm{m\,s^{-2}}$

$t = t$ (where t is the time the ball is in the air)

$s = 0$ (since the ball is back on the ground)

From equation [2.8]

$$0 = 20t + \tfrac{1}{2}(-10)t^2$$

i.e. $t = 0$ or $t = 4\ \mathrm{s}$

The required solution is $t = 4\ \mathrm{s}$. The other solution, $t = 0$, refers to the fact that the height of the ball was also zero when it was first projected.

QUESTIONS 2C

Take $g = 10\ \mathrm{m\,s^{-2}}$ where necessary.

1. A particle is moving in a straight line with a constant acceleration of $6.0\ \mathrm{m\,s^{-2}}$. As it passes a point, A, its speed is $20\ \mathrm{m\,s^{-1}}$. What is its speed 10 s after passing A?

2. A particle which is moving in a straight line with a velocity of $15\ \mathrm{m\,s^{-1}}$ accelerates uniformly for 3.0 s, increasing its velocity to $45\ \mathrm{m\,s^{-1}}$. What distance does it travel whilst accelerating?

3. A car starts to accelerate at a constant rate of $0.80\,\mathrm{m\,s^{-2}}$. It covers 400 m whilst accelerating in the next 20 s. What was the speed of the car when it started to accelerate?

4. A body of mass 3.0 kg, initially at rest, moves along a smooth horizontal surface under the effect of a horizontal force of 12 N. **(a)** Find the acceleration of the body. **(b)** Find the speed of the body after 5.0 s.

5. A car moving at $30\,\mathrm{m\,s^{-1}}$ is brought to rest with a constant retardation of $3.6\,\mathrm{m\,s^{-2}}$. How far does it travel whilst coming to rest?

6. A stone is dropped from the top of a cliff which is 80 m high. How long does it take to reach the bottom of the cliff?

7. A particle is projected vertically upwards at $30\,\mathrm{m\,s^{-1}}$. Calculate: **(a)** how long it takes to reach its maximum height, **(b)** the two times at which it is 40 m above the point of projection, **(c)** the two times at which it is moving at $15\,\mathrm{m\,s^{-1}}$.

8. A stone is fired vertically upwards from a catapult and lands 5.0 s later. What was the initial velocity of the stone? For how long was the stone at a height of 20 m or more?

9. A hot-air balloon is 21 m above the ground and is rising at $8.0\,\mathrm{m\,s^{-1}}$ when a sandbag is dropped from it. How long does it take the sandbag to reach the ground?

10. A stone is thrown vertically upwards at $10\,\mathrm{m\,s^{-1}}$ from a bridge which is 15 m above a river. **(a)** What is the speed of the stone as it hits the river? **(b)** With what speed would it hit the river if it were thrown downwards at $10\,\mathrm{m\,s^{-1}}$?

11. A bullet of mass 8.00×10^{-3} kg moving at $320\,\mathrm{m\,s^{-1}}$ penetrates a target to a depth of 16.0 mm before coming to rest. Find the resistance offered by the target, assuming it to be uniform.

2.4 MOTION UNDER GRAVITY

A body that is projected at an angle to the vertical moves along a curved (parabolic) path. In order to solve problems involving motion of this type, we consider the horizontal and vertical components of the motion separately. This is justified because the horizontal motion has no effect on the vertical motion and vice versa. To appreciate this, consider two bodies, A and B, projected horizontally off the edge of a table, and suppose that the velocity with which A is projected is greater than that of B (Fig. 2.5). Both A and B reach the ground at the same time even

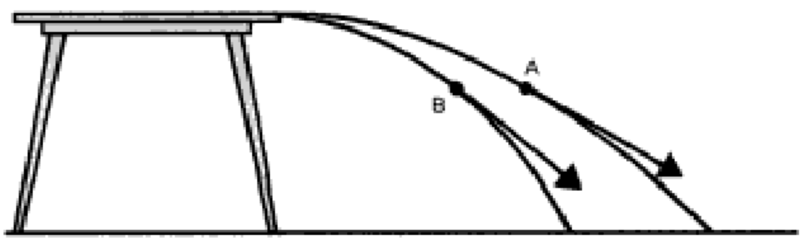


Fig. 2.5
To show the motion of two bodies projected horizontally under gravity

though their velocities of projection were different. This is because, initially, neither body had any vertical component of velocity (they were projected horizontally). The downward motions of both A and B are due to the effect of gravity, and this accelerates each at the same rate ($9.8\,\mathrm{m\,s^{-2}}$). Since they both start from rest (in terms of the vertical motion) and travel the same vertical distance, they reach the ground at the same time. In the absence of air resistance, each body retains its original horizontal component of velocity for the whole of its motion. The horizontal distance travelled by A is therefore greater than that travelled by B.

EXAMPLE 2.9

A body is projected with a velocity of $200\,\mathrm{m\,s^{-1}}$ at an angle of 30° above the horizontal. Calculate:

(a) the time taken to reach the maximum height,

(b) its velocity after 16 s.

(Assume $g = 10\,\mathrm{m\,s^{-2}}$ and ignore air resistance.)

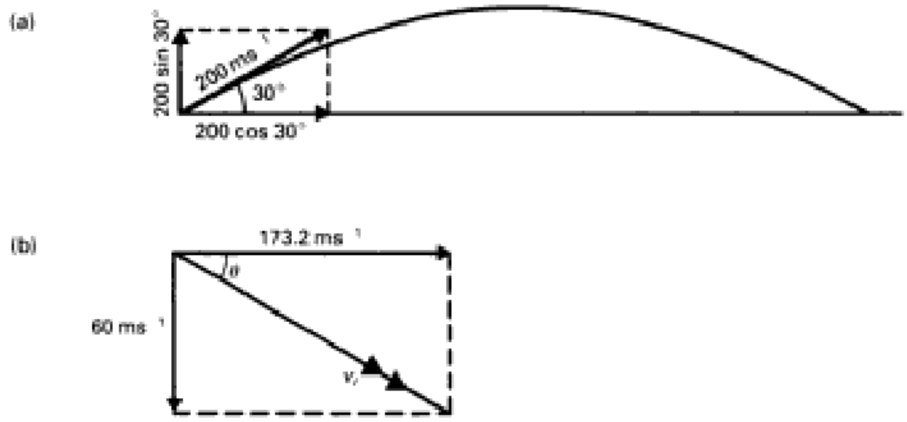


Fig. 2.6
Diagrams for Example 2.9

Solution

(a) Consider the vertical motion. In the notation of the last section:

$u = 200 \sin 30° = 100\,\mathrm{m\,s^{-1}}$

$v = 0$ (at the maximum height)

$a = -10\,\mathrm{m\,s^{-2}}$ (minus sign because 'up' has been taken to be positive)

$t = t$ (where t is the time taken to reach the maximum height)

From equation [2.6]

$$0 = 100 + (-10)t$$

i.e. $t = 10\,\mathrm{s}$

(b) Considering the vertical component of the motion:

$u = 100\,\mathrm{m\,s^{-1}}$

$v = v_y$

$a = -10\,\mathrm{m\,s^{-2}}$

$t = 16\,\mathrm{s}$

From equation [2.6]

$$v_y = 100 + (-10)16$$

i.e. $v_y = -60\,\mathrm{m\,s^{-1}}$

(The minus sign indicates that the body is moving downwards.)

The horizontal component of the velocity will still be 200 cos 30° ($= 173.2\,\mathrm{m\,s^{-1}}$) since, in the absence of air resistance, there is no horizontal component of acceleration. The actual velocity, v_r, is therefore as shown in Fig. 2.6(b), from which

$$v_r^2 = 60^2 + 173.2^2 \qquad \text{i.e.} \quad v_r = 183\,\text{m s}^{-1}$$

Also,

$$\tan\theta = 60/173.2 \qquad \text{i.e.} \quad \theta = 19.1^\circ$$

2.5 PARABOLIC MOTION

A body projected with a velocity v at an angle α above the horizontal has a vertical component of velocity of $v \sin\alpha$. Its vertical displacement, y, after time t is given by equation [2.8] as

$$y = (v\sin\alpha)t - \tfrac{1}{2}gt^2 \qquad [2.10]$$

At the same time, its horizontal displacement, x, due to its <u>constant</u> horizontal component of velocity of $v\cos\alpha$, is given by $s = vt$ as

$$x = (v\cos\alpha)t \qquad [2.11]$$

Eliminating t between equations [2.10] and [2.11] leads to

$$y = x\tan\alpha - g\frac{\sec^2\alpha}{2v^2}x^2$$

This is the equation of a parabola and it follows, therefore, that a body moving under the influence of gravity travels along a parabolic path. The path of a charged particle in a uniform electric field is also a parabola (see section 50.1).

Points to Bear in Mind when Attempting Questions 2D

To Find Time of Flight

Use $s = ut + \frac{1}{2}at^2$ for the vertical motion with $s = 0$.

To Find Time to Maximum Height

Use $v = u + at$ for the vertical motion with $v = 0$.

To Find Maximum Height

Use $v^2 = u^2 + 2as$ for the vertical motion with $v = 0$.

To Find Range

Find the time of flight, t, then substitute for t in $s = vt$ for the horizontal motion.

To Find Direction of Motion

Use $\tan\theta = v_y/v_x$ where θ is the angle the direction of motion makes with the horizontal, and v_y and v_x are the vertical and horizontal components of velocity respectively.

QUESTIONS 2D

1. A particle is projected with a speed of 25 m s^{-1} at 30° above the horizontal. Find: **(a)** the time taken to reach the highest point of the trajectory, **(b)** the magnitude and direction of the velocity after 2.0 s.

2. A particle is projected with a velocity of 30 m s^{-1} at an angle of 40° above a horizontal plane. Find: **(a)** the time for which the particle is in the air, **(b)** the horizontal distance it travels.

3. A pebble is thrown from the top of a cliff at a speed of 10 m s^{-1} and at 30° above the horizontal. It hits the sea below the cliff 6.0 s later. Find: **(a)** the height of the cliff, **(b)** the distance from the base of the cliff at which the pebble falls into the sea.

4. A pencil is accidentally knocked off the edge of a (horizontal) desk top. The height of the desk is 64.8 cm and the pencil hits the floor a horizontal distance of 32.4 cm from the edge of the desk. What was the speed of the pencil as it left the desk?

5. A particle is projected from level ground in such a way that its horizontal and vertical components of velocity are 20 m s^{-1} and 10 m s^{-1} respectively. Find: **(a)** the maximum height of the particle, **(b)** its horizontal distance from the point of projection when it returns to the ground, **(c)** the magnitude and direction of its velocity on landing.

6. An aeroplane moving horizontally at 150 m s^{-1} releases a bomb at a height of 500 m. The bomb hits the intended target. What was the horizontal distance of the aeroplane from the target when the bomb was released?

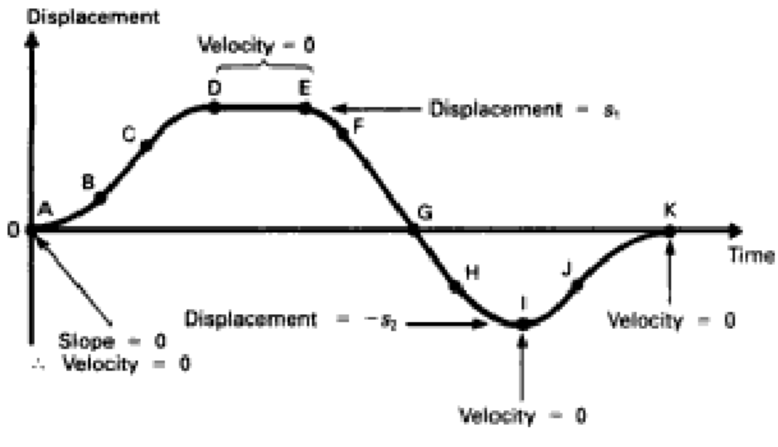
2.6 GRAPHICAL REPRESENTATION OF MOTION IN A STRAIGHT LINE

Graphs can be used to represent the motion of a body which is moving in a straight line. (The motion must be in a straight line because there is no means of representing more than two directions, e.g. forwards and backwards, on a graph.) The method is particularly useful when the body under consideration has a non-uniform acceleration, for the equations of motion (section 2.3) do not apply in such cases and even calculus methods are of no use if the acceleration varies with time in such a way that it cannot be expressed mathematically.

Displacement–Time Graphs

By definition, velocity is rate of change of displacement and therefore **the slope of a graph of displacement against time represents velocity**. Suppose that the displacement–time graph shown in Fig. 2.7 refers to the motion of a shunting engine. Bearing in mind that the slope of the graph represents velocity, we can make the following analysis of the motion of the engine:

Fig. 2.7
A displacement–time graph



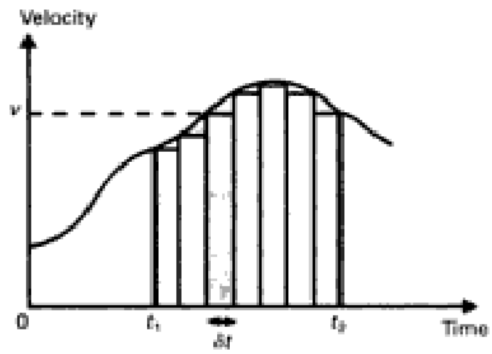
At A Stationary
A–B Accelerating (slope increasing)
B–C Moving with constant velocity (slope constant)
C–D Decelerating (slope decreasing)
D–E Stationary
E–F Accelerating and moving back towards the starting point
F–G Moving with constant velocity
At G Momentarily at the starting point
G–H Moving away from the starting point with constant velocity in the opposite direction to the original direction
H–I Decelerating
At I Momentarily stationary
I–J Accelerating and moving back towards the starting point
J–K Decelerating
At K Stationary at the starting point.

Note At the end of the period under consideration the engine is back at its starting point and therefore has zero displacement; the distance it has travelled, however, is $2s_1 + 2s_2$.

Velocity–Time Graphs

By definition, acceleration is rate of change of velocity and therefore **the slope of a graph of velocity against time represents acceleration. The area under such a graph represents distance.** We shall illustrate this by referring to the velocity–time graph in Fig. 2.8.

Fig. 2.8
To show that the area under a velocity–time graph represents distance



For a body which is moving with constant velocity, distance moved = velocity × time. It follows that if the velocity had the constant value of v during the time interval δt, the distance moved would be $v\,\delta t$. This is the area of the shaded strip, and therefore if the velocity varied with time according to the stepped line, the total distance moved in the interval from t_1 to t_2 would be the sum of the areas of the strips. By considering narrower and narrower strips we can make the stepped line follow the actual curve more and more closely. In the limit of infinitesimally narrow strips the sum of the areas of the strips is exactly equal to the area under the curve between t_1 and t_2, i.e. **the area under the curve between t_1 and t_2 represents the distance moved in the interval from t_1 to t_2.**

Suppose that a body moves in the manner represented by the velocity–time graph in 2.9. Bearing in mind that the slope of the graph represents acceleration, we can make the following analysis of the motion of the body:

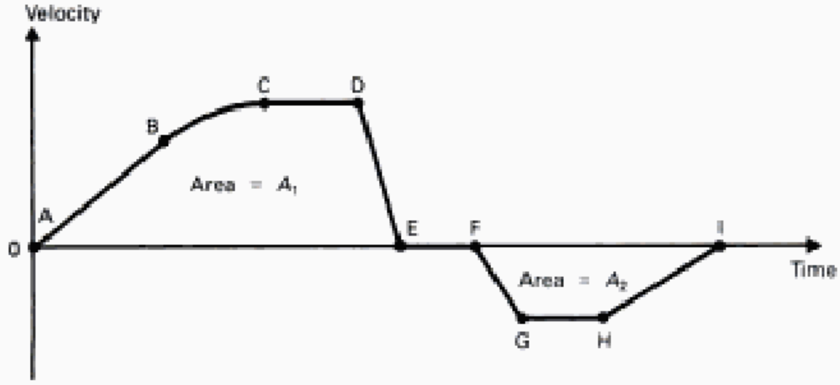


Fig. 2.9
A velocity–time graph

A–B	Moves from rest with a constant acceleration
B–C	Velocity still increasing, acceleration decreasing
C–D	Moving with constant velocity
D–E	Decelerating at a constant rate. Comes to rest
E–F	Stationary
F–G	Moving in the opposite direction to the original direction. Acceleration constant
G–H	Constant velocity
H–I	Decelerating at a constant rate. Comes to rest.

Note Total distance moved $= A_1 + A_2$. Net distance moved (i.e. magnitude of displacement $= A_1 - A_2$.

2.7 THE CONSERVATION OF LINEAR MOMENTUM

Suppose that two bodies, A and B, are involved in a collision (Fig. 2.10) and that there are no external forces acting. The force on A due to B, F_A, is, by Newton's third law, equal (in magnitude) to the force on B due to A, F_B. Therefore, by Newton's second law, each body experiences the same rate of change of momentum. Each force obviously acts for the same length of time as the other (i.e. for the duration of the collision), and therefore since the only forces that are acting are the internal forces F_A and F_B, the magnitudes of the changes of

Fig. 2.10
Collision of two bodies

momentum of the two bodies will be the same. The changes in momentum, however, are oppositely directed and therefore the total change in momentum is zero. The result can be extended to any number of bodies in any situation where the bodies interact only with themselves, i.e. where there are no external forces. It is known as the **principle of conservation of linear momentum** and can be stated as:

> The total linear momentum of a system of interacting (e.g. colliding) bodies, on which no external forces are acting, remains constant.

The experimental investigation of the conservation of linear momentum is dealt with in section 2.14.

EXAMPLE 2.10

A body, A, of mass 4 kg moves with a velocity of 2 m s^{-1} and collides head-on with another body, B, of mass 3 kg moving in the opposite direction at 5 m s^{-1}. After the collision the bodies move off together with velocity v. Calculate v.

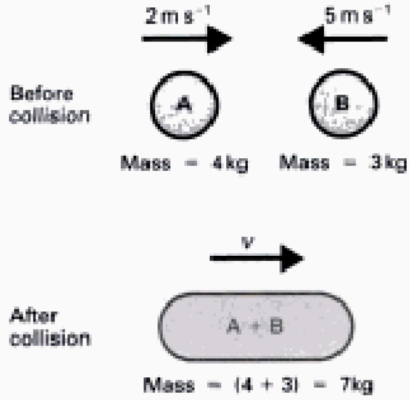


Fig. 2.11 Diagram for Example 2.10

Solution

Referring to Fig. 2.11 and taking momentum directed to the right to be positive, we find that

$$\text{Momentum of A before the collision} = 4 \times 2 = 8 \text{ kg m s}^{-1}$$

$$\text{Momentum of B before the collision} = 3 \times (-5) = -15 \text{ kg m s}^{-1}$$

$$\therefore \quad \text{The total momentum before the collision} = -7 \text{ kg m s}^{-1}$$

$$\text{Momentum of (A + B) after the collision} = 7v$$

By the principle of conservation of momentum

$$-7 = 7v$$

i.e. $v = -1 \text{ m s}^{-1}$

The minus sign indicates that the bodies move to the left (i.e. in the original direction of B) after the collision.

EXAMPLE 2.11

A bullet of mass 6.0×10^{-3} kg is fired from a gun of mass 0.50 kg. If the muzzle velocity of the bullet is 300 m s^{-1}, calculate the recoil velocity of the gun.

Solution

Initially, both the bullet and the gun are at rest and their total momentum is zero. After firing, the momentum of the bullet $= 6.0 \times 10^{-3} \times 300 = 1.8$ kg m s^{-1}. By the principle of conservation of linear momentum, the total momentum after firing is equal to that before firing, and therefore the gun must have a momentum of

$1.8\,\text{kg m s}^{-1}$ in the opposite direction to that of the bullet. If the recoil velocity of the gun is v, then

$$0.50\,v = 1.8$$

i.e. $$v = 3.6\,\text{m s}^{-1}$$

QUESTIONS 2E

1. A body of mass 6 kg moving at $8\,\text{m s}^{-1}$ collides with a stationary body of mass 10 kg and sticks to it. Find the speed of the composite body immediately after the impact.

2. A bullet of mass m is fired horizontally from a gun of mass M. Find the recoil velocity of the gun if the velocity of the bullet is v.

3. A flat truck of mass 400 kg is moving freely along a horizontal track at $3.0\,\text{m s}^{-1}$. A man moving at right angles to the track jumps on to the truck causing its speed to decrease by $0.50\,\text{m s}^{-1}$. What is the mass of the man?

4. A kitten of mass 0.60 kg leaps at 30° to the horizontal out of a toy truck of mass 1.2 kg causing it to move over horizontal ground at $4.0\,\text{m s}^{-1}$. At what speed did the kitten leap?

5. A particle of mass $5m$ moving with speed v explodes and splits into two pieces with masses of $2m$ and $3m$. The lighter piece continues to move in the original direction with speed $5v$ relative to the heavier piece. What is the actual speed of the lighter piece?

2.8 ELASTIC COLLISIONS

Whenever two bodies collide, their total momentum is conserved unless there are external forces acting on them. The total kinetic energy (see section 5.3), however, usually decreases, since the impact converts some of it to heat and/or sound and/or permanently distorts the bodies leaving them with an increased amount of potential energy.

A collision in which some kinetic energy is lost is known as an **inelastic collision**. A **completely inelastic** collision is one in which the bodies stick together on impact. A collision is **elastic** if there is no loss of kinetic energy.

2.9 NEWTON'S EXPERIMENTAL LAW OF IMPACT

The relative velocity with which two bodies separate from each other, after a collision, is related to their relative velocity of approach and a constant known as the **coefficient of restitution**, e, of the two bodies. The relationship is known as Newton's experimental law of impact and can be expressed as

$$\textbf{Speed of separation} = e \times \textbf{Speed of approach} \qquad [2.12]$$

The coefficient of restitution of the two bodies is defined by equation [2.12] and depends on their elastic properties and the natures of their surfaces. These same properties determine whether a collision is elastic, inelastic or completely inelastic and therefore it is possible to classify a collision according to the value of e that is associated with it (Table 2.1).

Table 2.1
Classification of collisions

Type of collision	e
Elastic	1
Inelastic	<1
Completely inelastic	0

EXAMPLE 2.12

A body, A, of mass 6 kg and moving at 9 m s^{-1} collides head-on with another body, B, of mass 3 kg and moving in the same direction as A at 4 m s^{-1}. If the velocities of A and B after the collision are respectively v_A and v_B and the coefficient of restitution of the bodies is 0.8, calculate v_A and v_B. Assume that no external forces act on the system.

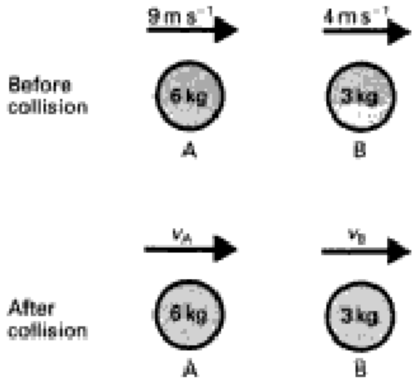


Fig. 2.12
Diagram for Example 2.12

Solution

Refer to Fig. 2.12. There are no external forces acting on the system, in which case momentum is conserved and we may put

$$(6 \times 9) + (3 \times 4) = 6v_A + 3v_B$$

i.e. $$22 = 2v_A + v_B \qquad [2.13]$$

Using Newton's experimental law of impact (equation [2.12]) we have

$$v_B - v_A = 0.8\,(9 - 4)$$

i.e. $$v_B - v_A = 4 \qquad [2.14]$$

Solving equations [2.13] and [2.14] simultaneously gives

$$v_A = 6\,\mathrm{m\,s^{-1}} \text{ and } v_B = 10\,\mathrm{m\,s^{-1}}$$

QUESTIONS 2F

1. A sphere, A, of mass 3.0 kg moving at 8.0 m s^{-1} collides directly with another sphere, B, of mass 5.0 kg moving in the opposite direction to A at 4.0 m s^{-1}. Find the velocities of the spheres immediately after the impact if $e = 0.30$.

2. A sphere of mass m moving with velocity u is involved in an elastic collision with a sphere of mass $2m$ moving along the same line with velocity $-u$. Find the velocities of the spheres immediately after the impact.

3. A ball is dropped onto horizontal ground from a height of 9.0 m. Find the height to which the ball rises: **(a)** on the first bounce, **(b)** on the second bounce. ($e = 0.70$.)

2.10 IMPULSE

The impulse of a constant force, F, acting for a time, Δt, is defined by

$$\textbf{Impulse} = F\,\Delta t \qquad [2.15]$$

Impulse is a vector quantity; its direction is the same as that of the force. It follows from equation [2.15] that the unit of impulse is the newton second (N s). Note that $1\ \text{N s} = 1\ \text{kg m s}^{-1}$.

Suppose that a force, F, causes the momentum of a body to change by $\Delta(mv)$ in a time Δt. By Newton's second law, force = rate of change of momentum, and therefore

$$F = \frac{\Delta(mv)}{\Delta t}$$

i.e. $$F\,\Delta t = \Delta(mv)$$

Therefore by equation [2.15]

$$\textbf{Impulse = Change in momentum} \qquad [2.16]$$

It can be shown that equation [2.16] applies to variable forces too.

The definition of impulse imposes no limit on the length of time for which the force may act. Nevertheless, the concept of impulse is normally used only in situations where a large variable force is acting for only a short time, for example a golf-club striking a ball or the blow of a hammer on a nail. Forces such as these are known as impulsive forces.

When a batsman strikes a cricket ball he 'follows through' in order to keep the bat in contact with the ball for as long a time as possible. It follows from equation [2.15] that this increases the impulse and therefore, by equation [2.16], produces a larger change in momentum and so increases the speed at which the ball leaves the bat.

Suppose now that the ball is caught by a fielder. In catching it the fielder has to reduce the momentum of the ball to zero. It follows from equation [2.16] that the impulse on his hand will be the same no matter how he catches the ball. However, by equation [2.15], he can reduce the force he feels by drawing his hands backwards to increase the time taken to effect the catch. Not only is this less painful, but it also reduces the likelihood of the ball bouncing out of his hands.

The impulse of a variable force, F, acting for a time, t, is defined by

$$\text{Impulse} = \int_0^t F\,dt \qquad [2.17]$$

Measuring the change in momentum that a variable force produces is usually much easier than measuring the way in which it varies with time. In practice, therefore, an impulse is more likely to be evaluated on the basis of equation [2.16] than equation [2.17].

2.11 FORCE–TIME GRAPHS

It follows from equation [2.17] that **the area under a graph of force against time represents impulse** (Fig. 2.13). It follows from equation [2.16] that **it also represents change in momentum**.

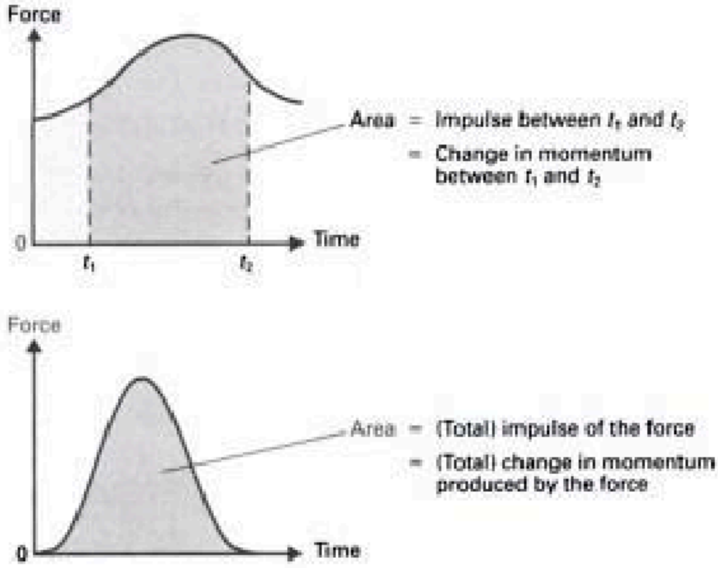


Fig. 2.13 Force–time graphs

EXAMPLE 2.13

A body of mass 4 kg is moving at $5\ \text{m s}^{-1}$ when it is given an impulse of 8 N s in the direction of its motion. (a) What is the velocity of the body immediately after the impulse? (b) If the impulse acts for 0.02 s, what is the average value of the force exerted on the body?

Solution

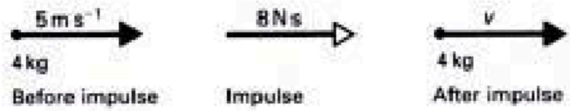


Fig. 2.14 Diagram for Example 2.13

(a) Refer to Fig. 2.14. Let v = velocity of body immediately after the impulse.

$$\text{Impulse} = \text{Change in momentum}$$

$$\therefore \quad 8 = 4v - 4 \times 5$$

$$\therefore \quad 8 = 4v - 20 \qquad \text{i.e.} \quad v = 7\ \text{m s}^{-1}$$

(b) Let F = average force

$$\text{Impulse} = F\,\Delta t$$

$$\therefore \quad 8 = F \times 0.02 \qquad \text{i.e.} \quad F = 4 \times 10^2\ \text{N}$$

QUESTIONS 2G

1. A particle of mass 6.0 kg moving at $8.0\,m\,s^{-1}$ due N is subjected to an impulse of 30 N s. Find the magnitude and direction of the velocity of the particle immediately afterwards if the direction of the impulse is: **(a)** due N, **(b)** due S.

2. A ball of mass 6.0×10^{-2} kg moving at $15\,m\,s^{-1}$ hits a wall at right angles and bounces off along the same line at $10\,m\,s^{-1}$. **(a)** What is the magnitude of the impulse of the wall on the ball? **(b)** The ball is estimated to be in contact with the wall for 3.0×10^{-2} s, what is the average force on the ball?

3. A body of mass 2.0 kg and which is at rest is subjected to a force of 200 N for 0.20 s followed by a force of 400 N for 0.30 s acting in the same direction. Find: **(a)** the total impulse on the body, **(b)** the final speed of the body.

4. Find the final speed of the body in question 3 by using $F = ma$ and $v = u + at$.

2.12 FRICTION

Static Friction

When the surface of a body moves or tends to move over that of another, each body experiences a frictional force. The frictional forces act along the common surface, and each is in such a direction as to oppose the relative motion of the surfaces.

Fig. 2.15 illustrates an arrangement which can be used to investigate frictional forces. Small masses are added, one at a time, to the scale-pan in order to increase P. At first P is small and the block does not move, but as more masses are added, eventually a point is reached at which the block starts to slide. This is interpreted by supposing that for small values of P the frictional force F is equal to P but that there is a maximum frictional force which can be brought into play. This is called the **limiting frictional force** and its value is equal to the value of P at which the block starts to move. The way in which the frictional force depends on the normal reaction R can be investigated by placing weights on the block. The effect of the area of contact can be studied by repeating the experiment with different faces of the block in contact with the table.

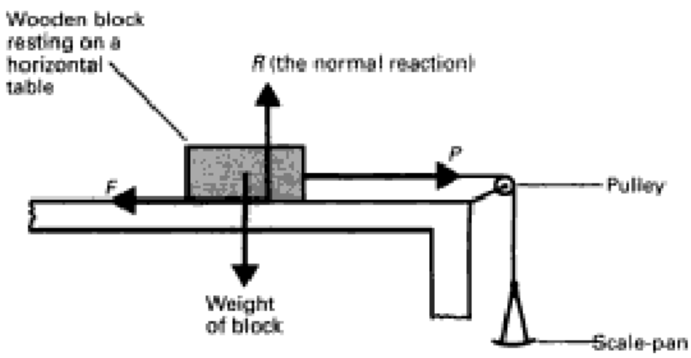


Fig. 2.15
Investigation of frictional forces

Sliding Friction

The frictional force which exists between two adjacent surfaces which are in relative motion is usually slightly less than the limiting frictional force between the surfaces and is called the **sliding** (or **dynamic** or **kinetic**) **frictional force**. This can be demonstrated by using the apparatus of Fig. 2.15 and giving the block a slight push each time a mass is added to the scale-pan. The value of P at which the block continues to move with constant velocity after being pushed is the value of the sliding frictional force and is less than the force required to produce motion when the block is not pushed.

The Laws of Friction

The results of experiments of the type described in Static Friction and Sliding Friction above are summarized in the laws of friction.

(i) The frictional force between two surfaces opposes their relative motion or attempted motion.

(ii) Frictional forces are independent of the area of contact of the surfaces.

(iii) For two surfaces which have no relative motion the limiting frictional force is directly proportional to the normal reaction.

For two surfaces which have relative motion the sliding frictional force is directly proportional to the normal reaction and is approximately independent of the relative velocity of the surfaces.

The Coefficients of Friction

The coefficient of limiting friction μ and the coefficient of sliding friction μ' are defined by

$$\mu = \frac{F}{R} \quad \text{and} \quad \mu' = \frac{F'}{R}$$

where F and F' are the limiting and sliding frictional forces respectively and R is the normal reaction. Both μ and μ' depend on the nature and the condition of the surfaces which are in contact but are independent of the area of contact. For steel on steel $\mu \approx 0.8$; for Teflon on Teflon $\mu \approx 0.04$. (The values given are approximate because even a mono-molecular layer of some surface impurity affects the experimental results.) If two surfaces are assumed to be perfectly smooth, there is no frictional force and $\mu = \mu' = 0$.

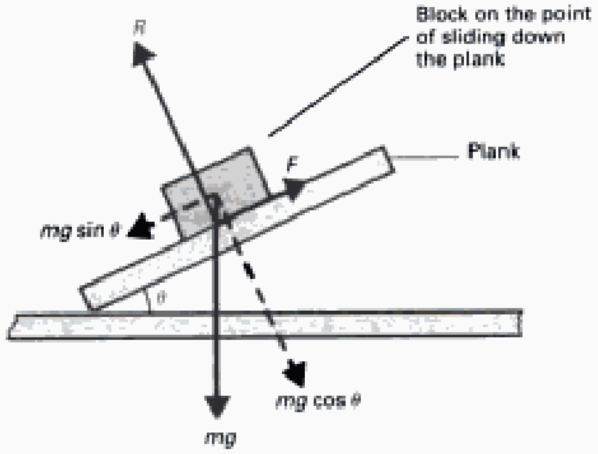


Fig. 2.16 Determination of the coefficient of limiting friction

The coefficient of limiting friction can be determined by carrying out an experiment of the type described in Static Friction above and measuring R and the minimum value of P that produces motion. The arrangement shown in Fig. 2.16 provides an alternative method. One end of the plank is raised gradually and the value of θ (the **angle of friction**) at which the block is on the point of slipping is measured. When the block is about to slip $\mu = F/R$, and therefore since

$$mg \sin \theta = F \quad \text{and} \quad mg \cos \theta = R$$

$$\mu = \frac{mg \sin \theta}{mg \cos \theta}$$

i.e. $\mu = \tan \theta$

An Explanation of the Laws of Friction

On a microscopic level, even a highly polished surface has bumps and hollows. It follows that when two surfaces are put together the actual area of contact is less than the apparent area of contact (Fig. 2.17).

Fig. 2.17
Magnified cross-section through two surfaces in contact

For example, it has been estimated that for steel on steel, the actual contact area can be as little as one ten-thousandth of the apparent area. The pressures at the contact points are very high, and it is thought that the molecules are pushed into such close proximity that the attractive forces between them weld the surfaces together at these points. These welds have to be broken before one surface can move over the other. Clearly, therefore, no matter in which direction the motion occurs there is a force which opposes it. This explains law (i).

If the apparent area of contact of a body is decreased by turning the body so that it rests on one of its smaller faces, the number of contact points is reduced. Since the weight of the body has not altered, there is increased pressure at the contact points and this flattens the bumps so that the total contact area and the pressure return to their original values. Thus, although the apparent area of contact has been changed, the actual area of contact has not. This explains law (ii).

The extent to which the bumps are flattened depends on the weight of the body. Therefore the greater the weight, the greater the actual area of contact. This explains law (iii), because the weight is equal to the normal reaction.

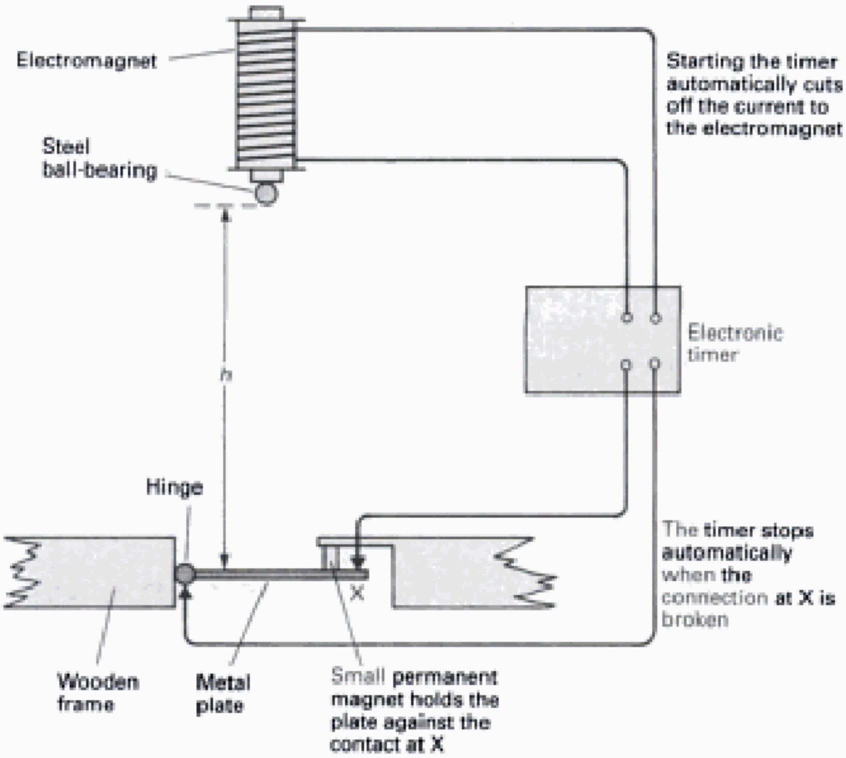
2.13 DETERMINATION OF THE ACCELERATION DUE TO GRAVITY (g) BY FREE FALL

The apparatus is shown in Fig. 2.18. The principle of the method is to measure the time, t, for a ball-bearing to fall from rest through a measured distance, h.

The circuitry is such that switching on the electronic timer automatically cuts off the current to the electromagnet and releases the ball-bearing. The bearing falls freely until it strikes the hinged metal plate. The impact causes the plate to swing downwards, breaking the electrical connection at X and stopping the timer. The timer therefore automatically registers the time of fall.

Once h has been measured (with an extending rule, say) the acceleration due to gravity, g, can be calculated. It follows from $s = ut + \frac{1}{2}at^2$ (equation [2.8]) with $s = h$, $u = 0$, $a = g$ and $t = t$, that $g = 2h/t^2$, hence g.

Fig. 2.18
Apparatus to determine ***g*** by free fall



Notes

(i) The timer should be capable of registering t with an uncertainty of ± 0.01 s or less.

(ii) h is measured from the bottom of the ball-bearing.

(iii) There may be a delay in releasing the ball-bearing due to residual magnetism in the electromagnet. The likelihood of this can be reduced by arranging that the bearing is held only weakly by the electromagnet to start with. This can be done by reducing the magnetizing current to the minimum that will hold the bearing, or by placing a piece of paper or thin card between the bearing and the electromagnet.

(iv) The experiment should be repeated a number of times and the average value of g found. Alternatively, the times of fall may be measured for a number of different values of h. Since $g = 2h/t^2$, $\sqrt{h} = (\sqrt{g/2})t$ and therefore the gradient of a graph of $\sqrt{h}$ against t is $\sqrt{g/2}$, allowing g to be found graphically. This has the advantage that the effect of any constant error in t (e.g. that due to the bearing not being released immediately the timer is started) is eliminated. (If there is an error of this type, the graph will not pass through the origin but the gradient will be unaffected.)

2.14 EXPERIMENTAL INVESTIGATION OF THE PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The principle of conservation of linear momentum can be investigated by means of two plastic vehicles riding on the cushion of air above a linear air-track (Fig. 2.19). The track is a hollow tube of triangular cross-section through which air is blown; the air emerges through holes in each side of the track. It has adjustable feet

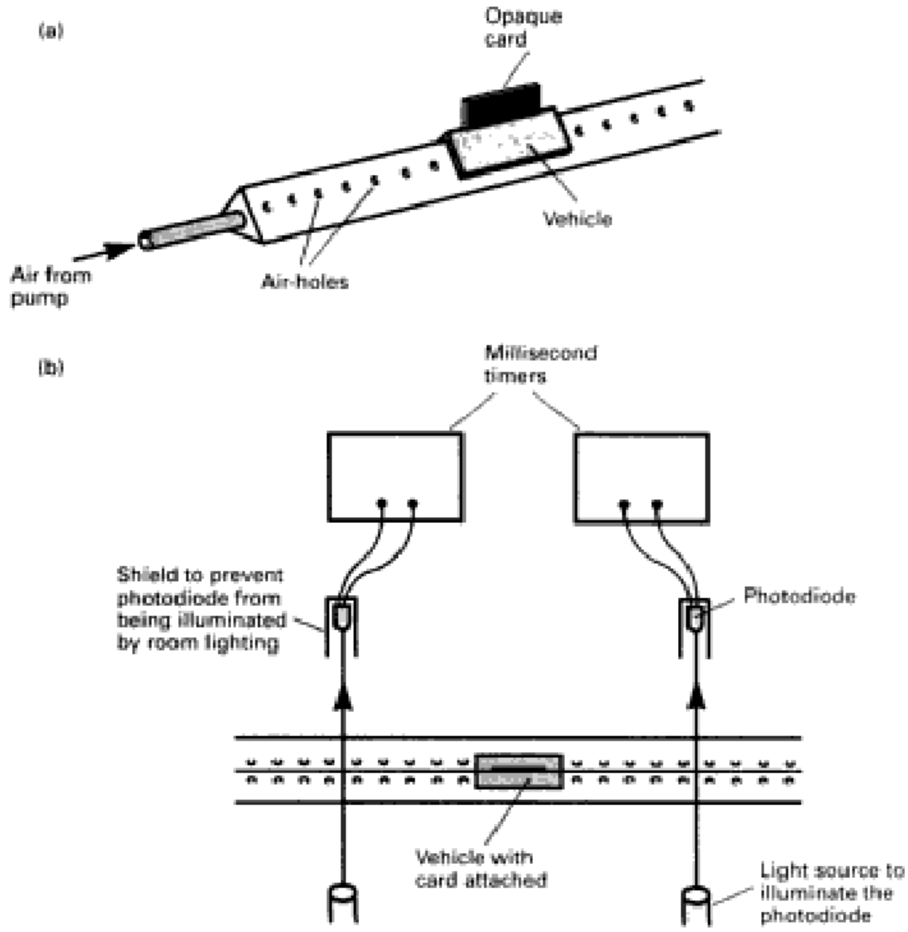


Fig. 2.19
(a) Vehicle on an air-track, (b) timing arrangement

allowing it to be made accurately horizontal so that the vehicles have no tendency to drift along it in either direction. A number of small (e.g. 50 g) masses may be attached to the vehicles. Each vehicle can carry an opaque card of known length (e.g. 10 cm) which is arranged to interrupt a beam of light falling on a photodiode. The circuitry is such that each of the millisecond timers is inoperative whilst light is falling on the photodiode to which it is connected. When a light beam is broken by the leading edge of a card the associated timer switches on and remains operative for as long as the card is in the beam. The timer therefore records the time for the vehicle to travel a distance equal to the length of the card and so allows the speed to be found.

Completely Inelastic Collision

Refer to Fig. 2.20. A is pushed towards B, which is stationary and has no card attached. A interrupts beam X and therefore its speed (u_A) before impact can be found. A pin on the front of A sticks in a small piece of plasticine on the back of B, and the vehicles then move together. The card on A interrupts beam Y allowing the (common) speed (v_{AB}) of A and B to be found.

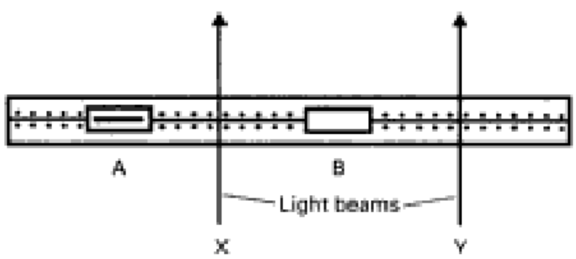


Fig. 2.20
Initial arrangement for inelastic collision

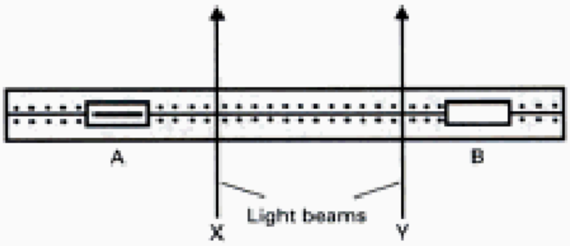


Fig. 2.21
Initial arrangement for elastic collision

Suppose the masses of A and B are m_A and m_B respectively. Momentum is conserved if $m_A u_A = (m_A + m_B) v_{AB}$. The experiment should be repeated for a number of different values of m_A, m_B and u_A.

Elastic Collision

Refer to Fig. 2.21. A and B (each carrying a card) are pushed towards each other so that they collide in the region between the beams. Since A will have passed through beam X and B will have passed through beam Y, their speeds before the collision can be found. Each vehicle has a stretched rubber band attached to its front end, and these act as buffers so that the collision is almost (perfectly) elastic (see section 2.8). It can be arranged that each vehicle reverses its direction of motion on impact. Since A then passes back through beam X and B passes through beam Y, their speeds after the collision can be found. The experiment requires two people – one to observe each timer.

Suppose the masses of A and B are m_A and m_B respectively, and their speeds are u_A and u_B before collision, and v_A and v_B after collision. Taking left to right as positive, the initial momentum is $m_A u_A - m_B u_B$ and the momentum after impact is $m_B v_B - m_A v_A$. Momentum is conserved if, within experimental error, $m_A u_A - m_B u_B = m_B v_B - m_A v_A$. The experiment should be repeated for a number of different values of m_A, m_B, u_A and u_B.

2.15 EXPERIMENTAL INVESTIGATION OF $F = ma$

Newton's second law in the form $F = ma$ can be investigated using the apparatus shown in Fig. 2.22. To compensate for friction, the slope of the runway is adjusted so that the trolley, when given a slight push, runs down it at constant speed (dots equally spaced on ticker-tape). The accelerating force is provided by means of an elastic thread attached to the rear of the trolley. The experimenter pulls on the

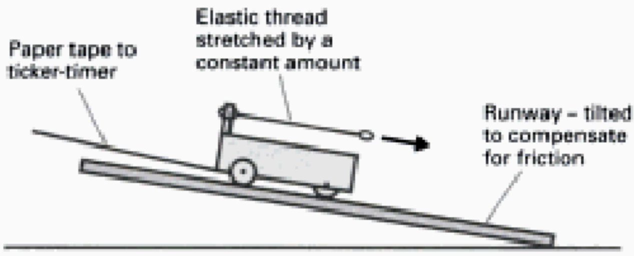


Fig. 2.22
Apparatus for investigating $F = ma$

thread and walks along keeping the length of the thread constant (equal to the length of the trolley, say). The effects of friction have been compensated for by tilting the track and therefore the net force on the trolley is that provided by the stretched thread. Since the thread is stretched by a constant amount, the trolley is being accelerated by a constant force. The acceleration of the trolley is found by analysing the spacings of the dots on the ticker-tape. (The dots are produced at intervals of $\frac{1}{50}$ s, from which the velocity, and hence the acceleration, can be calculated.)

The effect of doubling (or tripling) the accelerating force is investigated by using two (or three) identical threads in parallel with each other and stretched by the same amount as in the first experiment. The effect of doubling (or tripling) the mass is investigated by stacking two (or three) identical trolleys on top of each other.

The accelerating force is proportional to the number of threads and the mass is proportional to the number of trolleys. A graph of acceleration against (number of threads/number of trolleys) can therefore be expected to be a straight line through the origin (i.e. $a \propto F/m$).

Note The wheels of the trolleys are made from a low-density material so that very little of the accelerating force is 'wasted' in providing the angular acceleration of the wheels.

CONSOLIDATION

Newton's first law Every body continues in a state of rest or of uniform (unaccelerated) motion in a straight line unless acted on by some external force.

Newton's second law The rate of change of momentum of a body is directly proportional to the external force acting on the body and takes place in the direction of the force.

$$F = \frac{d}{dt}(mv)$$

becomes

$$F = ma \qquad \text{for constant mass}$$

The newton (N) is defined as that force which produces an acceleration of 1 m s^{-2} when it acts on a mass of 1 kg.

Newton's third law If A exerts a force on B, then B exerts an equal and oppositely directed force on A

$$s = vt \qquad \text{for constant velocity}$$

$$\left.\begin{aligned} v &= u + at \\ v^2 &= u^2 + 2as \\ s &= ut + \tfrac{1}{2}at^2 \\ s &= \tfrac{1}{2}(u+v)t \end{aligned}\right\} \quad \text{for constant acceleration}$$

Displacement–Time Graphs

Gradient = velocity

Velocity–Time Graphs

Gradient = acceleration

Area under graph = distance

The principle of conservation of linear momentum The total linear momentum of a system of interacting (e.g. colliding) bodies, on which no external forces are acting, remains constant.

An elastic collision is one in which there is no loss of kinetic energy.

Law of Impact

Speed of separation = e × Speed of approach

Impulse

Impulse of constant force = $F\,\Delta t$

Impulse of variable force = $\int_0^t F\,\mathrm{d}t$

Impulse = Change in momentum (for both constant and variable forces)

3

TORQUE

3.1 DEFINITION OF TORQUE

Consider a force acting on a rigid body (Fig. 3.1) so as to cause it to turn about an axis which is perpendicular to the paper and passes through O. The effect of the force is determined by its turning moment, a quantity which depends not only on the size and direction of the force but also on where it acts. **The turning moment (or torque)** is defined by

$$T = Fd \tag{3.1}$$

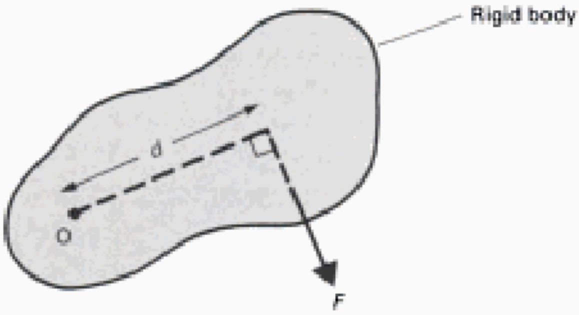
where

T = torque (or turning moment) (N m)

F = the magnitude of the force (N)

d = the perpendicular distance of the line of action of the force from the axis (m).

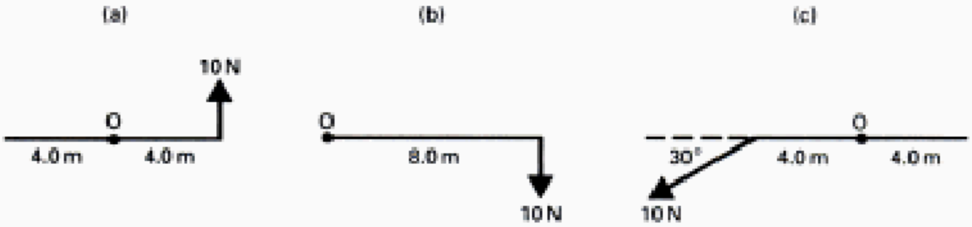
Fig. 3.1
Definition of torque



EXAMPLE 3.1

Find the moment of the 10 N force about the axis through O and perpendicular to the paper in each of the three situations shown in Fig. 3.2.

Fig. 3.2
Diagram for Example 3.1



Solution

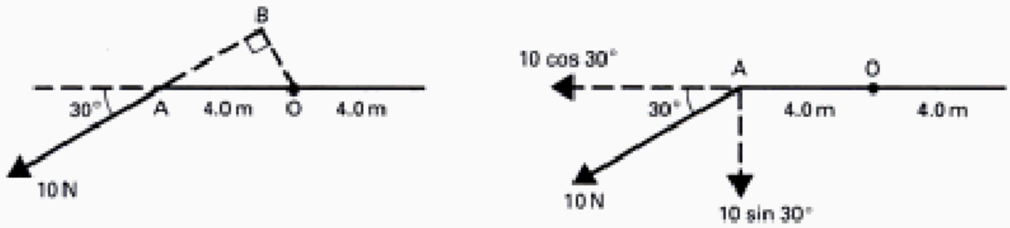
(a) Moment about O = 10 × 4.0 = 40 N m (anti-clockwise)

(b) Moment about O = 10 × 8.0 = 80 N m (clockwise)

(c) Refer to Fig. 3.3(a). Perpendicular distance of line of action of 10 N force from O = OB = OA sin 30° = 2.0 m

∴ Moment about O = 10 × 2.0 = 20 N m (anti-clockwise)

Fig. 3.3
Diagram for solution of Example 3.1(c)



Alternative Method

The 10 N force has components of 10 sin 30° and 10 cos 30° perpendicular and parallel to AO respectively (Fig. 3.3(b)).

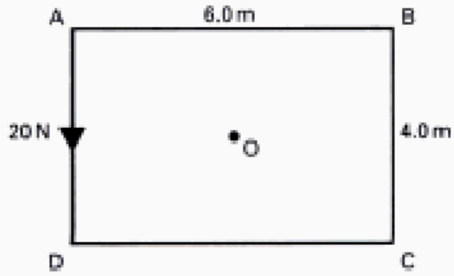
Moment about O of perpendicular component = 10 sin 30° × 4.0
= 20 N m
(anti-clockwise)

Moment about O of parallel component = 0

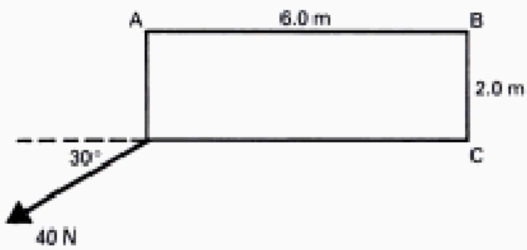
∴ Total moment about O = 20 N m (anti-clockwise)

QUESTIONS 3A

1. Find the moment of the 20 N force about axes perpendicular to the paper and through: **(a)** A, **(b)** B, **(c)** C, **(d)** D, **(e)** O where O is the centre of the rectangle



2. By resolving the 40 N force into two suitable components, or otherwise, find its moment about an axis perpendicular to the paper and through: **(a)** A, **(b)** B, **(c)** C.



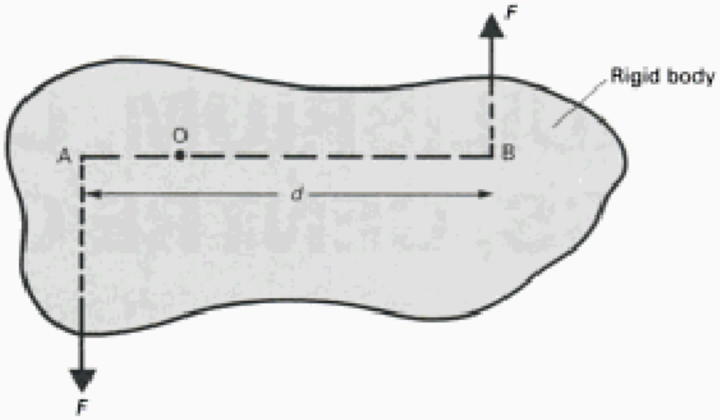
3.2 COUPLES

Two forces which are equal in magnitude and which are anti-parallel constitute a couple (Fig. 3.4).

Notes (i) There is no direction in which a couple can give rise to a resultant force, and therefore **a couple can produce a turning effect only** – it cannot produce translational motion.

(ii) Since a single force is bound to produce translation, it follows that **a couple cannot be represented by a single force**.

Fig. 3.4
Definition of a couple



3.3 TORQUE DUE TO A COUPLE

In Fig. 3.4,

$$\begin{aligned}\text{Total torque about O} &= F \times \text{OA} + F \times \text{OB} \\ &= F(\text{OA} + \text{OB}) \\ &= Fd\end{aligned}$$

Thus, the torque about O does not depend on the position of O and therefore it follows that:

The torque due to a couple is the same about any axis and is given by

$$\text{Torque due to a couple} = \text{One force} \times \text{Separation of forces} \quad [3.2]$$

4

EQUILIBRIUM, CENTRE OF MASS, CENTRE OF GRAVITY

4.1 THE CONDITIONS FOR EQUILIBRIUM

A body is in equilibrium if:

(a) the acceleration of its centre of mass is zero in all directions, and

(b) its angular acceleration is zero.

Neither of these conditions requires that the body is at rest – a body may move with constant velocity and rotate with constant angular velocity and still be in equilibrium!

It follows from (a) and (b) that

A body is in equilibrium if:

(i) the resultant force on its centre of mass is zero, **and**

(ii) the total torque about all axes is zero.

Statements (i) and (ii) are often referred to as **the conditions for equilibrium** and are more useful in problem solving than (a) or (b). It can be shown that for a body subject to coplanar forces only, condition (i) will have been fulfilled if the resultant force in any two directions in the plane of the forces is zero. Condition (ii) will have been fulfilled if the total torque about any one axis which is perpendicular to the plane of the forces is zero. Therefore

To prove that a system of coplanar forces is in equilibrium it is sufficient to show that:

1 the resultant force in any two directions in the plane of the forces is zero, **and**

2 the total torque about any one axis which is perpendicular to the plane of the forces is zero.

Neither one of these conditions is sufficient on its own to show that a body is in equilibrium. On the other hand, if a body is known to be in equilibrium, then we may make use of 1 or 2 or both 1 and 2. It also follows that

If a body is in equilibrium:

I the resultant force is zero in all directions, and

II the total torque is zero about any axis.

Notes (i) Conditions 1 and 2 are known as sufficient conditions because together they form the minimum set of conditions which is sufficient to ensure equilibrium under the action of coplanar forces*. Conditions I and II are known as necessary conditions, in the sense that each is necessarily true, rather than that it is necessary to show them to be true.

(ii) Statement II is sometimes called **the principle of moments**, and can also be expressed as

If a body is in equilibrium, the total clockwise moment about any axis is equal to the total anti-clockwise moment about the same axis.

(iii) Statement I and statement II (in both its forms) also apply when the equilibrium is due to non-coplanar forces.

(iv) To prove that a body acted on by non-coplanar forces is in equilibrium it is sufficient to show that:

the resultant force in any three mutually perpendicular directions is zero, and

the total torque about each of any three mutually perpendicular axes is zero.

(v) When solving problems in which a system of coplanar forces is known to be in equilibrium we may choose two directions and apply condition 1 in each direction, and we may choose one axis and apply condition 2. Thus we resolve twice and take moments once. This gives three independent equations and allows us to find the values of three unknowns. There are two alternatives – we may resolve once and take moments twice, or we may take moments about three axes which are not in line with each other. It is not possible to obtain more than three independent equations and therefore there is no point in, for example, resolving twice and taking moments twice.

Concurrent Forces

Concurrent forces are forces whose lines of action intersect at a single point. A little thought should convince the reader that it is impossible for such a system of forces to produce a torque about any axis if their resultant is zero. It follows that

Concurrent forces are in equilibrium if their resultant is zero.

To prove that concurrent coplanar forces are in equilibrium it is sufficient to show that 1 is true. If we know that a system of concurrent coplanar forces is in equilibrium, we use 1 alone when solving problems – there is no point using 2.

Notes (i) If a body is in equilibrium under the action of three non-parallel coplanar forces, the forces must be concurrent. (See section 4.2.)

*This is not the only set of minimum conditions, but it is the one most commonly used.

(ii) **A particle** is an object which has mass but which is small enough to be regarded as a point. It follows that a set of forces acting on a particle must be concurrent forces.

EXAMPLE 4.1

The system of forces in Fig. 4.1 is in equilibrium. Find P and Q.

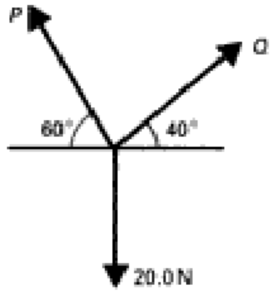


Fig. 4.1
Diagram for Example 4.1

Solution

We make use of condition 1 in the horizontal and vertical directions. Refer to Fig. 4.1

Resolving horizontally:

$$P \cos 60^\circ = Q \cos 40^\circ \qquad [4.1]$$

Resolving vertically:

$$P \sin 60^\circ + Q \sin 40^\circ = 20.0 \qquad [4.2]$$

By equation [4.1]

$$P = \frac{Q \cos 40^\circ}{\cos 60^\circ} \quad \text{i.e.} \quad P = 1.532\,Q \qquad [4.3]$$

Substituting for P in equation [4.2] gives

$$1.532\,Q \sin 60^\circ + Q \sin 40^\circ = 20.0$$

$$\therefore \quad 1.970\,Q = 20.0 \quad \text{i.e.} = 10.2\text{ N}$$

Substituting for Q in equation [4.3] gives

$$P = 15.6\text{ N}$$

Note We have resolved horizontally and vertically. It would have been quite reasonable to resolve perpendicular to P and perpendicular to Q. The advantage of this is that it gives an equation for Q which does not involve P and an equation for P which does not involve Q. The main disadvantage is that it is necessary to work out the angles that the forces make with these directions and although this is trivial, it leads to a rather messy diagram. It is by far the best method, though, when the unknown forces are at 90° to each other.

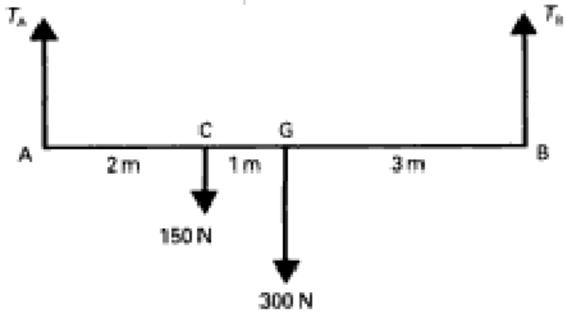
EXAMPLE 4.2

A uniform plank AB which is 6 m long and has a weight of 300 N is supported horizontally by two vertical ropes at A and B. A weight of 150 N rests on the plank at C where AC = 2 m. Find the tension in each rope.

Solution

The plank is uniform and therefore its weight acts at its mid-point, G, say. Let the tensions in the ropes at A and B be T_A and T_B respectively. Refer to Fig. 4.2.

Fig. 4.2
Diagram for Example 4.2



The plank is in equilibrium and therefore the clockwise moment about any point is equal to the anti-clockwise moment about the same point. (Condition 2.)

Taking moments about A gives

$$T_B \times 6 = 150 \times 2 + 300 \times 3$$

$$\therefore \quad 6T_B = 1200 \qquad \text{i.e.} \quad T_B = 200\ \text{N}$$

Resolving vertically gives

$$T_A + T_B = 150 + 300$$

$$\therefore \quad T_A + 200 = 450 \qquad \text{i.e.} \quad T_A = 250\ \text{N}$$

Notes

(i) As an alternative to resolving vertically, we could have taken moments about B to find T_A.

(ii) It is usually good policy to take moments about points where unknown forces are acting because this reduces the number of unknowns in each of the resulting equations.

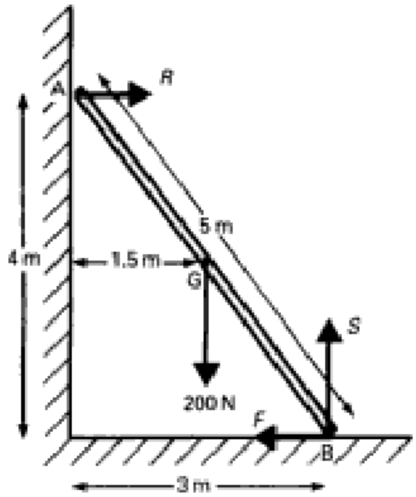
EXAMPLE 4.3

A uniform ladder which is 5 m long and has a mass of 20 kg leans with its upper end against a smooth vertical wall and its lower end on rough ground. The bottom of the ladder is 3 m from the wall. Calculate the frictional force between the ladder and the ground. ($g = 10\ \text{m s}^{-2}$.)

Solution

Refer to Fig. 4.3. The ladder is uniform and therefore its weight, $20 \times 10 = 200$ N, acts at its mid-point G, a distance of 1.5 m from the wall. The wall is smooth and therefore the only force acting at the top of the ladder is the normal reaction R. By Pythagoras the point A at which the ladder makes contact with the wall is 4 m above the ground. The forces acting at the bottom of the ladder are the normal reaction S and the frictional force F. If the ladder were to slip, its bottom end would move to the right; it follows that F acts to the left as shown.

Fig. 4.3
Diagram for Example 4.3



The ladder is in equilibrium and therefore there can be no resultant force in any direction. In particular there is no resultant vertical force, in which case

$$S = 200 \text{ N} \qquad [4.4]$$

Because the ladder is in equilibrium the total torque about any point is zero. In particular, the total (net) torque about A is zero and therefore

$$(F \times 4) + (200 \times 1.5) = S \times 3$$

i.e. $4F + 300 = 3S$

Therefore by equation [4.4]

$$4F + 300 = 600$$

i.e. $F = 75$ N

Note The reason that we have chosen to consider the torque about A, rather than some other point, is that this automatically excludes R – a force in which we have no interest. The reader is advised to convince himself that considering the torque about G and/or B and making use of the fact that $F = R$ also gives $F = 75$ N.

Points to Bear in Mind when Attempting Questions 4A

(a) Draw a clear diagram showing all the forces acting on the particle (or body) whose equilibrium is being considered.

(b) Draw diagrams in which the angles look something like the angles they represent. There is no need to use a protractor, but an angle of 30°, say, should look more like 30° than 45° or 60°.

(c) A smooth surface can exert a force only at right angles to itself – the **normal reaction**.

(d) The tension is the same in each section of a light string which passes over a smooth pulley or a smooth peg, or which passes through a smooth hole or a smooth ring.

(e) There is no point in resolving in more than two directions.

(f) It is often an advantage to resolve perpendicular to an unknown force.

(g) It is often an advantage to take moments about points where unknown forces are acting.

QUESTIONS 4A

1. Solve the problem in Example 4.1 by resolving perpendicular to P and/or Q.

2. Two forces, P and Q, act NW and NE respectively. They are in equilibrium with a force of 50.0 N acting due S and a force of 20.0 N acting due E. Find P and Q.

3. A particle whose weight is 50.0 N is suspended by a light string which is at 35° to the vertical under the action of a horizontal force F. Find: **(a)** the tension in the string, **(b)** F.

4. A particle of weight W rests on a smooth plane which is inclined at 40° to the horizontal. The particle is prevented from slipping by a force of 50.0 N acting parallel to the plane and up a line of greatest slope. Calculate: **(a)** W, **(b)** the reaction due to the plane.

5. Two light strings are perpendicular to each other and support a particle of weight 100 N. The tension in one of the strings is 40.0 N. Calculate the angle this string makes with the vertical and the tension in the other string.

6. A uniform pole AB of weight $5W$ and length $8a$ is suspended horizontally by two vertical strings attached to it at C and D where AC = DB = a. A body of weight $9W$ hangs from the pole at E where ED = $2a$. Calculate the tension in each string.

7. AB is a uniform rod of length 1.4 m. It is pivoted at C, where AC = 0.5 m, and rests in horizontal equilibrium when weights of 16 N and 8 N are applied at A and B respectively. Calculate: **(a)** the weight of the rod, **(b)** the magnitude of the reaction at the pivot.

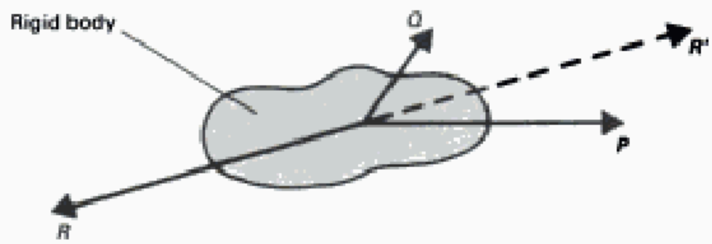
8. A uniform rod AB of length $4a$ and weight W is smoothly hinged at its upper end, A. The rod is held at 30° to the horizontal by a string which is at 90° to the rod and attached to it at C where AC = $3a$. Find: **(a)** the tension in the string, **(b)** the vertical component of the reaction at A, **(c)** the horizontal component of the reaction at A.

9. A sphere of weight 40 N and radius 30 cm rests against a smooth vertical wall. The sphere is supported in this position by a string of length 20 cm attached to a point on the sphere and to a point on the wall. Find: **(a)** the tension in the string, **(b)** the reaction due to the wall. (If you require a hint, turn to the answer.)

4.2 THE TRIANGLE OF FORCES

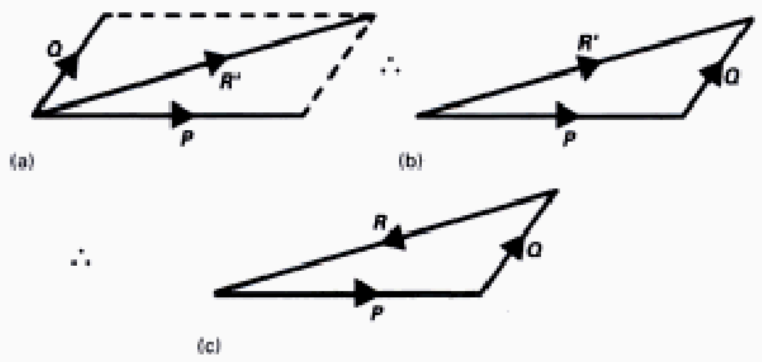
Suppose that a body is in equilibrium under the action of three non-parallel coplanar forces, $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ (Fig. 4.4). In order to satisfy condition (i) (p. 44), each force must be equal and opposite to the resultant of the other two. The system therefore reduces to one in which there are only two equal and opposite forces, ($\boldsymbol{R}$ and $\boldsymbol{R}'$, say, where $\boldsymbol{R}'$ is the resultant of $\boldsymbol{P}$ and $\boldsymbol{Q}$). Furthermore, these two forces ($\boldsymbol{R}$ and $\boldsymbol{R}'$) must be in line with each other, otherwise there would be a couple acting on the system and condition (ii) would not be satisfied. It follows that **$\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ must be concurrent**.

Fig. 4.4
Body acted on by three forces



Bearing in mind that $\boldsymbol{R}'$ is the resultant of $\boldsymbol{P}$ and $\boldsymbol{Q}$ and that $\boldsymbol{R} = -\boldsymbol{R}'$, leads to Figs. 4.5(a), (b) and (c). It follows from Fig. 4.5 that:

Fig. 4.5
The triangle of forces



If a body is in equilibrium under the action of three coplanar forces, then the forces can be represented in magnitude and direction by the sides of a triangle taken in order. This is known as **the triangle of forces**.

QUESTIONS 4B

1. Solve the problem in example 4.1 by using a triangle of forces.

4.3 THE POLYGON OF FORCES

The triangle of forces can easily be extended to any number of forces, in which case:

If a body is in equilibrium under the action of any number of forces, then the forces can be represented in magnitude and direction by the sides of a polygon taken in order

4.4 TYPES OF EQUILIBRIUM

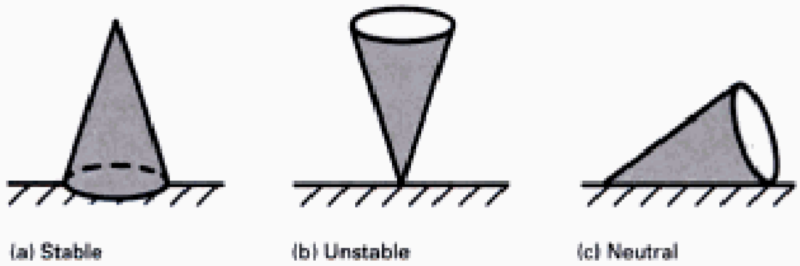
There are three types of equilibrium and these are illustrated by the cone shown in Fig. 4.6.

(i) **Stable equilibrium** A body is in stable equilibrium if it returns to its equilibrium position after it has been displaced slightly (Fig. 4.6(a)).

(ii) **Unstable equilibrium** A body is in unstable equilibrium if it does not return to its equilibrium position and does not remain in the displaced position after it has been displaced slightly (Fig. 4.6(b)).

(iii) **Neutral equilibrium** A body is in neutral equilibrium if it stays in the displaced position after it has been displaced slightly (Fig. 4.6(c)).

Fig. 4.6
Types of equilibrium

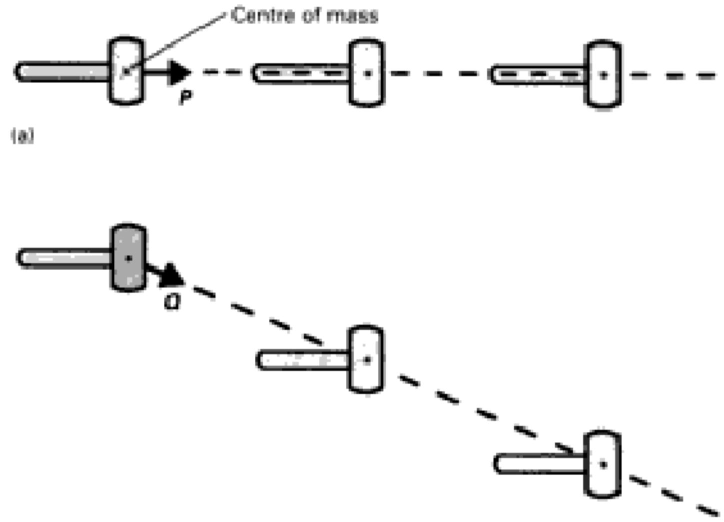


4.5 CENTRE OF MASS

The entire mass of a body can be considered to act at a single point, known as the **centre of mass** of the body. If a body is symmetrical and of uniform composition, the centre of mass is at the geometric centre of the body.

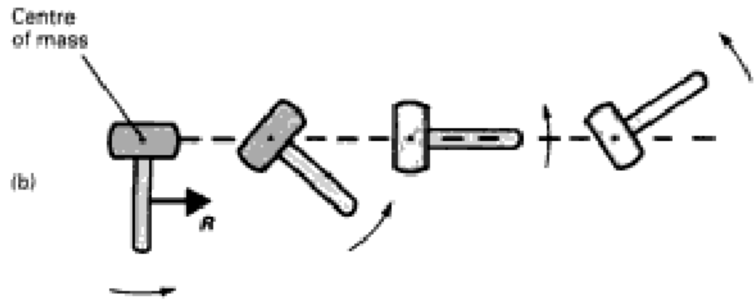
If a single force acts on a body and the line of action of the force passes through the centre of mass, the body will have a linear acceleration but no angular acceleration. Thus, a body which is accelerated from rest by such a force will move in a straight line without any rotation. As an example of this, imagine a stationary hammer resting on a frictionless surface. If forces such as ***P*** and ***Q*** are applied to the hammer (Figs 4.7(a) and (b)), it will move without rotation as shown.

Fig. 4.7
Effects of forces at centre of mass of a hammer



However, if a force such as ***R*** is applied to the hammer, its subsequent motion involves rotation because ***R*** does not act through the centre of mass (Fig. 4.8). Note that even when the body is rotating, the centre of mass moves along a straight line, i.e. the rotation takes place about the centre of mass. Thus, **in the absence of an actual pivot (e.g. an axle) a body behaves as if it is pivoted at its centre of mass and only at its centre of mass**.

Fig. 4.8
Effect of forces at centre of mass of a hammer



The motion of the centre of mass of a body cannot be affected by internal forces. Suppose that a space-ship, which is initially moving with uniform speed along a straight line, breaks into a number of pieces as a result of an explosion on board. No external force has acted on the mass of the space-ship and therefore the mass as a whole cannot (by Newton's second law) acquire an acceleration. Since the mass can be taken to be at the centre of mass, there can be no acceleration of the centre of mass. The pieces therefore move apart in such a way that the centre of mass continues to move with the original speed in the original direction.

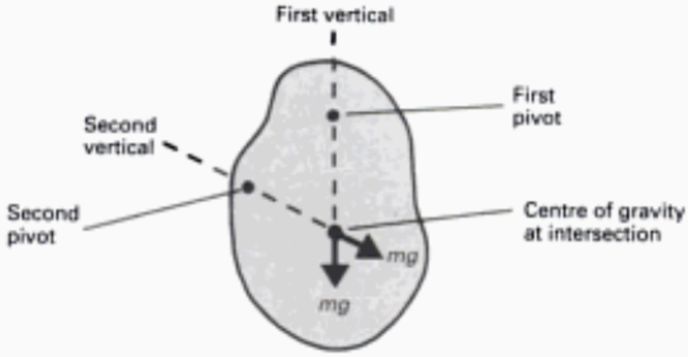
4.6 CENTRE OF GRAVITY

The centre of gravity of a body is the single point at which the entire weight of the body can be considered to act. In uniform gravitational fields (such as that of the Earth on a small body) the centre of gravity coincides with the centre of mass.

Since the weight of a body acts at its centre of gravity, **a freely suspended body hangs in such a way that its centre of gravity is vertically below the pivot.** This is the basis of the usual experimental determination of the position of the centre of gravity of a body (Fig. 4.9).

Centres of gravity (and therefore centres of mass) can also be located by calculation (Examples 4.4 to 4.7).

Fig. 4.9
Determination of centre of gravity



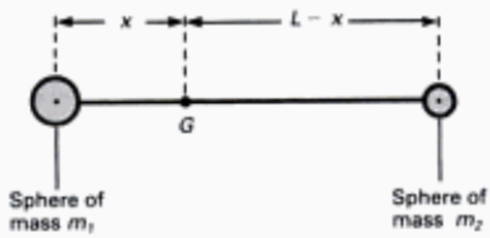
EXAMPLE 4.4

Calculate the position of the centre of gravity of a body which comprises two small spheres whose centres are connected by a straight rod of length L. The masses of the spheres are m_1 and m_2. The mass of the rod is very small and may be ignored.

Solution

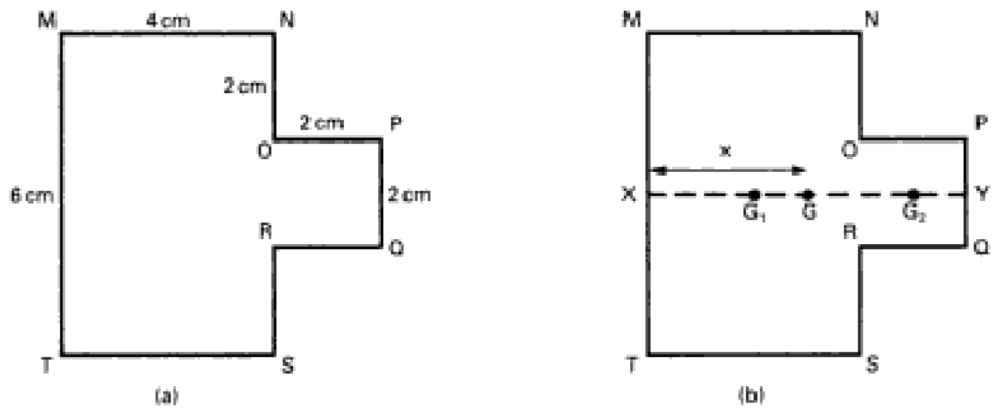
By symmetry, the centre of gravity of the system is at a point on the line joining the centres of gravity of two spheres. Since the centre of gravity of each sphere is at its centre, the centre of gravity of the whole system is at a point such as G (Fig. 4.10).

Fig. 4.10
Diagram for Example 4.4



The centre of gravity of a body is the point at which its weight acts, and therefore if the body were to be pivoted at its centre of gravity, there would be no gravitational torque about that point. Therefore,

Fig. 4.12
Diagram for Example 4.6



which are at their centres G_1 and G_2 respectively. The centre of gravity of the whole lamina must lie between G_1 and G_2. Let it be at G, a distance x from MT. Let w = the weight per unit area of the lamina.

Section	*Weight*	*Distance of centre of gravity from MT*
MNST	$24w$	2
OPQR	$4w$	5
MNPQRST	$28w$	x

Moment of whole about MT = Sum of moments of parts about MT

$$\therefore \quad 28wx = 24w \times 2 + 4w \times 5$$

$$\therefore \quad 28wx = 68w$$

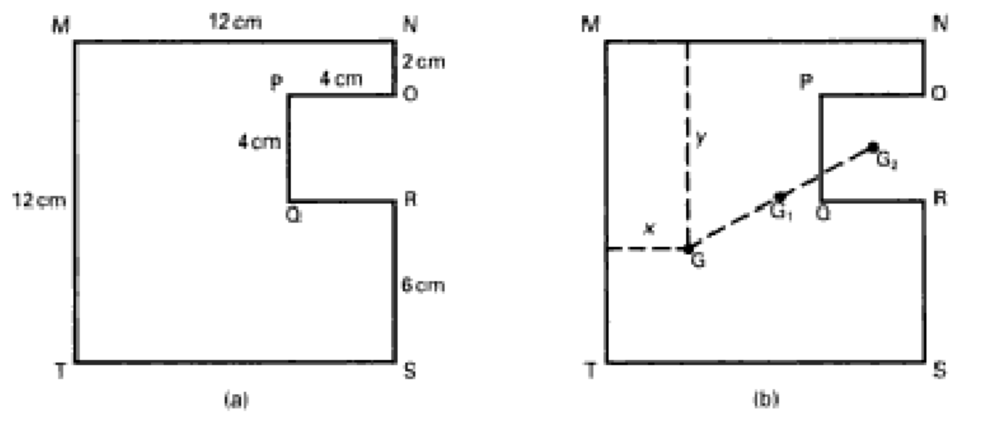
$$\therefore \quad x = \frac{68w}{28w} \qquad \text{i.e.} \quad x = 2.4 \text{ cm}$$

The centre of gravity is therefore 3 cm from MN and 2.4 cm from MT.

EXAMPLE 4.7

MNOPQRST is a uniform lamina whose dimensions are as shown in Fig. 4.13(a). Find the distance of its centre of gravity from MN and from MT.

Fig. 4.13
Diagram for Example 4.7



Solution

We regard the lamina as a square MNST from which a smaller square OPQR has been removed. The centres of gravity at these squares are at their centres G_1 and G_2 (Fig. 4.13(b)). Let the centre of gravity of the lamina be at G, a distance x from MT and a distance y from MN. Let w = the weight per unit area of the lamina.

Section	*Weight*	*Distance of centre of gravity from MT*	*Distance of centre of gravity from MN*
MNST	$144w$	6	6
OPQR	$16w$	10	4
MNOPQRST	$128w$	x	y

Moment of whole about MT = Sum of moments of parts about MT

$$\therefore \quad 144w \times 6 = 16w \times 10 + 128wx$$

$$\therefore \quad 864w = 160w + 128wx$$

$$\therefore \quad 704w = 128wx$$

$$\therefore \quad x = \frac{704w}{128w} \quad \text{i.e.} \quad x = 5.5 \text{ cm}$$

Moment of whole about MN = Sum of moments of parts about MN

$$\therefore \quad 144w \times 6 = 16w \times 4 + 128wy$$

$$\therefore \quad 864w = 64w + 128wy$$

$$\therefore \quad 800w = 128wy$$

$$\therefore \quad y = \frac{800w}{128w} \quad \text{i.e.} \quad y = 6.25 \text{ cm}$$

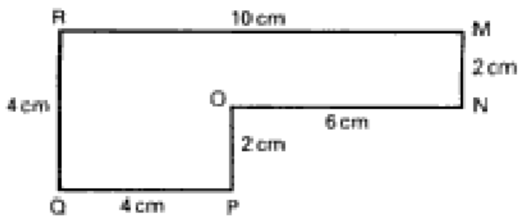
The centre of gravity is therefore 5.5 cm from MT and 6.25 cm from MN.

QUESTIONS 4C

1. A light square frame ABCD of side $10a$ has particles of mass m, $2m$, $3m$ and $4m$ at A, B, C and D respectively. Find the distance of the centre of gravity: **(a)** from AB, **(b)** from AD.

2. A non-uniform rod AB of weight 40 N and length 20 cm is supported by a pivot at C where AC = 14 cm. The rod rests in horizontal equilibrium when a weight of 30 N is attached to it at B. Find the distance of the centre of gravity of the rod from A.

3. MNOPQR is a uniform lamina. Find the distance of its centre of gravity: **(a)** from MR, **(b)** from MN.



4. A circular plate of uniform thickness and radius 12 cm has a circular hole of radius 4 cm cut out of it. The centre of the hole is 2 cm from the centre, O, of the plate. Find the distance of the centre of gravity from O.

CONSOLIDATION

If three coplanar forces are in equilibrium, the forces are bound to be concurrent.

If a particle is in equilibrium, the resultant force on it is zero in which case it must be at rest or moving in a straight line at constant speed.

If a body is in equilibrium, the resultant force on it is zero and the resultant torque is zero in which case it must be at rest or moving in a straight line at constant speed and if it is rotating, it must be doing so with a constant angular velocity.

To solve problems in which concurrent coplanar forces are known to be in equilibrium resolve in (up to) two directions and make use of the fact that the resultant force in each direction is zero.

To solve problems in which non-concurrent coplanar forces are known to be in equilibrium resolve twice and take moments once, or resolve once and take moments twice, or take moments three times.

The centre of gravity of a body is the point at which its weight can be taken to act.

5

WORK, ENERGY, POWER

5.1 WORK

If a body moves as a result of a force being applied to it, the force is said to be doing work on the body. The work done is given by

$$W = Fs \qquad [5.1]$$

where

W = the work done (joules, J)

F = the constant applied force (N)

s = the distance moved in the direction of the force (m).

It follows from equation [5.1] that a force is doing no work if it is merely preventing a body moving, because in such a circumstance $s = 0$. Thus, if a man lifts some object, he is doing work whilst actually lifting it; but he does no work in holding it above his head, say, once he has lifted it into that position. The man would, of course, become tired if he were to hold a heavy object for a long time but this is because he is having to keep his muscles under tension; it is not because he is doing work on the object.

Suppose that a constant force, F, acts on a body so as to move it in a direction other than its own (Fig. 5.1). The component of $\boldsymbol{F}$ in the direction of motion is $F \cos \theta$, in which case the work done, W, is given by

$$W = Fs \cos \theta$$

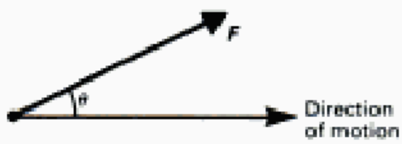


Fig. 5.1
Force at angle to motion

This situation can occur only if there is some other force preventing motion taking place in the direction of $\boldsymbol{F}$. For example, consider a man pulling a garden roller in the manner shown in Fig. 5.2. For convenience, the man is holding the handle at an angle θ to the horizontal and exerts a force $\boldsymbol{F}$ in the direction shown. The other force that acts on the roller is its weight, mg, and this of course, acts vertically downwards. The upward directed component of $\boldsymbol{F}$ will be less than the weight. Therefore there is no vertical motion and no work is done by the upward directed component of $\boldsymbol{F}$.

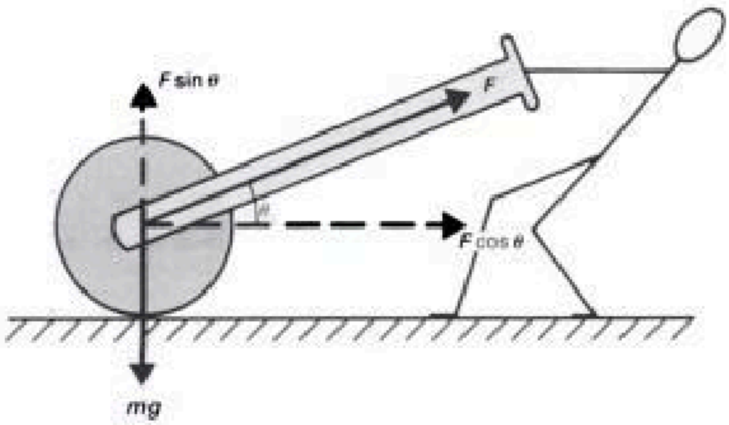


Fig. 5.2
Force diagram for a man pulling a roller

5.2 ENERGY

A body which is capable of doing work is said to possess energy. The amount of energy that a body has is equal to the amount of work that it can do (or what amounts to the same thing, the amount of work that must have been done on it to give it that energy).

Although it is often convenient to classify energy as being chemical energy or nuclear energy or heat energy, etc., there are basically only two types of energy – kinetic energy (KE) and potential energy (PE).

5.3 KINETIC ENERGY

The energy which a body possesses solely because it is moving is called kinetic energy.

The kinetic energy of a body can be defined as the amount of work it can do in coming to rest, or what amounts to the same thing, the amount of work that must have been done on it to increase its velocity from zero to the velocity it has. On this basis, if a body of mass m is moving with velocity v, then

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2$$

Kinetic energy is a positive, scalar quantity

To Show that Kinetic Energy $= \frac{1}{2}mv^2$ (Variable Force)

Suppose that a body of mass m moves a small distance δs under the action of a force F. Suppose also that, though the force may be varying, δs is so small that the force can be considered constant over the distance δs. The work done δW is given by equation [5.1] as

$$\delta W = F\,\delta s$$

If the force increases the velocity of the body from zero to v, the total work done W is given by

$$W = \int_{v=0}^{v=v} F\,\mathrm{d}s$$

Using Newton's second law (equation [2.2]) we can write F as

$$F = m\frac{dv}{dt}$$

where dv/dt is the acceleration of the body. Therefore

$$W = \int_{v=0}^{v=v} m\frac{dv}{dt}\,ds$$

Bearing in mind that $v = ds/dt$, we can write

$$W = \int_0^v mv\,dv$$

and therefore

$$W = [\tfrac{1}{2}mv^2]_0^v$$

i.e. $$W = \tfrac{1}{2}mv^2$$

By definition, the work done is the kinetic energy of the body, and therefore

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2$$

Note The kinetic energy of a body depends only on its mass and its velocity and as such, the kinetic energy is independent of the way in which the body acquired this velocity. In view of this, the result that has just been derived could have been obtained more simply by specifying that the body was accelerated by a constant force. This will now be done.

To Show that Kinetic Energy = $\frac{1}{2}mv^2$ (Constant Force)

If a body of mass m moves a distance s under the action of a constant force F, the work done W by the force is given by equation [5.1] as

$$W = Fs$$

If the (constant) acceleration is a, then from Newton's second law $F = ma$ and therefore

$$W = mas \qquad [5.2]$$

If the body has been accelerated from rest to some velocity v, then from equation [2.7]

$$v^2 = 0^2 + 2as$$

i.e. $$as = \frac{v^2}{2}$$

Therefore from equation [5.2]

$$W = \tfrac{1}{2}mv^2$$

and therefore by definition

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2$$

through a distance h the increase in potential energy is mgh. It follows that if the potential energy of the body is taken to be zero when it is on the ground, then its potential energy at a height h is mgh, i.e.

$$\text{Gravitational potential energy} = mgh$$

5.6 CONSERVATION OF MECHANICAL ENERGY

The principle of conservation of mechanical energy can be stated as:

In a system in which the only forces acting are associated with potential energy (e.g. gravitational and elastic forces) the sum of the kinetic and potential energies is constant

i.e. $$\text{KE} + \text{PE} = \text{a constant} \quad [5.3]$$

Note that, in particular, equation [5.3] does not apply when there are frictional forces present.

As an example of the application of equation [5.3], we shall use it to obtain an expression for the velocity acquired by a body of mass m in falling freely from rest at a height h in a vacuum (Fig. 5.3). As the body falls it loses gravitational potential energy and gains kinetic energy. It follows from equation [5.3] that

$$\text{KE gained} = \text{PE lost}$$

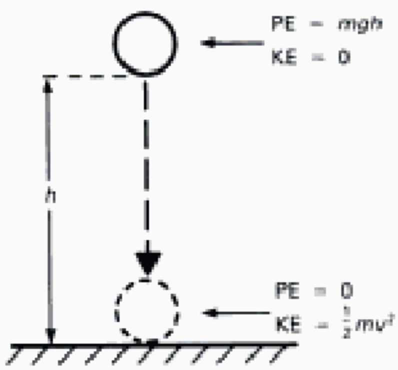


Fig. 5.3
Conversion of potential energy to kinetic energy

and therefore if the velocity of the body after it has fallen a distance h is v, then

$$\tfrac{1}{2}mv^2 = mgh$$

i.e. $$v = \sqrt{2gh}$$

The body comes to rest (at least momentarily) very soon after making contact with the ground. It does so because the Earth has exerted a force on it. The force is due to the solidity of the Earth, rather than to its gravitational properties. At the same time, the body exerts a force on the Earth, and both the body and the Earth become deformed. It is the kinetic energy which the body had immediately before the impact that has been used to produce these deformations. If they are permanent, the energy which created them is dissipated as heat and sound, and the body remains at rest on the ground. On the other hand, if the body and the Earth regain their original shapes, then they lose the elastic potential energy which they acquired at the impact and the body bounces. Some energy is bound to be dissipated as heat, and therefore the body has less than its original amount of kinetic energy and therefore does not reach its original height.

When friction is involved, and when work is done by external forces (i.e. forces other than those associated with potential energy) we make use of **the work-energy principle**:

$$\begin{pmatrix}\text{Work done by}\\ \text{external force}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{KE + PE}\end{pmatrix} + \begin{pmatrix}\text{Work done}\\ \text{against friction}\end{pmatrix} \quad [5.4(a)]$$

If work is done against external forces, equation [5.4(a)] becomes

$$\begin{pmatrix}\text{Decrease in}\\ \text{KE + PE}\end{pmatrix} = \begin{pmatrix}\text{Work done}\\ \text{against}\\ \text{external forces}\end{pmatrix} + \begin{pmatrix}\text{Work done}\\ \text{against friction}\end{pmatrix} \quad [5.4(b)]$$

In problems where there are sudden changes in velocity, e.g. where two bodies collide or where there is a sudden increase in the tension in a string (i.e. a jerk) some mechanical energy is converted to heat and/or sound*. In such circumstances, using the principle of conservation of mechanical energy or the work–energy principle (equations 5.4(a) and (b)) allows us to do no more than find out just how much energy has been converted in this way. For example if we know the height to which a bouncing ball rebounds, we can calculate the amount of mechanical energy converted to heat, etc., as a result of the impact, but energy considerations alone do not allow us to calculate the height to which the ball rebounds in the first place.

EXAMPLE 5.1

A car of mass 800 kg and moving at 30 m s^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000 N. Calculate the distance the car moves whilst coming to rest.

Solution

If the car travels a distance s in coming to rest, then by equation [5.1] the work done by the car against the retarding force

$$= 5000s$$

The kinetic energy ($\frac{1}{2}mv^2$) lost by the car in coming to rest

$$= \tfrac{1}{2} \times 800 \times 30^2$$

$$= 360\,000 \text{ J}$$

The work done against the retarding force is equal to the kinetic energy lost by the car, and therefore

$$5000s = 360\,000$$

i.e. $s = 72$ m

Alternatively, the solution could have been obtained by using Newton's second law (equation [2.2]) to calculate the value of the retardation which could then be used in equation [2.7] to find s.

*This is not true in the special case of an elastic collision – see section 2.8.

EXAMPLE 5.2

A small block (Fig. 5.4) is released from rest at A and slides down a smooth curved track. Calculate the velocity of the block when it reaches B, a vertical distance h below A.

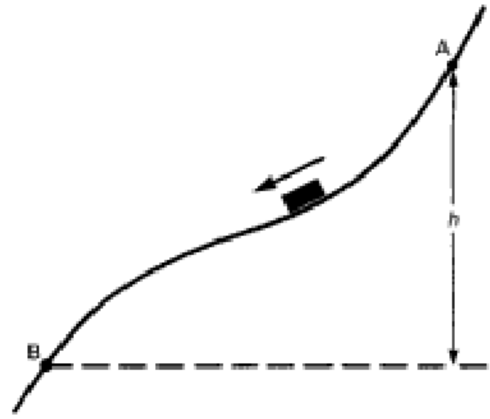


Fig. 5.4
Diagram for Example 5.2

Solution

Suppose that the mass of the block is m. The gravitational potential energy lost by the block in moving from A to B is mgh. If the velocity of the block on reaching B is v, then the kinetic energy gained by the block is $\frac{1}{2}mv^2$.

The track is smooth and therefore no work is done against friction, in which case

$$\text{KE gained} = \text{PE lost}$$

$$\therefore \quad \tfrac{1}{2}mv^2 = mgh$$

$$\text{i.e.} \quad v = \sqrt{2gh}$$

The problem has been solved by making use of the principle of conservation of mechanical energy. Unlike Example 5.1, it could not have been solved by using $F = ma$ and $v^2 = u^2 + 2as$ because the acceleration is not constant and therefore $v^2 = u^2 + 2as$ does not apply. A solution based on $F = ma$ is possible provided the equation of the curve is known, but it involves using calculus and is much more difficult than the solution given here.

Note that the speed at B does not depend on the particular shape of the curve. However the time to reach B does, and cannot be found by using energy considerations.

EXAMPLE 5.3

A car of mass 1.0×10^3 kg increases its speed from $10\,\text{m s}^{-1}$ to $20\,\text{m s}^{-1}$ whilst moving 500 m up a road inclined at an angle α to the horizontal where $\sin\alpha = \frac{1}{20}$. There is a constant resistance to motion of 300 N. Find the driving force exerted by the engine, assuming that it is constant. (Assume $g = 10\,\text{m s}^{-2}$.)

Solution

The work done by the engine is used to increase both the KE and PE of the car and to overcome the resistive force.

In moving 500 m along the road the car gains a vertical height of 500 sin $\alpha = 25$ m. Therefore

$$\text{PE gained} = 1.0 \times 10^3 \times 10 \times 25 = 2.5 \times 10^5 \text{ J}$$

$$\text{KE gained} = \tfrac{1}{2} \times 1.0 \times 10^3 \times 20^2 - \tfrac{1}{2} \times 1.0 \times 10^3 \times 10^2 = 1.5 \times 10^5 \text{ J}$$

$$\text{Work done against resistance} = 300 \times 500 = 1.5 \times 10^5 \text{ J}$$

By the work–energy principle,

Work done by engine = Increase in PE + Increase in KE + Work against resistance

$\therefore$ Work done by engine $= 2.5 \times 10^5 + 1.5 \times 10^5 + 1.5 \times 10^5 = 5.5 \times 10^5$ J

If the driving force of the engine is F, then (by $W = Fs$)

$$5.5 \times 10^5 = F \times 500 \qquad \text{i.e.} \quad F = 1.1 \times 10^3 \text{ N}$$

Alternatively, as with Example 5.1, the solution could have been obtained by using $F = ma$ and $v^2 = u^2 + 2as$.

QUESTIONS 5A

Questions 1 to 9 should be solved by using energy considerations (Assume $g = 10\,\text{m s}^{-2}$ where necessary.)

1. A car of mass 1.2×10^3 kg moves 300 m up a road which is inclined to the horizontal at an angle α where $\sin \alpha = \frac{1}{15}$. By how much does the gravitational PE of the car increase?

2. A particle is projected with speed v at an angle α to the horizontal. Find the speed of the particle when it is at a height h.

3. A car of mass 800 kg moving at $20\,\text{m s}^{-1}$ is brought to rest by the application of the brakes in a distance of 100 m. Calculate the work done by the brakes and the force they exert assuming that it is constant and that there is no other resistance to motion.

4. The speed of a dog-sleigh of mass 80 kg and moving along horizontal ground is increased from $3.0\,\text{m s}^{-1}$ to $9.0\,\text{m s}^{-1}$ over a distance of 90 m. Find: **(a)** the increase in the KE of the sleigh, **(b)** the force exerted on the sleigh by the dogs, assuming that it is constant and that there is no resistance to motion.

5. A simple pendulum consisting of a small heavy bob attached to a light string of length 40 cm is released from rest with the string at 60° to the downward vertical. Find the speed of the pendulum bob as it passes through its lowest point.

6. A car of mass 900 kg accelerates from rest to a speed of $20\,\text{m s}^{-1}$ whilst moving 80 m along a horizontal road. Find the tractive force (i.e. the driving force) exerted by the engine, assuming that it is constant and that there is a constant resistance to motion of 250 N.

7. A child of mass 20 kg starts from rest at the top of a playground slide and reaches the bottom with a speed of $5.0\,\text{m s}^{-1}$. The slide is 5.0 m long and there is a difference in height of 1.6 m between the top and the bottom. Find: **(a)** the work done against friction, **(b)** the average frictional force.

8. Two particles of masses 6.0 kg amd 2.0 kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest with the string taut. Find the speed of the particles when the heavier one has descended 2.0 m.

9. A ball of mass 50 grams falls from a height of 2.0 m and rebounds to a height of 1.2 m. How much kinetic energy is lost on impact?

5.7 POWER

The power of a machine is the rate at which it does work (alternatively, it is the rate at which it supplies* energy). The unit of power is the **watt** (W).

Thus

$$P = \frac{dW}{dt} \qquad [5.5]$$

where

P = the instantaneous power (W)

$\frac{dW}{dt}$ = the rate of working ($J\,s^{-1}$). Thus $1\,W = 1\,J\,s^{-1}$

If a machine is working at a steady rate,

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}} \qquad [5.6]$$

When the rate of working is not steady, equation [5.6] gives the average power.

Another useful expression for power can be obtained by combining equations [5.1] and [5.5]. Thus from equation [5.5]

$$P = \frac{dW}{dt}$$

Therefore, from equation [5.1]

$$P = \frac{d}{dt}(Fs)$$

If the force is constant

$$P = F\frac{ds}{dt}$$

i.e. $\boldsymbol{P = Fv}$

where P is the power output of a machine which is doing work by exerting a force F and moving the point of application of the force with velocity v. Equation [5.7] is useful in, say, calculating the force exerted by a car engine when the car is moving at a known velocity and the power being produced by the engine is also known.

*The machine has not, of course, actually produced the energy, it has merely converted it from another form.

EXAMPLE 5.4

A pump raises water through a height of 3.0 m at a rate of 300 kilograms per minute and delivers it with a velocity of $8.0\,\mathrm{m\,s^{-1}}$. Calculate the power output of the pump. (Assume $g = 10\,\mathrm{m\,s^{-2}}$.)

Solution

The work done by the pump is used to increase both the PE and the KE of the water. In one second the pump delivers $300/60 = 5.0$ kg of water. Therefore

$$\text{Increase in PE each second} = 5.0 \times 10 \times 3.0 = 150\ \mathrm{J}$$
$$\text{Increase in KE each second} = \tfrac{1}{2} \times 5.0 \times 8.0^2 = 160\ \mathrm{J}$$

Therefore

$$\text{Work done each second} = 150 + 160 = 310\ \mathrm{J}$$

Since work done per second is power, the power output of the pump is 310 W.

QUESTIONS 5B

Assume $g = 10\,\mathrm{m\,s^{-2}}$ where necessary.

1. A man of mass 75 kg climbs 300 m in 30 minutes. At what rate is he working?

2. A pump with a power output of 600 W raises water from a lake through a height of 3.0 m and delivers it with a velocity of $6.0\,\mathrm{m\,s^{-1}}$. What mass of water is removed from the lake in one minute?

3. What is the power output of a cyclist moving at a steady speed of $5.0\,\mathrm{m\,s^{-1}}$ along a level road against a resistance of 20 N.

4. What is the maximum speed at which a car can travel along a level road when its engine is developing 24 kW and there is a resistance to motion of 800 N?

5. A crane lifts an iron girder of mass 400 kg at a steady speed of $2.0\,\mathrm{m\,s^{-1}}$. At what rate is the crane working?

6. A man of mass 70 kg rides a bicycle of mass 15 kg at a steady speed of $4.0\,\mathrm{m\,s^{-1}}$ up a road which rises 1.0 m for every 20 m of its length. What power is the cyclist developing if there is a constant resistance to motion of 20 N?

6

CIRCULAR MOTION AND ROTATION

6.1 ANGULAR VELOCITY

Suppose that a particle (Fig. 6.1) moves from A to P along the arc AXP at a constant speed* in a time interval t. **The angular velocity,** ω, of the particle is given by

$$\omega = \frac{\theta}{t} \qquad [6.1]$$

where θ = the angle turned through in radians. (**The radian** (rad) is the SI unit of angle and is the angle subtended at the centre of a circle by an arc of the circumference equal in length to the radius of the circle.)

ω = the angular velocity of the particle about O (rad s^{-1})

t = the time taken (s).

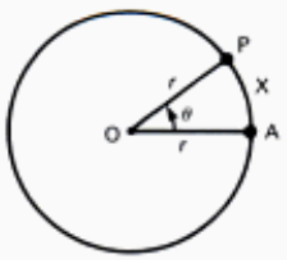
The period, T, of the rotational motion is the time taken for the particle to complete one revolution (i.e. to turn through 2π radians) and is given by equation [6.1] as

$$\omega = \frac{2\pi}{T}$$

i.e.

$$T = \frac{2\pi}{\omega} \qquad [6.2]$$

Fig. 6.1
Definition of angular velocity



*Note the use of the word 'speed' and not 'velocity'. The particle cannot have a constant velocity because its direction of motion is changing.

Note If the angular velocity is not constant, equation [6.1] is replaced by

$$\omega = \frac{d\theta}{dt}$$

where ω is the instantaneous angular velocity.

Referring again to Fig. 6.1, we can see that in time t the distance moved by the particle is the arc length AP and therefore its linear speed v is given by

$$v = \frac{\text{Arc length AP}}{t}$$

i.e. $$v = \frac{r\theta}{t}$$

Therefore, from equation [6.1]

$$v = \omega r \qquad [6.3]$$

6.2 CENTRIPETAL FORCE

If a body is moving along a circular path, there must be a force acting on it, for if there were not, it would move in a straight line in accordance with Newton's first law. Furthermore, if the body is moving at a constant speed, this force cannot (at any stage) have a component which is in the direction of motion of the body, for if it did it would be bound to either increase or decrease the speed of the body. The force that acts on the body must, therefore, be perpendicular to the direction of motion of the body and must therefore be directed towards the centre of the circular path. The force is known as a **centripetal force**.

If a brick is being whirled in a circle on one end of a piece of string, the centripetal force is provided by the tension in the string. If the string were to break, there would be no centripetal force and the brick would fly off at a tangent.

The centripetal force on an orbiting planet is gravitational; that on an electron moving round a nucleus is electrostatic.

6.3 CENTRIPETAL ACCELERATION

Because there is a resultant force on a body which is describing a circular path, the body must (by Newton's second law) have an acceleration. This acceleration must be in the same direction as the force, i.e. toward the centre of the circle. It is known as a **centripetal acceleration**. For a body which is moving with constant angular velocity, ω, along a circular path of radius, r, the magnitude of the centripetal acceleration can be shown to be given by

$$a = \omega^2 r \qquad [6.4]$$

where

a = the centripetal acceleration (m s^{-2}).

If the linear speed of the particle is v, then by equation [6.3]

$$a = \frac{v^2}{r} \qquad [6.5]$$

To Show that the Centripetal Acceleration $= \dfrac{v^2}{r}$

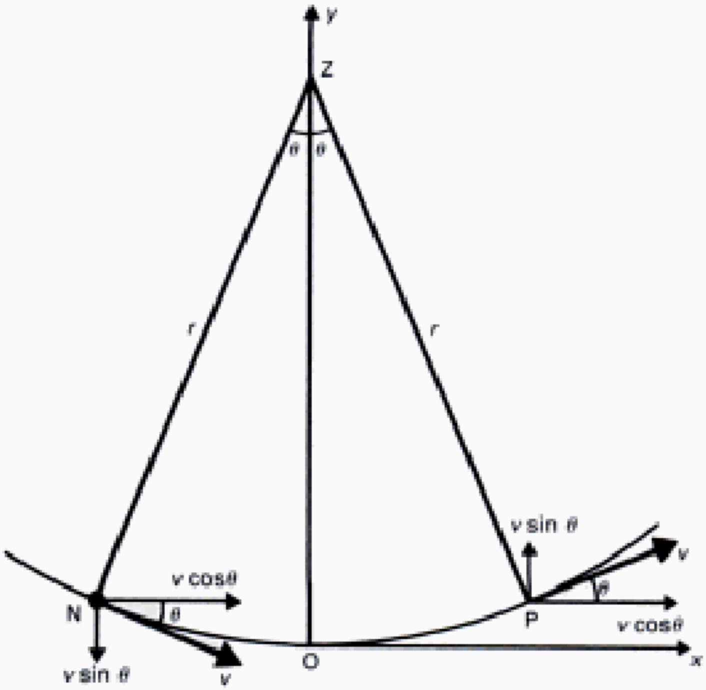


Fig. 6.2
To calculate the centripetal acceleration of a particle moving in a circle

Consider a particle moving with constant speed v along an arc NOP (Fig. 6.2). The x-component of velocity of the particle has the same value at P as at N and therefore its x-component of acceleration, a_x, is zero, i.e.

$$a_x = 0$$

As the particle moves from N to P its y-component of velocity changes by $2v \sin\theta$. If this takes place in a time interval, t, its y-component of acceleration, a_y, is given by

$$a_y = \frac{2v \sin\theta}{t} \qquad [6.6]$$

The speed of the particle along the arc is v, and therefore

$$t = \frac{\text{Arc length NOP}}{v}$$

i.e. $$t = \frac{2\theta r}{v}$$

Therefore from equation [6.6]

$$a_y = \frac{2v \sin\theta}{2\theta r/v}$$

i.e. $$a_y = \frac{v^2}{r}\,\frac{\sin\theta}{\theta}$$

If N and P are now taken to be coincident at O, then $\theta = 0$ and $\sin\theta/\theta$ has its limiting value of 1*, in which case

$$a_y = \frac{v^2}{r}$$

Thus at O, $a_x = 0$ and $a_y = v^2/r$ and therefore the acceleration is directed (entirely) along OZ, i.e. towards the centre of the circle. This result does not depend on the position of O and therefore in general:

> The acceleration of a particle moving with constant speed v along a circular path of radius r is v^2/r and is directed towards the centre of the circle.

Note This result is not approximate, it does not depend on the approximate relationship $\sin\alpha \approx \alpha$, but on the limiting value of $\sin\alpha/\alpha$ as α tends to zero, and this is exactly equal to unity.

EXAMPLE 6.1

A particle of mass 3.0 kg is attached to a point O on a smooth horizontal table by means of a light inextensible string of length 0.50 m. The string is fully extended and the particle moves on the table in a circular path about O with a constant angular velocity of 8.0 radians per second. Calculate the tension in the string.

Solution

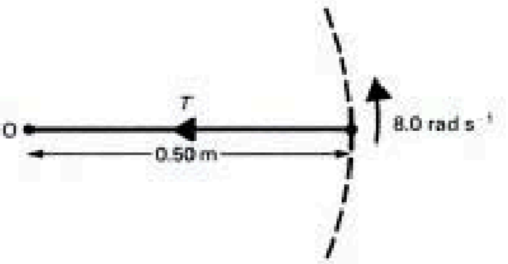


Fig. 6.3
Diagram for Example 6.1

Refer to Fig. 6.3. By Newton's second law

$$\text{Force} = \text{mass} \times \text{acceleration}$$

The 'acceleration' is the centripetal acceleration, $\omega^2 r$. The 'force' is the centripetal force and is provided by the tension, T, in the string. Therefore

$$T = 3.0 \times 8.0^2 \times 0.50$$

i.e. $$T = 96\ \text{N}$$

*It is a general result that for a small angle α

$$\sin\alpha \approx \alpha \quad \text{measured in radians}$$

i.e. $$\frac{\sin\alpha}{\alpha} \approx 1$$

In the limit as α tends to zero $(\sin\alpha)/\alpha = 1$

i.e. $$\lim_{\alpha \to 0}\left(\frac{\sin\alpha}{\alpha}\right) = 1$$

EXAMPLE 6.2

A small bead of mass m is threaded on a smooth circular wire of radius r and centre O, and which is fixed in a vertical plane. The bead is projected with speed u from the highest point, A, of the wire. Find the reaction on the bead due to the wire when the bead is at P, in terms of m, g, r, u and θ where $\theta = \mathrm{A\hat{O}P}$.

Solution

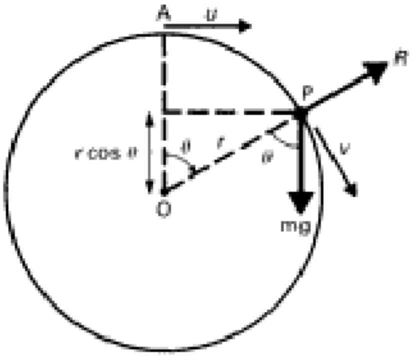


Fig. 6.4
Diagram for example 6.2

Refer to Fig. 6.4. Let the speed of the bead at $P = v$; let the reaction $= R$. Consider the motion from A to P. Since the wire is smooth, no work is done against friction and therefore

$$\text{Increase in KE} = \text{Decrease in PE}$$

$$\therefore \quad \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 = mg\,(r - r\cos\theta)$$

$$\therefore \quad v^2 = u^2 + 2gr\,(1 - \cos\theta)$$

Applying Newton's second law to the motion along PO gives

$$mg\cos\theta - R = m\frac{v^2}{r}$$

$$\therefore \quad mg\cos\theta - R = \frac{m}{r}[u^2 + 2gr\,(1 - \cos\theta)]$$

$$\therefore \quad R = mg\,(3\cos\theta - 2) - \frac{mu^2}{r}$$

QUESTIONS 6A

1. A particle moves along a circular path of radius 3.0 m with an angular velocity of 20 rad s^{-1}. Calculate: **(a)** the linear speed of the particle, **(b)** the angular velocity in revolutions per second, **(c)** the time for one revolution, **(d)** the centripetal acceleration.

2. A particle of mass 0.2 kg moves in a circular path with an angular velocity of 5 rad s^{-1} under the action of a centripetal force of 4 N. What is the radius of the path?

3. What force is required to cause a body of mass 3 g to move in a circle of radius 2 m at a constant rate of 4 revolutions per second?

4. An astronaut, as part of her training, is spun in a horizontal circle of radius 5 m. If she can withstand a maximum acceleration of 8g, what is the maximum angular velocity at which she can remain conscious?

5. A particle of mass 80 g rests at 16 cm from the centre of a turntable. If the maximum frictional force between the particle and the turntable is 0.72 N, what is the maximum angular velocity at which the turntable could rotate without the particle slipping?

6. The gravitational force on a satellite of mass m at a distance r from the centre of the Earth is $4.0 \times 10^{14}\ m/r^2$. Assuming that the Earth is a sphere of radius 6.4×10^3 km, find the period of revolution (in hours) of a satellite moving in a circular orbit at a height of 3.6×10^4 km above the Earth's surface.

7. A small bead is threaded on a smooth circular wire of radius r which is fixed in a vertical plane. the bead is projected from the lowest point of the wire with speed $\sqrt{6gr}$. Find the speed of the bead when it has turned through: **(a)** 60°, **(b)** 90°, **(c)** 180°, **(d)** 300°.

8. An aeroplane loops the loop in a vertical circle of radius 200 m, with a speed of 40 m s^{-1} at the top of the loop. The pilot has a mass of 80 kg. What is the tension in the strap holding him into his seat when he is at the top of the loop?

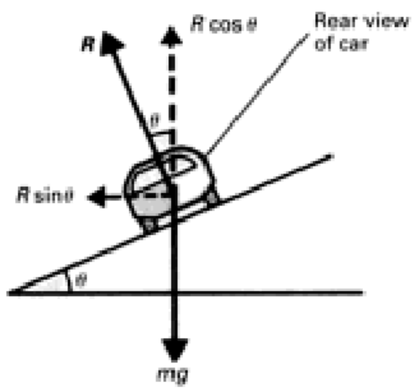
9. A bucket of water is swung in a vertical circle of radius r in such a way that the bucket is upside down when it is at the top of the circle. What is the minimum speed that the bucket may have at this point if the water is to remain in it?

6.4 VEHICLES GOING ROUND BENDS

The centripetal force required to cause a car to go round a bend on a level surface has to be provided by the frictional force exerted on the tyres by the road. The need to rely on friction is removed if the road is suitably 'banked' (Fig. 6.5). The normal reaction, $\boldsymbol{R}$, of the road on the car acquires a horizontal component ($R \sin \theta$) as a result of the banking. If the mass of the car is m and it is moving with constant speed v around a bend of radius r, the centripetal force needs to provide an acceleration of v^2/r, and therefore by Newton's second law (equation [2.2])

$$R \sin \theta = \frac{mv^2}{r} \qquad [6.7]$$

Fig. 6.5
Car going round a bend on a banked corner



Also, since there is no vertical acceleration,

$$R \cos \theta = mg \qquad [6.8]$$

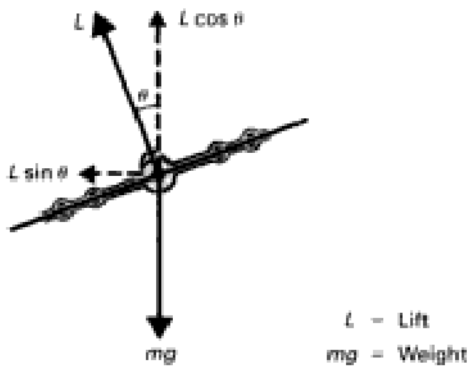
Dividing equation [6.7] by equation [6.8] leads to

$$\tan \theta = \frac{v^2}{gr} \qquad [6.9]$$

If a railway train rounds a bend on a level track, the centripetal force is provided by the push of the outer rail on the flanges of the wheels. This causes a certain amount of wear which could be avoided by banking the track. Equation [6.9] obviously applies to this situation too.

Equation [6.9] also gives the angle at which an aircraft should be banked in order to turn (Fig. 6.6).

Fig. 6.6
Aircraft banking



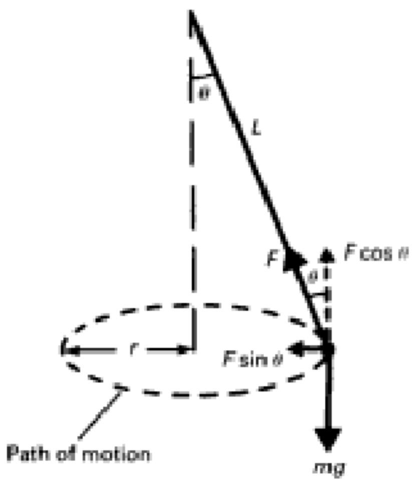
6.5 THE CONICAL PENDULUM

If a pendulum bob is displaced sideways and then given the appropriate velocity in a direction at right angles to its displacement, it will move in a horizontal circle and the string will sweep out a cone. Such an arrangement is shown in Fig. 6.7. There are two forces acting on the pendulum bob. These are:

(i) its weight (mg), and

(ii) the tension in the string (F).

Fig. 6.7
Action of a conical pendulum



The horizontal component of the tension ($F \sin \theta$) provides the necessary centripetal force. If the radius of the circular path is r and the speed of the bob is v, then from Newton's second law

$$F \sin \theta = \frac{mv^2}{r} \qquad [6.10]$$

There is no vertical acceleration, and therefore

$$F \cos \theta = mg \qquad [6.11]$$

Dividing equation [6.10] by equation [6.11] leads to

$$\tan\theta = \frac{v^2}{gr}$$

Note that this is the same expression (equation [6.9]) that governs the angle at which a turning vehicle must lean. This is not surprising – the forces acting in Fig. 6.7 have the same relationship with each other as those in Fig. 6.5.

Bearing in mind that $v = \omega r$ (equation [6.3]), we obtain

$$\tan\theta = \frac{\omega^2 r^2}{gr}$$

i.e. $$\tan\theta = \frac{\omega^2 r}{g}$$

Referring to Fig. 6.7, $r = L\sin\theta$, and therefore

$$\tan\theta = \frac{\omega^2 L\sin\theta}{g}$$

i.e. $$\frac{\sin\theta}{\cos\theta} = \frac{\omega^2 L\sin\theta}{g}$$

i.e. $$\omega = \sqrt{\frac{g}{L\cos\theta}}$$

Therefore, from equation [6.2], the period, T, is given by

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} \qquad [6.12]$$

EXAMPLE 6.3

A pendulum bob of mass 2.0 kg is attached to one end of a string of length 1.2 m. The bob moves in a horizontal circle in such a way that the string is inclined at 30° to the vertical. Calculate:

(a) the tension in the string,

(b) the period of the motion.

(Assume $g = 10\,\mathrm{m\,s^{-2}}$.)

Solution

The forces acting on the bob are shown in Fig. 6.8; m is the mass of the bob and F is the tension in the string. Since there is no vertical acceleration,

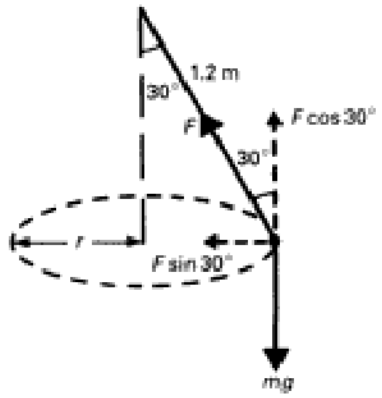
$$F\cos 30° = mg$$

i.e. $$F = \frac{mg}{\cos 30°}$$

i.e. $$F = \frac{2.0 \times 10}{0.8660} = 23.1$$

i.e. Tension = 23 N

Fig. 6.8
Diagram for Example 6.3



Applying Newton's second law to the horizontal motion gives

$$F \sin 30^\circ = m\omega^2 r$$

where ω is the angular velocity of the bob and r is the radius of the circular path. Noting that $r = 1.2 \sin 30^\circ$ leads to

$$F \sin 30^\circ = m\omega^2 \times 1.2 \sin 30^\circ$$

i.e. $$F = 1.2\, m\omega^2$$

Therefore, since $m = 2.0$ kg and $F = 23.1$ N,

$$\omega = \sqrt{\frac{23.1}{1.2 \times 2}}$$

i.e. $$\omega = 3.103 \text{ rad s}^{-1}$$

From equation [6.2], period $= 2\pi/\omega$, and therefore

$$\text{Period} = 2\pi/3.103 = 2.03$$

i.e. $$\text{Period} = 2.0 \text{ s}$$

The same result could have been obtained (though less satisfyingly) by substituting the relevant values into equation [6.12].

QUESTIONS 6B

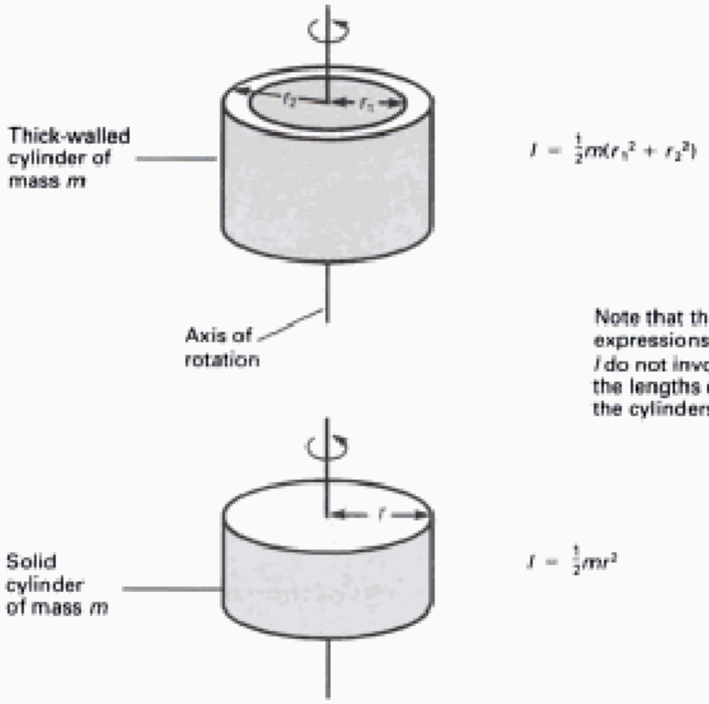
1. A particle of mass 0.20 kg is attached to one end of a light inextensible string of length 50 cm. The particle moves in a horizontal circle with an angular velocity of 5.0 rad s^{-1} with the string inclined at θ to the vertical. Find the value of θ.

2. A particle is attached by means of a light, inextensible string to a point 0.40 m above a smooth, horizontal table. The particle moves on the table in a circle of radius 0.30 m with angular velocity ω. Find the reaction on the particle in terms of ω. Hence find the maximum angular velocity for which the particle can remain on the table.

3. A particle of mass 0.25 kg is attached to one end of a light inextensible string of length 3.0 m. The particle moves in a horizontal circle and the string sweeps out the surface of a cone. The maximum tension that the string can sustain is 12 N. Find the maximum angular velocity of the particle.

4. A particle of mass 0.30 kg moves with an angular velocity of 10 rad s^{-1} in a horizontal circle of radius 20 cm inside a smooth hemispherical bowl. Find the reaction of the bowl on the particle and the radius of the bowl.

Fig. 6.11
Moments of inertia of cylindrical bodies



Note that the expressions for I do not involve the lengths of the cylinders

6.8 ANGULAR MOMENTUM

Consider a rigid body rotating with angular velocity ω about an axis which is perpendicular to the paper and which passes through O (Fig. 6.9). Consider a particle of the body which is at P and which has mass m_1 and is at a perpendicular distance r_1 from the axis. The body is rigid and therefore all of its particles have the same angular velocity, ω. Therefore, by equation [6.3] the linear velocity of P is ωr_1 and its linear momentum is $m_1\omega r_1$.

The angular momentum of a particle about an axis is the product of its linear momentum and the perpendicular distance of the particle from the axis.

Therefore the angular momentum about O of the particle at P is

$$m_1\omega {r_1}^2$$

If the rest of the body is made up of particles of masses $m_2, m_3, \ldots$, whose distances from the axis are $r_2, r_3, \ldots$, respectively, the total angular momentum of the body about O is given by

$$\begin{aligned}\text{Angular momentum} &= m_1\omega {r_1}^2 + m_2\omega {r_2}^2 + \ldots \\ &= \omega(m_1{r_1}^2 + m_2{r_2}^2 + \ldots)\end{aligned}$$

Therefore (from equation [6.13])

$$\text{Angular momentum} = I\omega \qquad [6.15]$$

The Principle of Conservation of Angular Momentum

The linear momentum of a body moving along a straight line stays constant as long as no resultant external force acts on it (section 2.7). On the other hand, if a body is rotating, it is its angular momentum that is conserved. This is known as **the principle of conservation of angular momentum** and it can be stated as:

> The total angular momentum of a system is constant unless an external torque acts on it.

The principle is readily demonstrated by a spinning skater. If she brings her arms in close to her body, her moment of inertia decreases (because some of her mass is now closer to her axis of rotation than it was previously) and her angular velocity increases to such an extent that her angular momentum ($I\omega$) is unchanged. It is left as an exercise for the reader to show that this results in the skater's rotational kinetic energy increasing in the same ratio as her angular velocity. The reader might also like to give some thought to what has provided this increase in energy.

6.9 THE ROTATIONAL FORM OF NEWTON'S SECOND LAW

If a torque is applied to a rigid body which is at rest, the body will start to rotate and will rotate with an ever increasing angular velocity, i.e. the application of the torque causes the body to have an angular acceleration. **Angular acceleration,** α, is defined as rate of change of angular velocity, i.e.

$$\alpha = \frac{d\omega}{dt}$$

where

α = angular acceleration (rad s^{-2}).

Thus, whereas in linear motion a force produces an acceleration which is related to the force through Newton's second law, in rotational motion a torque gives rise to an angular acceleration. The reader will not be surprised to learn therefore, that there is a rotational form of Newton's second law which relates torque and angular acceleration. It may be written as

$$T = I\alpha \qquad [6.16]$$

where

T = the applied torque (N m)

I = the moment of inertia of the body that is rotating (kg m^2)

α = the angular acceleration of the body (rad s^{-2}).

In a more general situation in which the moment of inertia is not constant this becomes

$$T = \frac{d}{dt}(I\omega) \qquad [6.17]$$

where $\frac{d}{dt}(I\omega)$ is the rate of change of angular momentum.

Equations [6.16] and [6.17] are the rotational forms of equations [2.2] and [2.1] respectively. Note that when there is no exernal torque (i.e. when $T = 0$) equation [6.17] reduces to $I\omega$ = a constant. Thus **the principle of conservation of angular momentum is merely a special case of the rotational form of Newton's second law.**

6.10 THE EQUATIONS OF ROTATIONAL MOTION

Equations [6.18]–[6.21] describe the motion of bodies which are moving with constant (uniform) angular acceleration.

$$\omega = \omega_0 + \alpha t \qquad [6.18]$$

$$\omega^2 = \omega_0{}^2 + 2\alpha\theta \qquad [6.19]$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad [6.20]$$

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t \qquad [6.21]$$

where

ω_0 = the angular velocity when $t = 0$

ω = the angular velocity at time t

α = the constant angular acceleration

θ = the angle turned through in time t. (Note that if the direction of motion reverses, θ is the net angle turned through.)

These equations are analogous to those which govern uniformly accelerated linear motion (equations [2.6]–[2.9]), with u, v, a and s replaced by ω_0, ω, α and θ respectively.

EXAMPLE 6.4

A flywheel is mounted on a horizontal axle which has a radius of 0.06 m. A constant force of 50 N is applied tangentially to the axle. If the moment of inertia of the system (flywheel + axle) is 4 kg m^2, calculate:

(a) the angular acceleration of the flywheel,

(b) the number of revolutions that the flywheel makes in 16 s assuming that it starts from rest.

Solution

The arrangement is shown in Fig. 6.12.

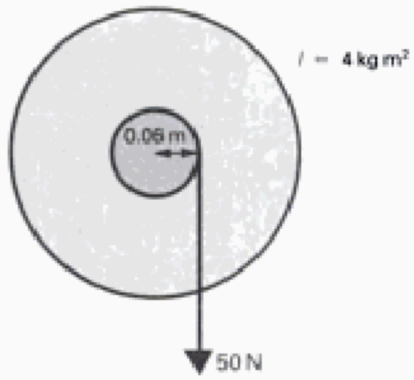


Fig. 6.12
Diagram for Example 6.4

(a) The torque, T, is given by

$$T = 50 \times 0.06 \qquad \text{(equation [3.1])}$$

i.e. $T = 3\,\text{N m}$

If $I(= 4\,\text{kg m}^2)$ is the moment of inertia of the system and α is the angular acceleration, then

$$T = I\alpha \qquad \text{(equation [6.16])}$$

$\therefore \quad 3 = 4\alpha$

i.e. $\alpha = 0.75\,\text{rad s}^{-2}$

(b) If θ is the angle turned through in time $t(= 16\,\text{s})$ and $\omega_0(= 0)$ is the initial angular velocity, then

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \text{(equation [6.20])}$$

$\therefore \quad \theta = 0 + \frac{1}{2} \times 0.75 \times 16^2$

i.e. $\theta = 96\,\text{rad}$

$1 \text{ revolution} = 2\pi \text{ rad}$

$\therefore$ Number of revolutions $= 96/2\pi \approx 15$

QUESTIONS 6C

1. A wheel of moment of inertia $0.30\,\text{kg m}^2$ mounted on a fixed axle accelerates uniformly from rest to an angular velocity of $60\,\text{rad s}^{-1}$ in 12 s. Find: **(a)** the angular acceleration, **(b)** the torque causing the wheel to accelerate, **(c)** the number of revolutions in this 12 s period.

2. A flywheel with a moment of inertia of $5.0\,\text{kg m}^2$ moves from rest under the action of a torque of 3.0 N m. Find: **(a)** the angular acceleration, **(b)** the angular velocity after 10 revolutions.

6.11 WORK DONE BY A TORQUE

Consider a rigid body turning through an angle θ as a result of a force F being applied to it (Fig. 6.13). Suppose that the axis of rotation passes through O and is

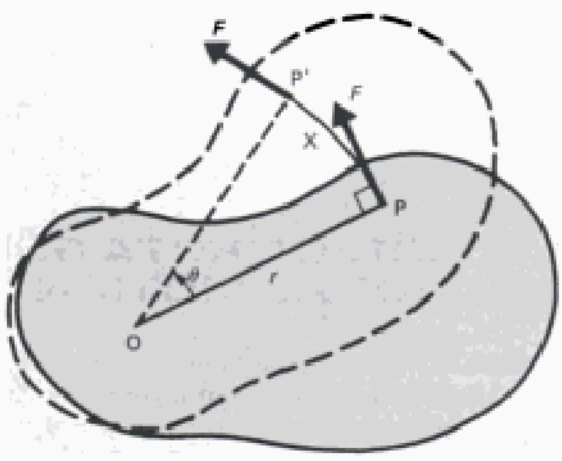


Fig. 6.13
Calculation of work done by a torque

perpendicular to the paper. Suppose also, that the perpendicular distance of the line of action of the force from the axis is constant and is equal to r. The force therefore gives rise to a constant torque, T, given by equation [3.1] as

$$T = Fr$$

As the body turns, the point of application of the force moves from P to P′ along the arc PXP′, and so moves a distance $r\theta$. The work done, W, is given by equation [5.1] as

$$W = Fr\theta$$

i.e. $$W = T\theta \qquad [6.22]$$

where

W = the work done (J)

T = the constant torque (N m)

θ = the angle turned through (rad).

Notes (i) The work done can have increased the rotational kinetic energy of the body, and/or have been used to overcome any resistive forces (e.g. friction) that are present.

(ii) If the torque is not constant, equation [6.22] is replaced by

$$W = \int T \, d\theta$$

QUESTIONS 6D

1. A constant force of 30 N is applied tangentially to the rim of a wheel mounted on a fixed axle and which is initially at rest. The wheel has a moment of inertia of 0.20 kg m^2 and a radius of 15 cm.
   (a) What is the torque acting on the wheel?
   (b) Find the work done on the wheel in 10 revolutions.
   (c) Assuming that no work is done against friction, use energy considerations to find the angular velocity of the wheel after 10 revolutions.

2. A disc and a hoop roll down a slope. They have the same mass and the same radius.
   (a) Which has the greater moment of inertia?
   (b) Does one loose more PE than the other?
   (c) Which acquires the greater speed?

6.12 COMPARISON OF ROTATIONAL AND TRANSLATIONAL MOTION

Each of the quantities that is used in the treatment of rotational motion is analogous to one that features in the description of translational motion. The various 'pairs' are listed in Table 6.1

ω = the angular velocity of the flywheel when the mass reaches the floor,

n_1 = the number of revolutions of the flywheel whilst the mass is falling, and

f = the work done against friction during each revolution, then

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + n_1 f \qquad [6.23]$$

If the flywheel makes a further n_2 revolutions before coming to rest, the work done against friction is $n_2 f$. This is done at the expense of the kinetic energy of the flywheel, and therefore

$$\tfrac{1}{2}I\omega^2 = n_2 f$$

Substituting for f in equation [6.23] gives

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + \frac{n_1}{n_2}\left(\tfrac{1}{2}I\omega^2\right) \qquad [6.24]$$

The velocity with which the mass hits the floor is twice its average velocity, and therefore

$$v = \frac{2h}{t} \quad \text{and} \quad \omega = \frac{2h}{rt}$$

where t is the time the mass takes to reach the ground and r is the radius of the axle. Substituting for v and ω in equation [6.24] and rearranging gives

$$I = mr^2\left(\frac{gt^2}{2h} - 1\right)\left(\frac{n_2}{n_1 + n_2}\right)$$

The mass m is found by using a balance; r and h are measured with vernier calipers and a metre rule respectively; t is measured with a stopwatch; g is known; and n_1 and n_2 are counted – hence I.

7

SIMPLE HARMONIC MOTION

7.1 DEFINITION, EQUATIONS, EXPLANATIONS

There are many types of vibration but perhaps the most common is that which is known as simple harmonic motion (SHM). It is important not only because there are many examples of it but also because all other vibrations can be treated as if they are composed of simple harmonic vibrations. It is the way in which the acceleration of a body depends on its position that determines the particular type of vibration a body is undergoing.

If a body moves in such a way that its acceleration is directed towards a fixed point in its path and is directly proportional to its distance from that point, the body is moving with simple harmonic motion.

It follows that the 'fixed point' is the equilibrium position, i.e. the position at which the body would come to rest if it were to lose all of its energy.

If a body is vibrating with simple harmonic motion, its motion can be described by an equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad [7.1]$$

where

$\frac{d^2x}{dt^2}$ = the acceleration of the body ($m\ s^{-2}$)

x = the displacement of the body from its equilibrium position, i.e. from the 'fixed point' in its path (m)

ω^2 = a positive constant (s^{-2})

Notes (i) The minus sign in equation [7.1] ensures that the acceleration is always directed towards the equilibrium position, as required by the definition.

(ii) The constant of proportionality is written as ω^2 (rather than ω) because of the connection with circular motion – see section 7.2.

Integrating equation [7.1] leads to

$$v = \pm\, \omega\sqrt{a^2 - x^2} \qquad [7.2]$$

and

$$x = a \cos \omega t \qquad [7.3]$$

where

v = the velocity of the body at time t (m s^{-1})

a = **the amplitude** of the motion, i.e. the maximum displacement from the equilibrium position (m).

Note Equation [7.3] requires that $x = a$ when $t = 0$. An alternative expression for x is $x = a \sin \omega t$; this requires that $x = 0$ when $t = 0$, i.e. that the motion is taken to start from the equilibrium position rather than the point of maximum positive displacement.

Timing may commence when the body is at any point of its oscillation. To take account of this the expression

$$x = a \sin (\omega t + \varepsilon)$$

is used, in which ε, the **initial phase angle** or **epoch**, is a constant expressed in radians and given by $\sin \varepsilon = x_0/a$, where x_0 is the value of x at $t = 0$. The reader should confirm that this reduces to $x = a \sin (\omega t + \pi/2) = a \cos \omega t$ when $x = a$ at $t = 0$, and reduces to $x = a \sin (\omega t + 0) = a \sin \omega t$ when $x = 0$ at $t = 0$.

The period T of the motion (i.e. the time for one complete oscillation) is given by

$$T = 2\pi/\omega \qquad [7.4]$$

Note that **for any particular system the period is independent of the amplitude**. For example, if the amplutide of oscillation of a simple pendulum is increased, its average speed increases and there is no change in the time it takes to complete an oscillation.*

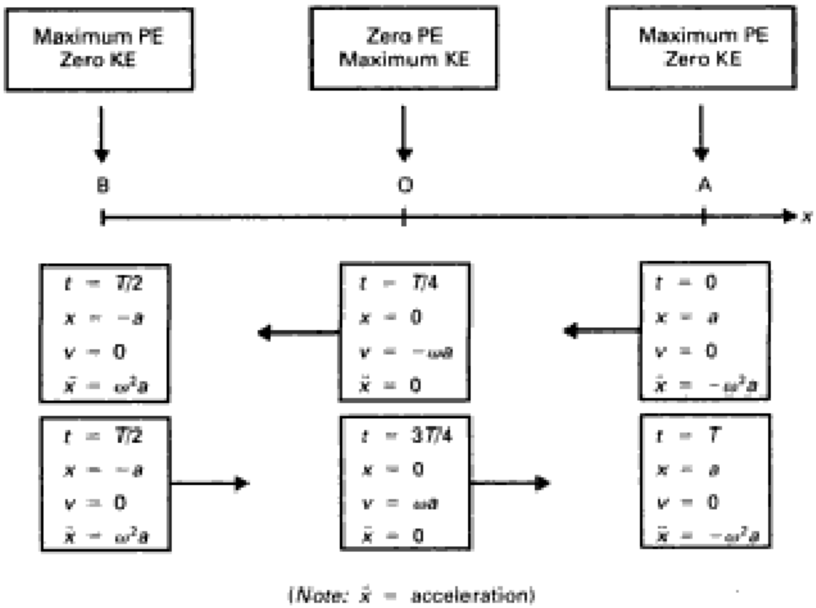
Fig. 7.1 illustrates the positional dependence of some of the parameters which are associated with the motion of a body whose oscillations are simple harmonic. The positive direction of x is, as usual, toward the right and therefore whenever the body is moving to the left its velocity is negative. Also, the acceleration is negative when it is directed towards the left. (Note that a negative acceleration must not be confused with retardation; the body slows down only when its acceleration is in the opposite direction to its velocity.)

As the body moves from A towards its equilibrium position, O, its speed increases and reaches a maximum at O. During this time its acceleration decreases to zero. Although there is no force acting on the body when it is at O, its inertia carries it through to B. From O to B there is a retarding force; the speed of the body decreases and is momentarily zero at B. As the body moves back to O its speed increases, and then decreases again from O to A.

Bearing in mind that the acceleration of a body of constant mass is proportional to the force acting on it, we see that a body which is moving with simple harmonic motion does so because there is a force acting on it which is proportional to the displacement of the body from its equilibrium position and is directed towards that

*The motion of a simple pendulum is not exactly simple harmonic and therefore this statement is only approximately true.

Fig. 7.1
Characteristics of SHM



position. Such is the case, for example, of a body oscillating on the end of a spring which obeys Hooke's law (section 11.1). If the body is pulled down below its equilibrium position and then released, a net upward force acts on it because the tension in the spring is greater than the weight of the body. The greater the downward displacement, the greater the tension and therefore the greater the upward directed force. When the body is above its equilibrium position its weight is greater than the tension and so the resultant force is downwards.

EXAMPLE 7.1

A particle is moving with SHM of period 8.0 s and amplitude 5.0 m. Find: (a) the speed of the particle when it is 3.0 m from the centre of its motion, (b) the maximum speed, (c) the maximum acceleration.

Solution

The equations for speed and acceleration involve ω; our first step therefore is to find ω.

$$T = 2\pi/\omega$$

$$\therefore \quad \omega = 2\pi/T = 2\pi/8.0 = 0.785\ s^{-1}$$

(a) $$v = \pm\omega\sqrt{a^2 - x^2}$$

$$\therefore \quad v = \pm 0.785\sqrt{5.0^2 - 3.0^2} = \pm 3.14\ \text{m s}^{-1}$$

i.e. Speed 3.0 m from centre $= 3.1\ \text{m s}^{-1}$

(b) It follows from equation [7.2] with $x = 0$ that the maximum speed v_{max} is given by

$$v_{max} = \omega a$$

$$\therefore \quad v_{max} = 0.785 \times 5.0 = 3.93\ \text{m s}^{-1}$$

i.e. Maximum speed $= 3.9\ \text{m s}^{-1}$

(c) It follows from equation [7.1] that the magnitude of the acceleration is greatest when $x = \pm a$ and is given by

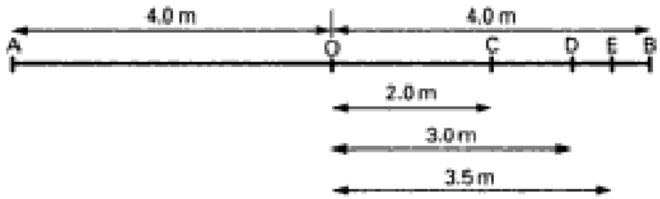
$$\text{maximum acceleration} = \omega^2 a$$
$$= (0.785)^2 \times 5.0 = 3.08\,\text{m s}^{-2}$$

i.e. Maximum acceleration $= 3.1\,\text{m s}^{-2}$

EXAMPLE 7.2

A particle is moving with SHM of period 24 s between two points, A and B (Fig. 7.2). Find the time taken for the particle to travel from: (a) A to B, (b) O to B, (c) O to C, (d) D to B, (e) C to E.

Fig. 7.2
Diagram for Example 7.2



Solution

Let t_{AB} = time from A to B, t_{BC} = time from B to C, etc.

(a) t_{AB} = time for half an oscillation = 12 s

(b) t_{OB} = time for quarter of an oscillation = 6 s

(c) Amplitude (a) = OA = OB = 4.0 m. $\omega = 2\pi/T = 2\pi/24 = 0.262\,\text{s}^{-1}$. Therefore by

$$x = a \sin \omega t$$
$$OC = 4.0 \sin (0.262\, t_{OC})$$
$$\therefore \quad 2.0 = 4.0 \sin (0.262\, t_{OC})$$
$$\therefore \quad \sin (0.262\, t_{OC}) = 0.5$$

(Note the use of radians, *not* degrees.)

$$\therefore \quad 0.262\, t_{OC} = \sin^{-1} (0.5) = \frac{\pi}{6} = 0.524\,\text{rad}$$
$$\therefore \quad t_{OC} = 2.0\,\text{s}$$

(d) $t_{DB} = t_{BD}$. Therefore by

(x is the displacement from O, it is not the distance from D to B.)

$$x = a \cos \omega t$$
$$OD = 4.0 \cos (0.262\, t_{DB})$$
$$\therefore \quad 3.0 = 4.0 \cos (0.262\, t_{DB})$$
$$\therefore \quad \cos (0.262\, t_{DB}) = 0.75$$
$$\therefore \quad 0.262\, t_{DB} = \cos^{-1} (0.75) = 0.723\,\text{rad}$$
$$\therefore \quad t_{DB} = 2.8\,\text{s}$$

(e) $t_{CE} = t_{OE} - t_{OC}$. Since t_{OC} has already been found, it remains to find t_{OE}. By

$$x = a \sin \omega t$$

$$OE = 4.0 \sin (0.262\, t_{OE})$$

$$\therefore \quad 3.5 = 4.0 \sin (0.262\, t_{OE})$$

$$\therefore \quad \sin (0.262\, t_{OE}) = 0.875$$

$$\therefore \quad 0.262\, t_{OE} = \sin^{-1} (0.875) = 1.065 \text{ rad}$$

$$\therefore \quad t_{OE} = 4.1 \text{ s}$$

$$\therefore \quad t_{CE} = t_{OE} - t_{OC} = 4.1 - 2.0 = 2.1 \text{ s}$$

QUESTIONS 7A

1. A particle is moving with SHM of period 16 s and amplitude 10 m. Find the speed of the particle when it is 6.0 m from its equilibrium position.

2. How far is the particle in question 1 from its equilibrium position 1.5 s after passing through it? What is its speed at this time?

3. A tuning fork has a frequency of 256 Hz. What is the maximum speed of the tips of the prongs if they each oscillate with SHM of amplitude 0.40 mm. (Assume that the tips of the prongs move in straight lines.)

4. A particle moves with SHM of period 4.0 s and amplitude 4.0 m. Its displacement from the equilibrium position is x. Find the time taken for it to travel: **(a)** from $x = 4.0$ m to $x = 3.0$ m, **(b)** from $x = -4.0$ m to $x = 3.0$ m, **(c)** from $x = 0$ to $x = 3.0$ m, **(d)** from $x = 1.0$ m to $x = 3.0$ m.

5. A particle moving with SHM has a speed of 8.0 m s^{-1} and an acceleration of 12 m s^{-2} when it is 3.0 m from its equilibrium position. Find: **(a)** the amplitude of the motion, **(b)** the maximum velocity, **(c)** the maximum acceleration.

7.2 RELATIONSHIP BETWEEN SHM AND CIRCULAR MOTION

Consider a particle P moving with constant angular velocity ω around the circumference of a circle of radius a (Fig. 7.3). Consider, in particular, the motion of N, the point at which the perpendicular from P meets the diameter AOB.

The acceleration of P is $\omega^2 a$ (see section 6.3) and is directed towards O. It follows that the x-component of the acceleration of P is

$$-\omega^2 a \cos \theta$$

(The acceleration is directed towards the left, i.e. it is in the negative direction of x, hence the inclusion of the minus sign.) Since N is always vertically below P, its acceleration d^2x/dt^2 is equal to the x-component of the acceleration of P and therefore is given by

$$\frac{d^2x}{dt^2} = -\omega^2 a \cos \theta \qquad [7.5]$$

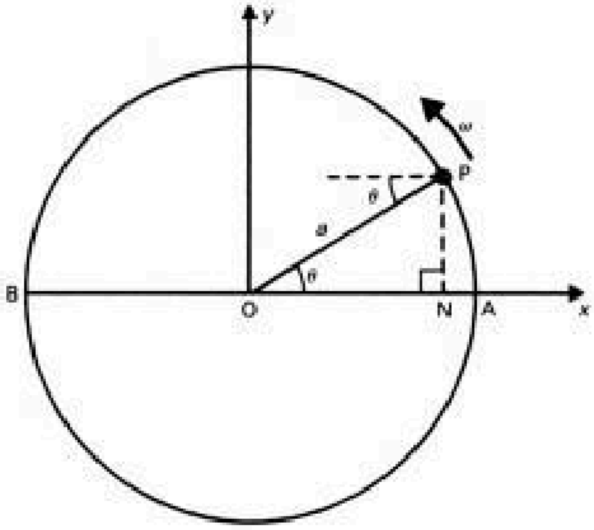


Fig. 7.3
To illustrate the relationship between SHM and circular motion

But $\cos\theta = ON/a$, and ON is equal to x, the displacement of N, and therefore $\cos\theta = x/a$. Substituting for $\cos\theta$ in equation [7.5] gives

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Since this is the equation of motion of a particle which is moving with simple harmonic motion, the motion of N is simple harmonic. N completes one cycle in the time it takes P to complete one revolution, and therefore

$$\text{Period of rotation of P} = \text{Period of oscillation of N} = \frac{2\pi}{\omega}$$

7.3 THE SIMPLE PENDULUM

Consider a pendulum bob, P, of mass m attached to a light, inextensible string of length L. Suppose that the pendulum is suspended from a fixed point, O, and that when the string is at an angle θ to the vertical the velocity of the bob is v (Fig. 7.4).

The forces acting on the bob are its weight, mg, and the tension, F, in the string.

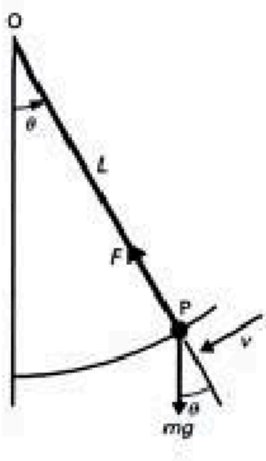


Fig. 7.4
To determine the period of oscillation of a simple pendulum

Consider the motion perpendicular to PO. By Newton's second law

$$mg \sin\theta = m\frac{dv}{dt}$$

The velocity, v, can be written in terms of the angular velocity $d\theta/dt$ as

$$v = -L\frac{d\theta}{dt}$$

(The minus sign is necessary because θ is measured from the vertical and therefore v is in the direction of θ decreasing.) Substituting for v gives

$$mg \sin\theta = -mL\frac{d^2\theta}{dt^2}$$

i.e. $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

If the amplitude of oscillation is small, θ is small and therefore $\sin\theta \approx \theta$, in which case, to a reasonable approximation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \qquad [7.6]$$

Since both g and L are positive constants, so also is g/L and therefore equation [7.6] is of the same form as equation [7.1]. It follows that, to a reasonable approximation, the motion of a simple pendulum is simple harmonic. Further, the positive constant ω^2 of equation [7.1] is equal to g/L and therefore, by equation [7.4], the period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad [7.7]$$

Notes

(i) Equation [7.7] does not involve m and therefore **the period of oscillation of a simple pendulum is independent of its mass**. This can be shown to be true no matter what the amplitude of oscillation.

(ii) Even for amplitudes of oscillation of as much as 15° the period calculated on the basis of equation [7.7] is accurate to within $\frac{1}{2}$%

7.4 DETERMINATION OF *g* USING A SIMPLE PENDULUM

A reasonably accurate determination of the acceleration due to gravity g can be made by measuring the period of oscillation and the length of a simple pendulum.

The pendulum, in the form of a small lead sphere attached to a suitable length (about 1 m) of sewing thread, should be suspended in the manner shown in Fig. 7.5. The wooden blocks should have well-defined right-angled edges at X so that there is no possibility of the pendulum swinging about more than one point.

Once the apparatus has been assembled the procedure outlined below should be followed.

(i) Measure the length L. (Note that the measurement is made to the centre of the bob.) Take care not to stretch the thread.

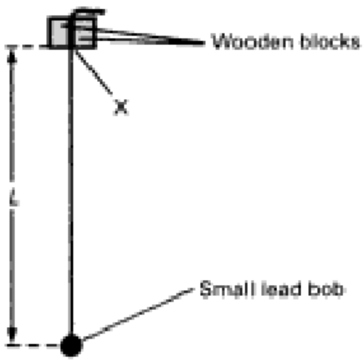


Fig. 7.5
Apparatus for the determination of ***g***

(ii) Displace the pendulum through about 5° and then release it so that it executes oscillations of small amplitude.

(iii) Determine the period T by using a stop-watch to time 50 oscillations. By using 50 oscillations, rather than one, the error that arises through being unable to start and stop the watch when the pendulum is exactly in the intended position is greatly reduced. The error which remains is minimized by making the timings to the mid-point of the motion because that is where the speed of the pendulum is greatest.

(iv) Repeat (i), (ii) and (iii) for about five more values of L.

From the theory of the simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\therefore \quad T^2 = \frac{4\pi^2}{g}L$$

It follows that the gradient of a graph of T^2 against L is $4\pi^2/g$, in which case g can be determined by plotting such a graph and measuring its gradient.

Notes (i) A graph of T against $\sqrt{L}$ has a gradient of $2\pi/\sqrt{g}$ and therefore such a graph could have been used to determine g. The reason for choosing to plot T^2 against L is that the graph is linear and its gradient is $4\pi^2/g$ even if there is an error in the measurement of L, providing it is a constant error.

(ii) The approximation made in deriving equation [7.7] leads to an error of less than 0.05% if the amplitude of oscillation does not exceed 5°. This is insignificant compared with the errors that are likely to be involved in the measurements of L and T.

7.5 A BODY ON A SPRING

The extension of a spring which obeys Hooke's law (section 11.1) is proportional to the tension which has produced it. Therefore

$$\text{Tension} = k \times \text{extension} \qquad [7.8]$$

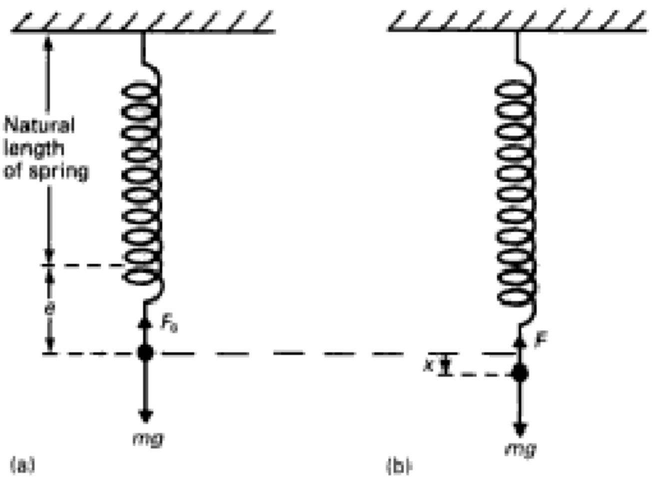
where

k = a constant of proportionality which is known as the **spring constant**. It is equal to the tension required to produce unit extension ($N\,m^{-1}$).

Suppose that a suspended spring which obeys Hooke's law has a body of mass m attached to its lower end. In Fig. 7.6(a) the body is at rest in its equilibrium position. There can be no resultant force acting on the body and therefore the tension F_0 is given by

$$F_0 = mg$$

Fig. 7.6
Oscillation of a body on a spring



It follows from equation [7.8] that since the extension is e

$$mg = ke \qquad [7.9]$$

Suppose now that the body is displaced downwards through a distance x (Fig. 7.6(b)). The body is no longer in equilibrium and feels a net upward force of $(F - mg)$, where F is the instantaneous value of the tension in the spring. Therefore by Newton's second law

$$F - mg = -m\frac{d^2x}{dt^2} \qquad [7.10]$$

(The minus sign is present because the resultant force on the body is directed upwards and therefore acts so as to decrease x.) By equation [7.8], since the total extension is now $(e + x)$

$$F = k(e + x)$$

Therefore from equation [7.10]

$$k(e + x) - mg = -m\frac{d^2x}{dt^2}$$

But, from equation [7.9], $mg = ke$, and therefore

$$kx = -m\frac{d^2x}{dt^2}$$

i.e.
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad [7.11]$$

Since both k and m are positive constants, so also is k/m and therefore equation [7.11] may be written as

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where ω^2 is a positive constant equal to k/m. This equation represents simple harmonic motion and therefore the motion of the body is simple harmonic. Since $\omega^2 = k/m$, equation[7.4] gives the period of the motion as

$$\text{Period} = 2\pi\sqrt{\frac{m}{k}} \qquad [7.12]$$

Except in the idealized case of a spring of zero mass, it is necessary to take account of the fact that the spring itself oscillates. It can be shown that m needs to be replaced by $(m + m_s)$, where m_s is a constant known as the **effective mass** of the spring. (Note that m_s is less than the actual mass of the spring because it is the lowest coil which oscillates with the full amplitude of the suspended body.) With this modification then, equation [7.12] becomes

$$\text{Period} = 2\pi\sqrt{\frac{m + m_s}{k}} \qquad [7.13]$$

7.6 DETERMINATION OF *g* BY USING A MASS ON AN OSCILLATING SPRING

From equation [7.9], $m = ke/g$. Substituting for m in equation [7.13] and squaring leads to

$$T^2 = 4\pi^2\left(\frac{ke/g + m_s}{k}\right)$$

where T is the period of oscillation. Removing the brackets gives

$$T^2 = \frac{4\pi^2}{g}e + \frac{4\pi^2 m_s}{k} \qquad [7.14]$$

Thus a graph of T^2 against e is linear (Fig. 7.7) and has a gradient of $4\pi^2/g$, and therefore enables g to be determined. Such a graph can be obtained by adding a number of different masses to the spring and measuring the static extension e which each produces, together with the corresponding period of oscillation T.

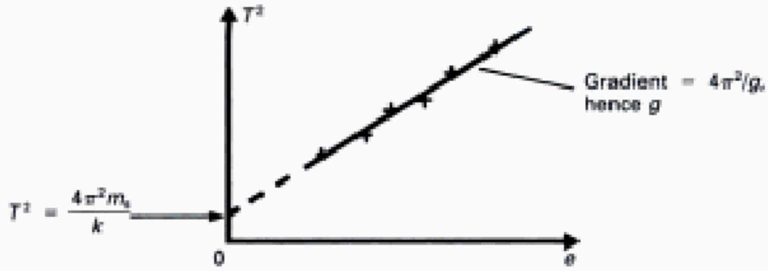


Fig. 7.7
T^2 against e for a body oscillating on a spring

Note When $e = 0$, $T^2 = 4\pi^2 m_s/k$, and therefore m_s can be found providing k is known. The value of k is found by plotting m against e since, by equation [7.9], the gradient of such a graph is k/g.

7.7 THE WORK DONE IN STRETCHING A SPRING

The tension, F, in a spring whose extension is x and which obeys Hooke's law is given by

$$F = kx$$

where k is the spring constant. If the extension is increased by δx where δx is so small that F can be considered constant, then (by equation [5.1]) the work done, δW, is given by

$$\delta W = F\,\delta x$$

i.e. $$\delta W = kx\,\delta x$$

The total work done in increasing the extension from 0 to x, i.e. the **elastic potential energy** stored in the spring when its extension is x, is given by W, where

$$W = \int_0^x kx\,\mathrm{d}x$$

i.e. $$W = \tfrac{1}{2}kx^2$$

Substituting for k from $F = kx$ gives $W = \frac{1}{2}Fx$; substituting for x gives $W = F^2/(2k)$, i.e.

$$W = \tfrac{1}{2}kx^2 = \tfrac{1}{2}Fx = \frac{F^2}{2k}$$

CONSOLIDATION

A body is moving with SHM if its acceleration is directed towards a fixed point in its path and is directly proportional to its distance from that point.

The amplitude of the motion (a) is the maximum displacement from the equilibrium position.

If a body is moving with SHM, its motion can be described by an equation of the form

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x \qquad [7.1]$$

The converse is also true, and therefore if we are required to show that a body is moving with SHM, it is sufficient to show that its motion is described by an equation of the same form as equation [7.1].

$$v = \pm\omega\sqrt{a^2 - x^2} \qquad [7.2]$$

$$x = a\cos\omega t \qquad (\text{if } x = a \text{ when } t = 0)$$

$$x = a\sin\omega t \qquad (\text{if } x = 0 \text{ when } t = 0)$$

$$T = 2\pi/\omega \qquad f = 1/T$$

Magnitude of maximum acceleration $= \omega^2 a$ (at $x = \pm a$)

Magnitude of maximum velocity $= \omega a$ (at $x = 0$)

To obtain v in terms of t substitute for x in equation [7.2].

The period (T) is independent of the amplitude (a).

The first step in solving many SHM problems is to find ω.

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(Simple pendulum)}$$

$$T = 2\pi\sqrt{\frac{m + m_s}{k}} \qquad \text{(Mass on a spring)}$$

$$W = \tfrac{1}{2}kx^2 = \tfrac{1}{2}Fx = \frac{F^2}{2k}$$

8

GRAVITATION AND GRAVITY

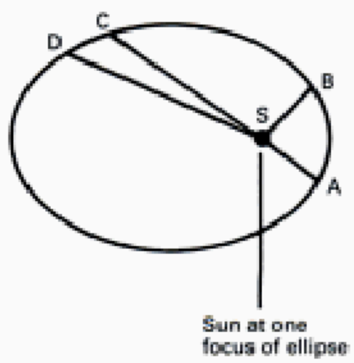
8.1 KEPLER'S LAWS

Throughout the last few decades of the sixteenth century, Tycho Brahe made precise measurements of the positions of the planets and various other bodies in the Solar System. Johannes Kepler made a detailed analysis of the measurements, and by 1619 had announced three laws which describe planetary motion.

1. The orbit of each planet is an ellipse which has the Sun at one of its foci.
2. Each planet moves in such a way that the (imaginary) line joining it to the Sun sweeps out equal areas in equal times.
3. The squares of the periods of revolution of the planets about the Sun are proportional to the cubes of their mean distances from it.

Fig. 8.1 illustrates law 2 but gives an exaggerated idea of the eccentricity of most planetary orbits. With the exceptions of Mercury and Pluto, the planets follow very nearly circular paths.

Fig. 8.1
Illustration of Kepler's second law



The average speed of the planet between A and B is greater than between C and D and in such a way that

Area ABS = Area CDS

8.2 NEWTON'S LAW OF UNIVERSAL GRAVITATION

About fifty years after Kepler's laws had been announced, Isaac Newton showed that any body which moves about the Sun in accordance with Kepler's second law must be acted on by a force which is directed towards the Sun. He was able to show that if this force is inversely proportional to the square of the distance of the body from the Sun, then the body must move along a path which is a conic section (i.e. elliptical, circular, parabolic or hyperbolic). Newton then showed that when the path is elliptical or circular the period of revolution is given by Kepler's third law. Thus, a centrally directed inverse square law of attraction is consistent with all three of Kepler's laws. Newton proposed that the planets are held in their orbits by just such a force. He further proposed that it is the same type of force which maintains the Moon in its orbit about the Earth, and which the Earth exerts on a body when it causes it to fall to the ground. Extending these ideas, Newton proposed that every body in the Universe attracts every other with a force which is inversely proportional to the square of their separation. His next step was to turn his attention to the masses of the bodies involved.

According to Newton's third law, if the Earth exerts a force on a body, then that same body must exert a force of equal magnitude on the Earth. Newton knew that the force exerted on a body by the Earth is proportional to the mass of the body. He saw no reason why the body should behave any differently from the Earth, in which case the force exerted on the Earth by the body must be proportional to the mass of the Earth. Since the two forces are equal, a change in one must be accompanied by an equal change in the other. It follows that each force must be proportional to the product of the Earth's mass and the mass of the body.

The ideas of the last two paragraphs are summarised in **Newton's law of universal gravitation**.

> Every particle in the Universe attracts every other with a force which is proportional to the product of their masses and inversely proportional to the square of their separation.

Thus

$$F = G\frac{m_1 m_2}{r^2} \qquad [8.1]$$

where

F = the gravitational force of attraction between two particles whose masses are m_1 and m_2, and which are a distance r apart

G = a constant of proportionality known as the **universal gravitational constant** ($= 6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$).

Note Equation [8.1] is concerned with particles (i.e. point masses) but, in the circumstances listed below, it can also be used for bodies of masses m_1 and m_2 whose centres are a distance r apart.

(a) It is valid for two bodies of any size provided that they each have spherical symmetry. (The Sun and the Earth is a good approximation.)

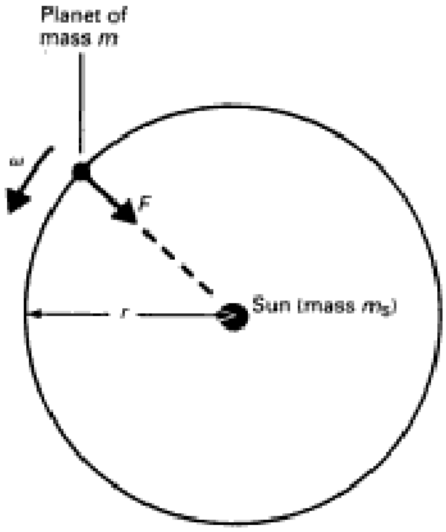
(b) It is a good approximation when one body has spherical symmetry and the other is small compared with the separation of their centres (e.g. the Earth and a brick).

(c) It is a good approximation when neither body has spherical symmetry, but where both are small compared with the separation of their centres (e.g. two bricks a few metres apart).

8.3 TO SHOW THAT KEPLER'S THIRD LAW IS CONSISTENT WITH $F = G\frac{m_1 m_2}{r^2}$

Consider a planet of mass m moving about the Sun in a circular* orbit of radius r. Suppose that the mass of the Sun is m_s and that the angular velocity of the planet is ω (Fig. 8.2).

Fig. 8.2
Force on planet in circular motion around the Sun



The force F which provides the centripetal acceleration $\omega^2 r$ (section 6.3) is given by Newton's second law as

$$F = m\omega^2 r \qquad [8.2]$$

By Newton's law of universal gravitation (equation [8.1])

$$F = G\frac{mm_s}{r^2}$$

Therefore, from equation [8.2]

$$G\frac{mm_s}{r^2} = m\omega^2 r$$

But $\omega = 2\pi/T$, where T is the period of revolution of the planet, and therefore

$$G\frac{mm_s}{r^2} = m\frac{4\pi^2}{T^2}r$$

i.e. $$T^2 = \frac{4\pi^2}{Gm_s}r^3$$

Since G, m_s and π have the same values no matter which planet is being considered,

$$T^2 \propto r^3$$

This is Kepler's third law; it has been derived on the basis of Newton's law of universal gravitation and therefore the two laws are consistent.

*The mathematics required to treat the general case of an elliptical orbit is beyond the scope of this book but leads to the same result.

8.4 NEWTON'S TEST OF THE INVERSE SQUARE LAW

The last two sections make it clear that Newton's law of universal gravitation is consistent with Kepler's laws of planetary motion. However, the forces which hold the planets in their orbits are due, in every case, to the Sun. In order to show that gravitational attraction is universal, Newton needed to test it in circumstances which did not involve the Sun. The obvious test was to apply his ideas to the Earth–Moon system.

If a body of mass m is at the surface of the Earth, the force acting on the body is its weight mg. This same force is given by the law of universal gravitation as

$$G\frac{mm_E}{r_E^2}$$

where m_E and r_E are respectively the mass and radius of the Earth. Therefore

$$G\frac{mm_E}{r_E^2} = mg$$

i.e.

$$G = \frac{gr_E^2}{m_E} \qquad [8.3]$$

The law of universal gravitation gives the force exerted by the Earth on the Moon in its orbit as

$$G\frac{m_M m_E}{r_M^2}$$

where m_M is the mass of the Moon and r_M is the radius of its orbit. It is this force which provides the Moon's centripetal acceleration $\omega^2 r_M$, and therefore

$$G\frac{m_M m_E}{r_M^2} = m_M\,\omega^2\,r_M$$

i.e.

$$G\frac{m_E}{r_M^2} = \omega^2 r_M$$

But ω, the angular velocity of the Moon, is equal to $2\pi/T$ where T is its period of revolution about the Earth, and therefore

$$G\frac{m_E}{r_M^2} = \frac{4\pi^2\,r_M}{T^2}$$

Substituting for G from equation [8.3] leads to

$$g = \frac{4\pi^2\,r_M^3}{T^2\,r_E^2} \qquad [8.4]$$

The value of r_E which was available to Newton was poor by present-day standards. Even so, equation [8.4] gave a value for g that was sufficiently close to the accepted value for Newton to conclude that the Earth exerted the same type of force on the Moon as the Sun did on the planets.

QUESTIONS 8A

1. Find the gravitational force of attraction between two 10 kg particles which are 5.0 cm apart.
($G = 6.7 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$.)

2. The average orbital radii about the Sun of the Earth and Mars are 1.5×10^{11} m and 2.3×10^{11} m respectively. How many (Earth) years does it take Mars to complete its orbit?

3. Calculate the mass of the Earth by considering the force it exerts on a particle of mass m at its surface.

(Radius of Earth = 6.4×10^3 km, $g = 9.8\,\text{m s}^{-2}$, $G = 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$.)

8.5 THE MASS OF THE EARTH

In 1798, a hundred and twenty one years after Newton had proposed the law of universal gravitation, Henry Cavendish made the first laboratory determination of the value of G.* Once this had been done, it was possible to obtain a value for the mass of the Earth on the basis of equation [8.3]. (Newton had worked the calculation in the opposite direction. He estimated G by using a value for m_E which was based on a guess at the density of the Earth.)

8.6 DEPENDENCE OF THE ACCELERATION DUE TO GRAVITY ON DISTANCE FROM THE CENTRE OF THE EARTH

The density of the Earth varies with depth but is largely independent of direction. It follows that the Earth can be treated as being made up of a large number of spherical shells of uniform density. This is useful, for it can be shown that:

(i) the acceleration due to gravity **outside** a spherical shell of uniform density is the same as it would be if the entire mass of the shell were concentrated at its centre, and

(ii) the acceleration due to gravity at all points **inside** a spherical shell of uniform density is zero.

These results will now be used to obtain expressions for the acceleration due to gravity both above and below the surface of the Earth.

Outside the Earth (i.e. $r > r_E$)

The acceleration due to gravity, g, at the surface of the Earth is given by rearranging equation [8.3] as

$$g = G\frac{m_E}{{r_E}^2} \qquad [8.5]$$

where m_E and r_E are respectively the mass and the radius of the Earth.

It follows from (i) that the acceleration due to gravity at a point outside the Earth has the value it would have if the entire mass of the Earth were at its centre. Therefore, by analogy with equation [8.5], the acceleration due to gravity g' at a distance r from the centre of the Earth when $r > r_E$ is given by

$$g' = G\frac{m_E}{r^2} \qquad [8.6]$$

*Cavendish had two lead spheres attached (one) to each end of a horizontal beam suspended by a silvered copper wire. When two larger spheres were brought up to the smaller ones, they deflected under the action of the gravitational force and twisted the suspension. The wire had been calibrated previously so that the strength of the force could be determined by measuring the angle through which the wire was twisted. An improved version of the experiment was performed by Boys in 1895.

Dividing equation [8.6] by equation [8.5] leads to

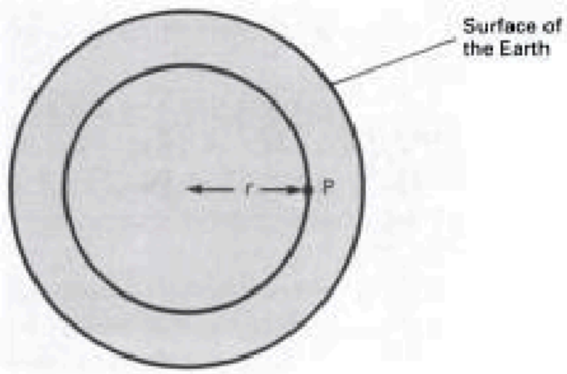
$$g' = \frac{r_E^2}{r^2} g \qquad [8.7]$$

Inside the Earth (i.e. $r < r_E$)

Consider a point P (Fig. 8.3) which is inside the Earth and at a distance r from its centre. From (ii) the acceleration due to gravity g' at P is due only to the sphere of radius r. If the mass of this sphere is m, then by analogy with equation [8.5]

$$g' = G\frac{m}{r^2} \qquad [8.8]$$

Fig. 8.3
To calculate the acceleration due to gravity inside the Earth



If the Earth is assumed to have uniform density*, ρ, then

$$m = \tfrac{4}{3}\pi r^3\rho \quad \text{and} \quad m_E = \tfrac{4}{3}\pi r_E{}^3\rho$$

and therefore

$$\frac{m}{m_E} = \frac{r^3}{r_E{}^3}$$

Substituting for m in equation [8.8] gives

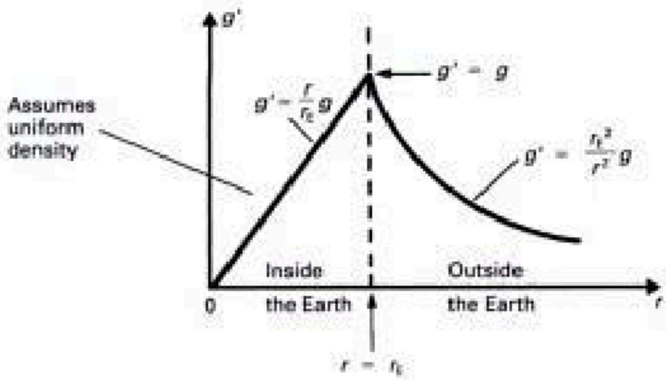
$$g' = G\frac{m_E r}{r_E{}^3}$$

Replacing $Gm_E/r_E{}^2$ by g (equation [8.5]) gives

$$g' = \frac{r}{r_E} g \qquad [8.9]$$

The variation of g' as a function of r on the basis of equation [8.7] and equation [8.9] is shown in Fig. 8.4. Note that each of these equations reduces to $g' = g$ when $r = r_E$.

Fig. 8.4
Variation of **g** with distance from Earth's core



*As has been stated at the beginning of this section, the density of the Earth is not uniform. It is, however, normal practice, at this level, to assume that it is in carrying out this calculation.

8.7 ESCAPE VELOCITY

If a ball is thrown upwards from the surface of the Earth, its speed decreases from the moment it is projected due to the retarding effect of the Earth's gravitational field. The height which the ball ultimately attains depends on the speed with which it is projected – the greater the speed, the greater the height. If the ball were to be required to escape from the Earth, it would have to be projected with a velocity which is at least great enough for the ball to reach infinity before coming to rest. The **minimum** velocity that achieves this is known as the **escape velocity**.

Consider a body of mass m being projected upwards with velocity v from the Earth's surface. When the body is at a distance r from the centre of the Earth it will feel a gravitational force of attraction, F, due to the Earth and given by equation [8.1] as

$$F = G\frac{mm_E}{r^2}$$

where m_E is the mass of the Earth. In moving a small distance δr against this force, the work done δW at the expense of the kinetic energy of the body is given by equation [5.1] as

$$\delta W = G\frac{mm_E}{r^2}\,\delta r$$

Therefore the total work done, W, in moving from the Earth's surface (where $r = r_E$) to infinity (where $r = \infty$) is given by

$$W = \int_{r_E}^{\infty} G\frac{mm_E}{r^2}\,dr$$

i.e. $$W = Gmm_E\left[\frac{-1}{r}\right]_{r_E}^{\infty}$$

i.e. $$W = G\frac{mm_E}{r_E}$$

If the body is to be able to do this amount of work (and so escape), it needs to have at least this amount of kinetic energy at the moment it is projected. The escape velocity v is therefore given by

$$\tfrac{1}{2}mv^2 = G\frac{mm_E}{r_E}$$

i.e. $$v = \sqrt{\frac{2Gm_E}{r_E}} \qquad [8.10]$$

Substituting known values into equation [8.10] leads to $v \approx 11\ \text{km s}^{-1}$.

Notes (i) This calaculation applies only to bodies which are not being driven, i.e. to projectiles. A body which is in powered flight does not have to rely on its initial kinetic energy to overcome the Earth's gravitational attraction, and therefore need never reach the escape velocity.

(ii) The escape velocity does not depend on the direction of projection. This is because the kinetic energy a body loses in reaching any particular height depends only on the height concerned and not on the path taken to reach it.

8.8 SATELLITE ORBITS

Consider a satellite of mass m orbiting the Earth with speed v along a circular path of radius r. The centripetal force, mv^2/r, is provided by the gravitational attraction of the Earth, and is given by Newton's law of universal gravitation as Gmm_E/r^2, where m_E is the mass of the Earth. Therefore

$$\frac{mv^2}{r} = G\frac{mm_E}{r^2}$$

i.e.
$$v = \sqrt{\frac{Gm_E}{r}} \qquad [8.11]$$

The orbital period, T, is given by $T = 2\pi r/v$, and therefore by equation [8.11]

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}} \qquad [8.12]$$

Since both G and m_E are constants, it follows from equations [8.11] and [8.12] that **both the speed and the orbital period of an Earth satellite depend only on the radius of its orbit**. Two situations are of particular interest.

An Orbit Close to the Earth's Surface

To a good approximation, for a satellite which is less than about 200 km above the Earth we may put $r = r_E$, where r_E is the radius of the Earth. Substituting for r in equations [8.11] and [8.12] gives

$$v = \sqrt{\frac{Gm_E}{r_E}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

Taking $G = 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$, $m_E = 6.0 \times 10^{24}\,\text{kg}$ and $r_E = 6.4 \times 10^6\,\text{m}$, we find

$$v = 7.9\,\text{km s}^{-1} \quad \text{and} \quad T = 85\,\text{minutes}$$

Geostationary (synchronous) Orbit

A satellite with an orbital period of 24 hours will always be at the same point above the Earth's surface (providing, of course, it is above the equator and is moving in the same direction as the Earth is rotating). Satellites of this type can be used to relay television signals and telephone messages (by radio link) from one point on the Earth's surface to another. Examples of these communications satellites are Syncom 2, Syncom 3 and Early Bird.

Substituting $G = 6.7 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$, $m_E = 6.0 \times 10^{24}\,\text{kg}$ and $T = 24$ hours ($= 8.64 \times 10^4\,\text{s}$) in equation [8.12] gives

$$r = 42\,400\,\text{km}$$

Having calculated r we can use equation [8.11] (or $2\pi r = vT$) to calculate v. We find

$$v = 3.1\,\text{km s}^{-1}$$

Since $r_E = 6.4 \times 10^6\,\text{m}$, the height above the Earth's surface of the geostationary orbit is

$$42\,400 - 6400 = 36\,000\,\text{km}$$

Notes

(i) Many artificial satellites actually move in elliptical orbits. For example, the orbit of Sputnik 1 took it from less than 250 km to over 900 km above the surface of the Earth. This is not so far from circular as it might at first seem. The radius of the Earth is about 6400 km, and therefore the semi-major axis of the ellipse was only about 10% bigger than the semi-minor axis.

(ii) The Moon, of course, is an Earth satellite and therefore equations [8.11] and [8.12] also apply to the Moon.

(iii) When a satellite is to be placed in orbit it is first carried to the desired height by rocket. It is then given the necessary tengential velocity (v) by firing rocket engines which are aligned parallel to the Earth's surface. If the satellite is still moving upwards when it reaches orbital height, it needs also to be given a downward directed thrust at this stage.

8.9 THE VARIATION OF *g* WITH LATITUDE

The acceleration due to gravity at the equator is 9.78 m s^{-2}, whereas at the poles it is 9.83 m s^{-2}. There are two main causes of this variation.

(i) The equatorial radius of the Earth is greater than the polar radius. Therefore a body at the equator is slightly further away from the centre of the Earth and consequently feels a smaller gravitational attraction. This accounts for 0.02 m s^{-2} of the observed difference of 0.05 m s^{-2}.

(ii) Because the Earth rotates, its gravitational pull on a body at the equator has to provide the body with a centripetal acceleration of 0.03 m s^{-2}. This does not apply at the poles.

8.10 WEIGHTLESSNESS

Consider an object of mass m hanging from a spring balance which is itself hanging from the roof of a lift (Fig. 8.5). The body is subjected to a downward directed force mg due to the Earth, and an upward directed force T, say, due to the tension in the spring. The net downward force is $(mg - T)$, and therefore by Newton's second law

$$mg - T = ma \qquad [8.13]$$

where a is the downward directed acceleration of the body.

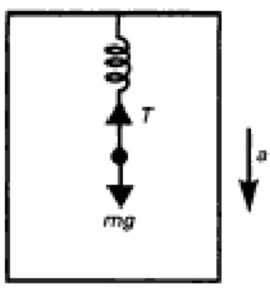


Fig. 8.5
Object suspended by a spring in a lift

If the lift is stationary or is moving with constant speed, $a = 0$ and therefore, by equation [8.13], $T = mg$, i.e. the balance registers the weight of the body as mg. However, if the lift is falling freely under gravity, both it and the body have a downward directed acceleration of g, i.e. $a = g$. It follows from equation [8.13] that $T = 0$, i.e. the balance registers the weight of the body as zero. It is usual to refer to a body in such a situation as being **weightless**. The term should be used with care; a gravitational pull of magnitude mg acts on the body whether it is in free fall or not, and therefore, in the strictest sense it has weight even when in free fall. The reason it is said to be weightless is that, whilst falling freely, it exerts no force on its support. Similarly, a man standing on the floor of a lift would exert no force on the floor if the lift were in free fall. In accordance with Newton's third law, the floor of the lift would exert no upward push on the man and therefore he would not have the sensation of weight.

An astronaut in an orbiting spacecraft has a centripetal acceleration equal to g', where g' is the acceleration due to gravity at the height of the orbit. The spacecraft has the same centripetal acceleration. The astronaut therefore has no acceleration relative to his spacecraft, i.e. he is weightless.

Note A body is weightless in the strictest sense only at a point where there is no gravitational field. An example of such a point is to be found between the Earth and the Moon where the two gravitational fields cancel.

8.11 GRAVITATIONAL POTENTIAL AND POTENTIAL ENERGY

The gravitational potential at a point in a gravitational field is defined as being numerically equal to the work done in bringing a unit mass from infinity (where the potential is zero) to that point.

Thus

$$U = \frac{W}{m} \qquad [8.14]$$

where

U = the gravitational potential at some point ($\mathrm{J\,kg^{-1}}$)

W = the work done in bringing a mass m from infinity to that point.

It has been shown in section 8.7, that the work which has to be done to take a mass m from the surface of the Earth to infinity is Gmm_E/r_E, where m_E and r_E are respectively the mass and the radius of the Earth. The work required to accomplish the reverse process, i.e. to bring the same body from infinity to the surface of the Earth, is therefore $-Gmm_E/r_E$. It follows from equation [8.14] that the gravitational potential U_E at the surface of the Earth is given by

$$U_E = -G\frac{m_E}{r_E} \qquad [8.15]$$

Notes (i) The minus sign in equation [8.15] indicates that the gravitational potential at the surface of the Earth is less than that at infinity. It follows that a body at infinity would 'fall' towards the Earth; a body on the Earth does not 'fall' to infinity.

(ii) It follows from equation [8.15] that in general

$$U = -G\frac{m}{r}$$

where U is **the gravitational potential** due to a body of mass m at a point outside the body and at a distance r from its centre. (This assumes that the body has spherical symmetry and/or is small compared with r.)

(iii) **The gravitational potential energy** of a body of mass m at a point where the gravitational potential is U is given by

$$\mathbf{PE} = m\mathbf{U}$$

This follows from the definition of gravitational potential and because the potential at infinity is zero.

8.12 GRAVITATIONAL FIELD STRENGTH

The gravitational field strength at a point in a gravitational field is defined as the force per unit mass acting on a mass placed at that point.

Thus

$$g = \frac{F}{m}$$

where

g = gravitational field strength ($\text{N kg}^{-1} = \text{m s}^{-2}$)

F = the force acting on a mass m

The same symbol (g) is used for gravitational field strength as for acceleration due to gravity. This is because they are one and the same thing, i.e. the field strength at a point in a gravitational field is equal to the gravitational acceleration of any mass placed at that point. We shall illustrate this for the particular case of the Earth. The gravitational force acting on a mass m at the Earth's surface is its weight, mg, where g is the acceleration due to gravity, and therefore the force per unit mass, i.e. the gravitational field strength, is mg/m – which is also g.

Note Gravitational field strength is a vector quantity. Its direction is that in which a mass would move under the influence of the field, i.e. towards a point of lower gravitational potential.

The gravitational field of the Earth is shown in Fig. 8.6. Note that it is a radial field directed towards the Earth and that it is stronger (field lines closer together) close to the Earth's surface than it is farther away. Over a limited area of the Earth's surface (an area that can be considered flat) the field can be considered to be uniform (field lines equally spaced) – see Fig. 8.7.

Field Strength Due to a Point Mass

The force on a point mass m' at a distance r from a point mass m is given by Newton's law of universal gravitation as F where

$$F = G\frac{mm'}{r^2}$$

Fig. 8.6
The gravitational field of the Earth

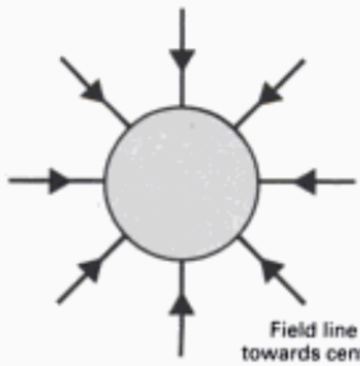
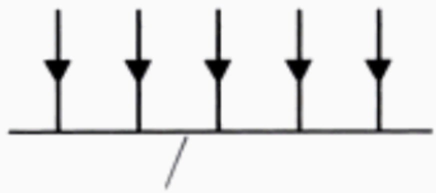


Fig. 8.7
The gravitational field of the Earth over a small area



The force per unit mass, i.e. the gravitational field strength, g, at a distance r from the point mass m is therefore given by

$$g = \frac{F}{m'}$$

i.e. $$g = G\frac{m}{r^2} \qquad [8.16]$$

Notes (i) Equation [8.16] also applies outside an extended body of mass m at a distance r from its centre providing the body has spherical symmetry and/or is small compared with r.

(ii) When m = the mass of the Earth, m_E, and r = the radius of the Earth, r_E, equation [8.16] becomes

$$g = G\frac{m_E}{r_E^2} \qquad [8.17]$$

where g is the field strength (acceleration due to gravity) at the surface of the Earth. We arrived at this same result in section 8.6 (equation [8.5]) by thinking of g as the acceleration due to gravity. Equation [8.17] is particularly useful for it is often necessary when solving examination questions to express g in terms of G and vice versa.

8.13 THE ANALOGY BETWEEN GRAVITY AND ELECTRICITY

Electric field strength is force per unit charge – gravitational field strength is force per unit mass. The definition of electrical potential involves the work done in bringing a unit (positive) charge from infinity – that of gravitational potential involves the work done in bringing a unit mass from infinity. The reader may find it useful to compare results (i) and (ii) of section 8.6 with the way in which the electric field of a hollow sphere varies with distance from the centre of the sphere (Fig. 39.8).

Table 8.1. Gravitational and electrical quantities compared

Gravitational quantity	Electrical quantity
$U = \frac{W}{m}$	$V = \frac{W}{Q}$
$g = \frac{F}{m}$	$E = \frac{F}{Q}$
$g = -\frac{dU}{dx}$	$E = -\frac{dV}{dx}$

Table 8.2. Comparison of effects of point masses and point charges

Gravitational quantity	Electrical quantity
$U = -G\frac{m}{r}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
$g = G\frac{m}{r^2}$	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
$F = G\frac{m_1 m_2}{r^2}$	$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$

In Table 8.1 expressions for gravitational potential and field strength are given together with the analogous electrical expressions. Table 8.2 compares the gravitational field strength and the gravitational potential at a distance r from a point mass m with the analogous electrical quantities at a distance r from a point charge Q. Note also the similarity between the expression for the gravitational force between two point masses with that for the electrical force between two point charges in vacuum.

Notes

(i) There is no gravitational analogue of electrical permittivity, i.e. the gravitational force between two masses does not depend on the medium in which they are situated.

(ii) The gravitational force, unlike the electrical force, is always attractive.

8.14 TO SHOW THAT $g = -dU/dx$

Suppose that a particle of mass m is moved by a force F from A to B in a gravitational field of strength g (Fig. 8.8). Suppose also that AB $= \delta x$, where δx is so small that F can be considered constant between A and B.

The work done δW in going from A to B is given by

$$\delta W = F\delta x$$

By definition $g = -F/m$ (the minus sign is necessary because g and F are oppositely directed), therefore

$$\delta W = -mg\delta x \qquad [8.18]$$

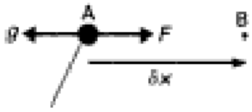


Fig. 8.8
To establish the relationship between ***g*** and ***U***

By definition, the increase in gravitational potential, δU, in moving the mass m from A to B is given by

$$\delta U = \delta W/m$$

Therefore, by equation [8.18]

$$\delta U = -g\delta x$$

Therefore, in the limit

$$g = -\mathrm{d}U/\mathrm{d}x$$

EXAMPLE 8.1

Use the following notation: m_E = mass of Earth, r_E = radius of Earth, g = acceleration due to gravity at surface of Earth, G = universal gravitational constant.

(a) Write down an expression for the gravitational potential at the Earth's surface.

(b) By how much would the gravitational PE of a body of mass m increase if it were moved from the Earth's surface to infinity.

(c) Hence find an expression for the minimum velocity with which a body could be projected from the Earth's surface and never return: (i) which involves G, (ii) which involves g

Solution

(a) $$\text{Gravitational potential} = -G\frac{m_E}{r_E}$$

(b) The gravitational potential at infinity is zero and therefore 'in moving to infinity' the potential increases by Gm_E/r_E. Therefore

$$\text{Increase in PE of mass } m = G\frac{mm_E}{r_E}$$

(c) (i) If a projectile of mass m is to have just sufficient KE to reach infinity from the Earth's surface, then (since decrease in KE = increase in PE) its velocity v on leaving the surface must be given by

$$\tfrac{1}{2}mv^2 = G\frac{mm_E}{r_E} \qquad \text{i.e.} \quad v = \sqrt{\frac{2Gm_E}{r_E}}$$

Since the projectile has just sufficient KE to reach infinity, this is the minimum velocity that will prohibit its return, i.e.

$$\text{Minimum velocity} = \sqrt{\frac{2Gm_E}{r_E}} \qquad [8.19]$$

(ii) The acceleration due to gravity, g, at the surface of the Earth is given by

$$g = G\frac{m_E}{r_E^2} \qquad [8.17]$$

$$\therefore \quad G\frac{m_E}{r_E} = gr_E$$

Substituting in equation [8.19] gives

$$\text{Minimum velocity} = \sqrt{2gr_E}$$

EXAMPLE 8.2

Using the notation of Example 8.1, find expressions for: (a) the gravitational PE of a satellite of mass m orbiting the Earth at a distance r from its centre, (b) the KE of the satellite, (c) the total energy of the satellite. (d) Explain how each of these quantities would change if the orbit of the satellite were so low that it encountered a considerable amount of air resistance.

Solution

(a) At a distance r from the centre of the Earth

$$\text{Gravitational potential} = -G\frac{m_E}{r}$$

$$\therefore \quad \text{Gravitational PE of satellite} = -G\frac{m_E m}{r} \qquad [8.20]$$

(b) By the law of universal gravitation and Newton's second law

$$G\frac{m_E m}{r^2} = m\frac{v^2}{r}$$

$$\therefore \quad \text{KE of satellite } (= \tfrac{1}{2}mv^2) = G\frac{m_E m}{2r} \qquad [8.21]$$

(c)
$$\text{Total energy} = \text{PE} + \text{KE}$$
$$= -G\frac{m_E m}{r} + G\frac{m_E m}{2r}$$

$$\text{i.e.} \quad \text{Total energy} = -G\frac{m_E m}{2r} \qquad [8.22]$$

(d) If the satellite encountered air resistance, it would do work against friction and therefore its total energy would decrease, i.e. become more negative. It follows from equation [8.22] that r would decrease and therefore by equation [8.20] the PE would decrease (i.e. become more negative); it follows from equation [8.2] that the KE would increase.

CONSOLIDATION

Newton's law of universal gravitation Every particle in the universe attracts every other with a force which is proportional to the product of their masses and inversely proportional to the square of their separation.

$$F = G\frac{m_1 m_2}{r^2}$$

QUESTIONS ON SECTION A

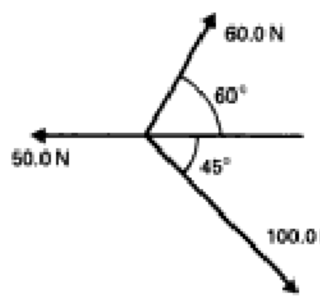
Assume $g = 10\,\mathrm{m\,s^{-2}} = 10\,\mathrm{N\,kg^{-1}}$ unless otherwise stated.

MECHANICS (Chapters 1–5)

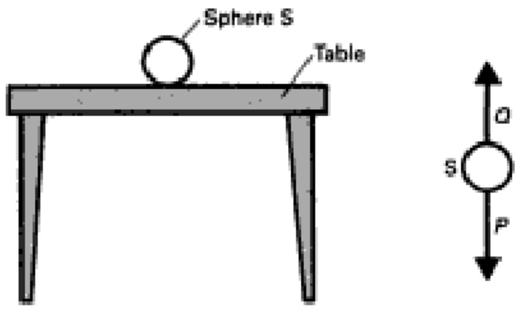
A1 Find, both graphically and by calculation, the horizontal and vertical components of a force of 50 N which is acting at 40° to the horizontal.

A2 The horizontal and vertical components of a force are respectively 20 N and 30 N. Calculate the magnitude and direction of the force.

A3 Calculate the magnitude and direction of the resultant of the forces shown in the figure below.



A4 The diagrams below show a sphere, S, resting on a table and a free-body diagram on which the forces acting on the sphere have been marked.



We know from Newton's third law of motion that forces occur in equal and opposite pairs. On which body does the force which pairs with force P act? Give its direction.

On which body does the force which pairs with force Q act? Give its direction.

A force F_1 acts on an object O and this same force F_1 forms a Newton's third law pair with a second force, F_2. State *two* ways in which F_1 and F_2 are similar and *two* ways in which they differ. [L, '91]

A5 Raindrops of mass 5×10^{-7} kg fall vertically in still air with a uniform speed of $3\,\mathrm{m\,s^{-1}}$. If such drops are falling when a wind is blowing with a speed of $2\,\mathrm{m\,s^{-1}}$, what is the angle which the paths of the drops make with the vertical? What is the kinetic energy of a drop? [S]

A6 A ship initially at rest accelerates steadily on a perfectly smooth sea. How would you attempt to estimate the value of the acceleration from within the ship? You cannot see out, but you have available all the apparatus normally found in a school laboratory. (You may assume that the acceleration is not less than $1\,\mathrm{m\,s^{-2}}$.) [W]

A7 A light string carrying a small bob of mass 5.0×10^{-2} kg hangs from the roof of a *moving* vehicle.

(a) What can be said about the motion of the vehicle if the string hangs vertically?

(b) The vehicle moves in a horizontal straight line from left to right, with a constant acceleration of $2.0\,\mathrm{m\,s^{-2}}$.

- **(i)** Show in a sketch the forces acting on the bob.
- **(ii)** By resolving horizontally and vertically or by scale drawing, determine the angle which the string makes with the vertical.

(c) The vehicle moves down an incline making an angle of 30° with the horizontal with a constant acceleration of $3.0\,\mathrm{m\,s^{-2}}$. Determine the angle which the string makes with the vertical. [J, '92]

A8 A hose with a nozzle 80 mm in diameter ejects a horizontal stream of water at a rate of $0.044\,m^3\,s^{-1}$. With what velocity will the water leave the nozzle? What will be the force exerted on a vertical wall situated close to the nozzle and at right-angles to the stream of water, if, after hitting the wall,
(a) the water falls vertically to the ground,
(b) the water rebounds horizontally?
(Density of water $= 1000\,kg\,m^{-3}$.) [AEB, '79]

A9 What is the connection between force and momentum? A helicopter of total mass 1000 kg is able to remain in a stationary position by imparting a uniform downward velocity to a cylinder of air below it of effective diameter 6 m. Assuming the density of air to be $1.2\,kg\,m^{-3}$, calculate the downward velocity given to the air. [J]

A10 An astronaut is outside her space capsule in a region where the effect of gravity can be neglected. She uses a gas gun to move herself relative to the capsule. The gas gun fires gas from a muzzle of area $160\,mm^2$ at a speed of $150\,m\,s^{-1}$. The density of the gas is $0.800\,kg\,m^{-3}$ and the mass of the astronaut, including her space suit, is 130 kg. Calculate:
(a) the mass of gas leaving the gun per second,
(b) the acceleration of the astronaut due to the gun, assuming that the change in mass is negligible. [J, '92]

A11 Sand is poured at a steady rate of $5.0\,g\,s^{-1}$ on to the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.20 m on to the pan and it does not bounce off the pan then, neglecting any motion of the pan, calculate the reading on the balance 10 s after the sand first hits the pan. [W, '92]

A12 A pebble is dropped from rest at the top of a cliff 125 m high. How long does it take to reach the foot of the cliff, and with what speed does it strike the ground? With what speed must a second pebble be thrown vertically downwards from the cliff top if it is to reach the bottom in 4 s? (Ignore air resistance.) [S]

A13 A stone thrown horizontally from the top of a vertical cliff with velocity $15\,m\,s^{-1}$ is observed to strike the (horizontal) ground at a distance of 45 m from the base of the cliff. What is **(a)** the height of the cliff, **(b)** the angle the path of the stone makes with the ground at the moment of impact? [S]

A14 A ball is thrown vertically upwards and caught by the thrower on its return. Sketch a graph of *velocity* (taking the upward direction as positive) against *time* for the whole of its motion, neglecting air resistance. How, from such a graph, would you obtain an estimate of the height reached by the ball? [L]

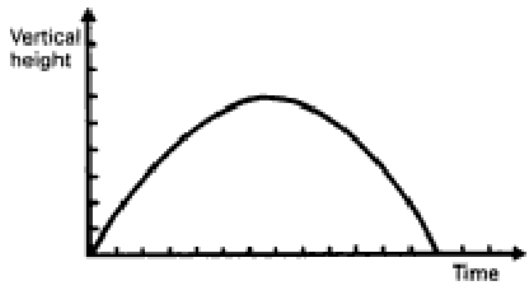
A15 A bus travelling steadily at $30\,m\,s^{-1}$ along a straight road passes a stationary car which, 5 s later, begins to move with a uniform acceleration of $2\,m\,s^{-2}$ in the same direction as the bus.
(a) How long does it take the car to acquire the same speed as the bus?
(b) How far has the car travelled when it is level with the bus? [W, '92]

A16 A cricketer throws a ball of mass 0.20 kg directly upwards with a velocity of $20\,m\,s^{-1}$, and catches it again 4.0 s later.
Draw labelled sketch graphs to show
(a) the velocity,
(b) the kinetic energy,
(c) the height
of the ball against time over the stated 4.0 s period. Your graphs must show numerical values of the given quantities. [S]

A17 A 'hammer' thrown in athletics consists of a metal sphere, mass 7.26 kg, with a wire handle attached, the mass of which can be neglected. In a certain attempt it is thrown with an initial velocity which makes an angle of 45° with the horizontal and its flight takes 4.00 s.

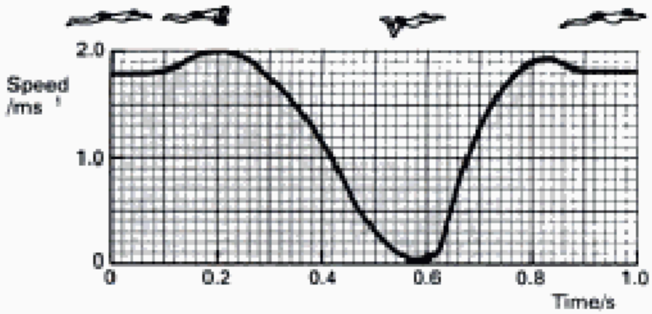
Find the horizontal distance travelled and the kinetic energy of the sphere just before it strikes the ground, stating any assumptions and approximations you make in order to do so. [S]

A18 The graph shown represents the variation in vertical height with time for a ball thrown upwards and returning to the thrower.



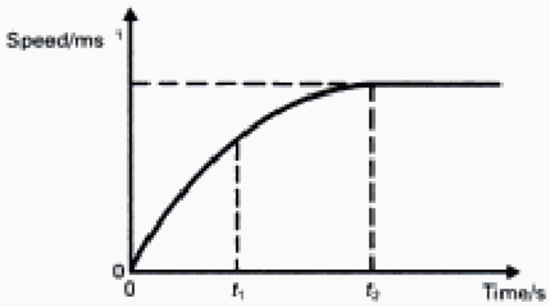
From this graph how could a *velocity* against *time* graph be constructed? Sketch the likely form of such a graph. [L]

A19 The diagram shows the speed–time graph for a swimmer performing one complete cycle of the breast stroke.



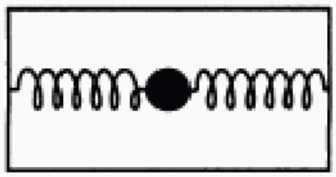
(a) Determine the maximum acceleration of the swimmer. Explain how you arrive at your answer.
(b) Without making any further calculations *sketch* a labelled acceleration–time graph for the same time interval as that shown in the diagram.
(c) Use the graph to estimate the distance travelled in one complete cycle of the stroke. Show your working clearly.
[AEB, '89]

A20 The graph shows how the speed of a car varies with time as it accelerates from rest.



(a) State:
  (i) the time at which the acceleration is a maximum,
  (ii) how you could use the graph to find the distance travelled between times t_1 and t_2.
(b) The driving force produced by the car engine can be assumed to be constant. Explain, in terms of the forces on the car, why
  (i) the acceleration is not constant,
  (ii) the car eventually reaches a constant speed.

(c) The diagram shows a simple version of an instrument used to measure acceleration, in which a mass is supported between two springs in a box so that when one spring is extended, the other is compressed. At rest, the mass is in the position shown.



Redraw the diagram showing the position of the mass when the box accelerates to the right. Explain why the mass takes up this position. [J, '91]

A21 **(a)** A ball is thrown vertically upwards from the surface of the Earth with an initial velocity u. Neglecting frictional forces, sketch a graph to show the variation of the velocity v of the ball with time t as the ball rises and then falls back to Earth. What information contained in the graph enables you to determine: **(i)** the gravitational acceleration, **(ii)** the maximum height to which the ball rises?
(b) If the frictional forces in the air were not negligible, how, in the above situation, would: **(i)** *the initial deceleration* of the ball, **(ii)** *the maximum height* reached by the ball, be affected? [AEB, '79]

A22 **(a)** Define *acceleration*. Explain how it is possible for a body to be undergoing an acceleration although its speed remains constant.
(b) A ball is placed at the top of a slope as shown in Fig. 1.

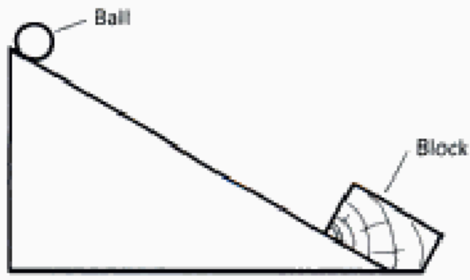


Fig. 1

A block is fixed rigidly to the lower end of the slope. The ball of mass 0.70 kg is released at time $t = 0$ from the top of the incline and v, the velocity of the ball down the slope, is found to vary with t as shown in Fig. 2.

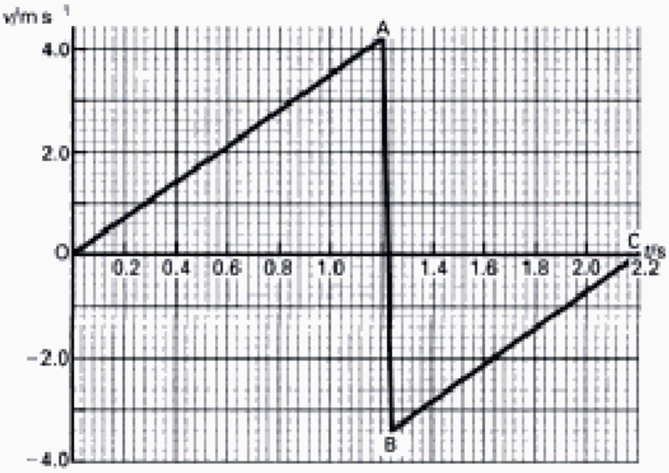


Fig. 2

(i) Describe qualitatively the motion of the ball during the periods OA, AB and BC.

(ii) Calculate: **(1)** the acceleration of the ball down the incline, **(2)** the length of the incline, **(3)** the mean force experienced by the ball during impact with the block.

(iii) Discuss whether the collision between the block and the ball is elastic. [C, '92]

A23 A rocket is caused to ascend vertically from the ground with a constant acceleration, a. At a time, t_s, after leaving the ground the rocket motor is shut off.

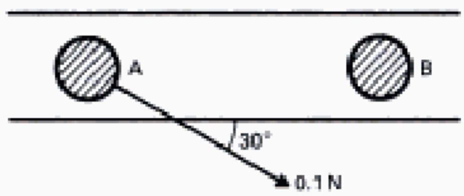
(a) Neglecting air resistance and assuming the acceleration due to gravity, g, is constant, sketch a graph showing how the velocity of the rocket varies with time from the moment it leaves the ground to the moment it returns to ground. In your sketch represent the ascending velocity as positive and the descending velocity as negative. Indicate on your graph **(i)** t_s, **(ii)** the time to reach maximum height, t_h, **(iii)** the time of flight, t_f.

(b) Account for the form of each portion of the graph and explain the significance of the area between the graph and the time axis from zero time to **(i)** t_s, **(ii)** t_h, **(iii)** t_f.

(c) Either by using the graph or otherwise, derive expressions in terms of a, g and t_s for **(i)** t_h, **(ii)** the maximum height reached, **(iii)** t_f. [J]

A24 A sphere of mass 3 kg moving with velocity $4\,\mathrm{m\,s^{-1}}$ collides head-on with a stationary sphere of mass 2 kg, and imparts to it a velocity of $4.5\,\mathrm{m\,s^{-1}}$. Calculate the velocity of the 3 kg sphere after the collision, and the amount of energy lost by the moving bodies in the collision. [S]

A25 The diagram shows a body of mass 2 kg resting in a frictionless horizontal gully in which it is constrained to move. It is acted upon by the force shown for 5 s after which time it strikes and sticks to the body B of mass 3 kg, the force being removed at this instant. What will the speed of the combined masses be? [L]



A26 A railway truck of mass 4×10^4 kg moving at a velocity of $3\,\mathrm{m\,s^{-1}}$ collides with another truck of mass 2×10^4 kg which is at rest. The couplings join and the trucks move off together. What fraction of the first truck's initial kinetic energy remains as kinetic energy of the two trucks after the collision? Is energy conserved in a collision such as this? Explain your answer briefly. [L]

A27 **(a)** **(i)** Define *linear momentum*.

(ii) State *the principle of conservation of linear momentum* making clear the condition under which it can be applied.

(b) A spacecraft of mass 20 000 kg is travelling at $1500\,\mathrm{m\,s^{-1}}$. Its rockets eject hot gases at a speed of $1200\,\mathrm{m\,s^{-1}}$ relative to the spacecraft. During one burn, the rockets are fired for a 5.0 s period. In this time the speed of the spacecraft increases by $3.0\,\mathrm{m\,s^{-1}}$.

(i) What is the acceleration of the spacecraft?

(ii) Assuming that the mass of fuel ejected is negligible compared with the mass of the spacecraft determine the distance travelled during the burn. Give your answer to four significant figures.

(iii) What is the thrust produced by the rocket?

(iv) Determine the mass of gas ejected by the rocket during the burn.

[AEB, '92]

A28 A bullet of mass 0.020 kg is fired horizontally at $150\,m\,s^{-1}$ at a wooden block of mass 2.0 kg resting on a smooth horizontal plane. The bullet passes through the block and emerges undeviated with a velocity of $90\,m\,s^{-1}$. Calculate:

(a) the velocity acquired by the block.

(b) the total kinetic energy before and after penetration and account for their difference. [W, '91]

A29 Distinguish between an *elastic collision* and an *inelastic collision*.

A particle A of mass m moving with an initial velocity u makes a 'head-on' collision with another particle B of mass $2m$, B being initially at rest. In terms of u, calculate the final velocity of A if the collision is **(i)** elastic, **(ii)** inelastic, (assume that the two particles adhere on collision). [AEB, '79]

A30 A puck collides perfectly inelastically with a second puck originally at rest and of three times the mass of the first puck. What proportion of the original kinetic energy is lost, and where does it go? [W]

A31 State the law of conservation of linear momentum.

A proton of mass 1.6×10^{-27} kg travelling with a velocity of $3 \times 10^{7}\,m\,s^{-1}$ collides with a nucleus of an oxygen atom of mass 2.56×10^{-26} kg (which may be assumed to be at rest initially) and rebounds in a direction at 90° to its incident path. Calculate the velocity and direction of motion of the recoil oxygen nucleus, assuming the collision is elastic and neglecting the relativistic increase of mass. [O & C*]

A32 **(a)** In an experiment to investigate the nature of different types of collision, a trolley of mass 1.6 kg was given a push towards a second trolley of mass 0.8 kg travelling more slowly but in the same direction. The speeds of both trolleys before and after collision were measured. The results for two different types of collisions were as follows:

Type A collision	*Speed before*	*Speed after*
1.6 kg trolley	$0.70\,m\,s^{-1}$	$0.30\,m\,s^{-1}$
0.8 kg trolley	$0.10\,m\,s^{-1}$	$0.89\,m\,s^{-1}$
Type B collision		
1.6 kg trolley	$0.60\,m\,s^{-1}$	$0.37\,m\,s^{-1}$
0.8 kg trolley	$0.10\,m\,s^{-1}$	$0.57\,m\,s^{-1}$

Describe a technique for measuring the speed of a trolley before and after a collision, showing how the speed is calculated.

Show that the results given above are consistent with the principle of conservation of linear momentum. Why should the speeds be measured *immediately* before and after the collisions?

(b) Distinguish clearly between *elastic* and *inelastic* collisions. Determine the nature of each of the collisions in (a) above, supporting your choice with appropriate calculations.

Describe how: **(i)** an elastic collision, **(ii)** an inelastic collision, could be simulated experimentally using the two trolleys and any necessary additional apparatus. [L]

A33 **(a)** A linear air-track is a length of metal track along which objects (gliders) can move with negligible friction, supported on a cushion of air. A glider of mass 0.40 kg is stationary near one end of a level air-track and an air-rifle is mounted close to the glider with its barrel aligned along the track. A pellet of mass 5.0×10^{-4} kg is fired from the rifle and sticks to the glider which acquires a speed of $0.20\,m\,s^{-1}$. Calculate the speed with which the pellet struck the glider.

(b) Describe an experimental arrangement you would use to verify the above result.

(c) A student using an air-track fails to level it correctly. The speed of a glider along the track near its centre is $0.20\,m\,s^{-1}$ and when it has moved a further distance of 0.90 m it is $0.22\,m\,s^{-1}$. Determine the angle made by the track to the horizontal.

(d) A glider reaches the end of a level air-track and rebounds from a rubber band stretched across the track. Assuming that, whilst in contact with the band, the force exerted on the glider is proportional to the displacement of the band from the point of impact, sketch a graph showing how the velocity of the glider varies with time. Explain the shape of the graph. [J]

A34 **(a)** The law of conservation of momentum suggests momentum is conserved in any collision. A tennis ball dropped onto a hard floor rebounds to about 60% of its initial height. State how momentum is conserved in this event.

(b) **(i)** A top class tennis player can serve the ball, of mass 57 g, at an initial horizontal speed of $50\,\mathrm{m\,s^{-1}}$. The ball remains in contact with the racket for 0.050 s. Calculate the average force exerted on the ball during the serve.

(ii) *Sketch* a graph showing how the horizontal acceleration of the ball might possibly vary with time during the serve, giving the axes suitable scales.

(iii) Explain how this graph would be used to show that the speed of the ball on leaving the racket is $50\,\mathrm{m\,s^{-1}}$. [AEB, '90]

A35 **(a)** **(i)** What is meant by the term *linear momentum*?

(ii) State the law of conservation of linear momentum.

(iii) Explain how force is related to linear momentum.

(b) Consider the following:

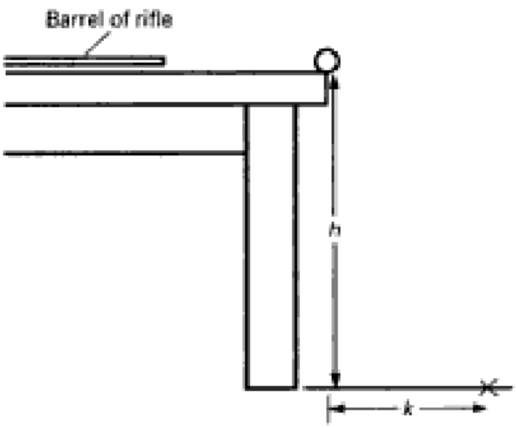
(i) a vehicle in space changes its direction by firing a rocket motor,

(ii) a dart is thrown at a board and sticks to the board,

(iii) a ball is dropped to the floor and rebounds.

In each case discuss how the law of conservation of linear momentum may be applied.

(c) A student devises the following experiment to determine the velocity of a pellet from an air rifle.



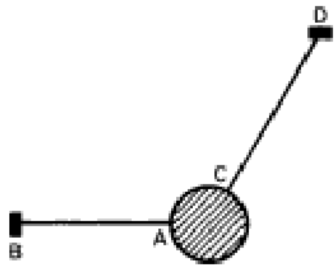
A piece of plasticine of mass M is balanced on the edge of a table such that it just fails to fall off. A pellet of mass m is fired horizontally into the plasticine and remains embedded in it. As a result the plasticine reaches the floor a horizontal distance k away. The height of the table is h.

(i) Show that the horizontal velocity of the plasticine with pellet embedded is $k\left(\frac{g}{2h}\right)^{1/2}$

(ii) Obtain an expression for the velocity of the pellet before impact with the plasticine. [S]

A36 A particle A is suspended as shown in the figure by two strings, which pass over smooth pulleys, and are attached to particles B and C. At A, the strings are at right angles to each other, and make equal angles with the horizontal. If the mass of B is 1 kg, what are the masses of A and C? [S]

A37 State the conditions that a rigid body may be in equilibrium under the action of three forces.



A uniform sphere of mass 2 kg is kept in position as shown by two strings AB and CD. AB is horizontal, and CD is inclined at 30° to the vertical. Calculate the tension in each string. [S]

A38 A uniform beam AB of length 5 m and which weighs 200 N is supported horizontally by two vertical ropes X and Y at A and B respectively. Calculate the tensions in the ropes if a man weighing 700 N stands on the beam at a distance of 2 m from A.

moving glider passes sensor number 1, an electronic timer is started. As the glider passes each sensor, the time taken for the glider to travel from sensor number 1 is recorded. A glider of mass 0.40 kg is given a push along the track. As it passes sensor number 5, it collides with, and sticks to, a stationary glider of mass 0.60 kg. The recorded times at each sensor are shown below.

Sensor number	1	2	3	4	5	6	7	8
Time s	0	0.66	1.32	1.98	2.64	4.31	5.98	7.65

(a) Calculate the speed just before and the speed just after the collision.
(b) Show that momentum is conserved in the collision.
(c) Calculate the kinetic energy before the collision and the kinetic energy after the collision.
Account for the difference. [J, '91]

A51 A motor car collides with a crash barrier when travelling at 100 km h^{-1} and is brought to rest in 0.1 s. If the mass of the car and its occupants is 900 kg calculate the average force on the car by a consideration of momentum.

Because of the seat belt, the movement of the driver, whose mass is 80 kg, is restricted to 0.20 m, relative to the car. By a consideration of energy calculate the average force exerted by the belt on the driver. [J]

A52 This question is about the design of experiments to measure the speed of an air-gun pellet.

The speed of the pellet is known to be about 40 m s^{-1} and the mass of the pellet is about 0.5 g.

(a) One student suggests that momentum ideas might be used. It is proposed that the pellet be fired into a trolley of mass M and that the speed v of the trolley after impact be determined by finding the time it takes a card to cross the path of a light beam. The light beam illuminates a photodiode which controls a timer. The timer can record the time the light beam is cut off to the nearest 0.01 s. The system is shown in Fig. 1.

(i) Explain how the speed of the pellet could be obtained from the proposed measurements.

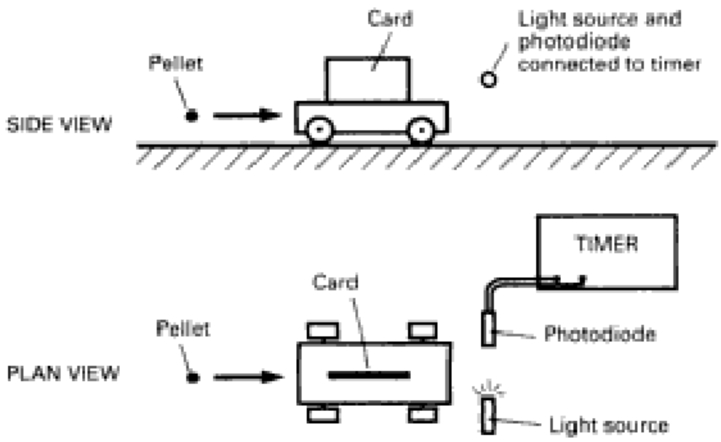


Fig. 1

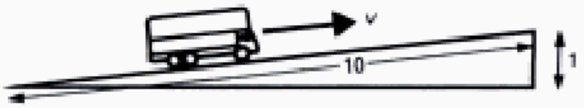
(ii) It is decided that the final speed should be about 0.1 m s^{-1}. Deduce a suitable mass for the trolley.
(iii) Assuming that the final speed is to be determined to a precision of about 2% give a suitable length for the card. Explain how you arrived at your answer.
(iv) It is suggested that the answer is not accurate because the trolley will not be 'friction-free'. How will this affect the final result?
(v) State and explain whether you would expect the result to be more accurate or less accurate if the card were made longer. [AEB, '91]

A53 Explain what is meant by *kinetic energy*, and show that for a particle of mass m moving with velocity v, the kinetic energy is $\frac{1}{2}mv^2$.
A steel ball is:
(a) projected horizontally with velocity v, at a height h above the ground,
(b) dropped from a height h and bounces on a fixed horizontal steel plate.
Neglecting air resistance, and using suitable sketch graphs, explain how the kinetic energy of the ball varies in (a) with its height above the ground, and in (b) with its height above the plate. [J]

A54 A model railway truck P, of mass 0.20 kg and a second truck, Q, of mass 0.10 kg are at rest on two horizontal straight rails, along which they can move with negligible friction. P is acted on by a horizontal force of 0.10 N which makes an angle of 30° with the track. After P has travelled 0.50 m, the force is removed and P then collides with and sticks to Q. Calculate:

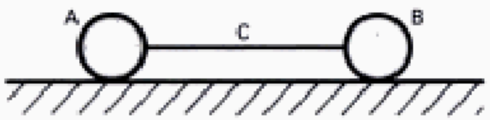
(a) the work done by the force,
(b) the speed of P before the collision,
(c) the speed of the combined trucks after the collision. [J, '90]

A55 A lorry of mass 3.5×10^4 kg attains a steady speed v while climbing an incline of 1 in 10



with its engine operating at 175 kW. Find v. (Neglect friction.) [W, '90]

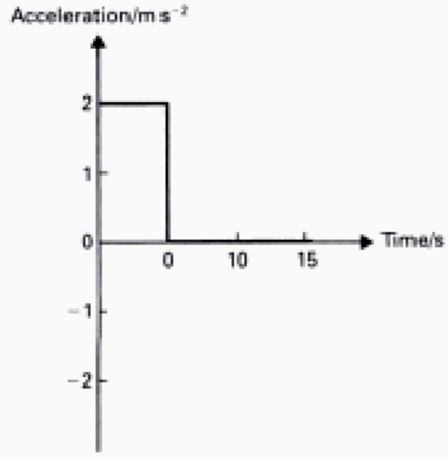
A56 A particle A of mass 2 kg and a particle B of mass 1 kg are connected by a light elastic string C, and initially held at rest 0.9 m apart on a smooth horizontal table with the string in tension. They are then simultaneously released. The string releases 12 J of energy as it contracts to its natural length. Calculate the



velocity acquired by each of the particles. Where do the particles collide? [S]

A57 State Newton's laws of motion, and show how the principle of conservation of linear momentum may be derived from them.
A particle of mass 3 kg and a particle Q of mass 1 kg are connected by a light elastic string and initially held at rest on a smooth horizontal table with the string in tension. They are then simultanously released. The string gives up 24 J of energy as it contracts to its natural length. Calculate the velocity acquired by each of the particles, assuming no energy is lost.
A helicopter of mass 810 kg supports itself in a stationary position by imparting a downward velocity v to all the air in a circle of area 30 m^2. Given that the density of air is 1.20 $kg\,m^{-3}$, calculate the value of v. What is the power needed to support the helicopter in this way, assuming no energy is lost? [S]

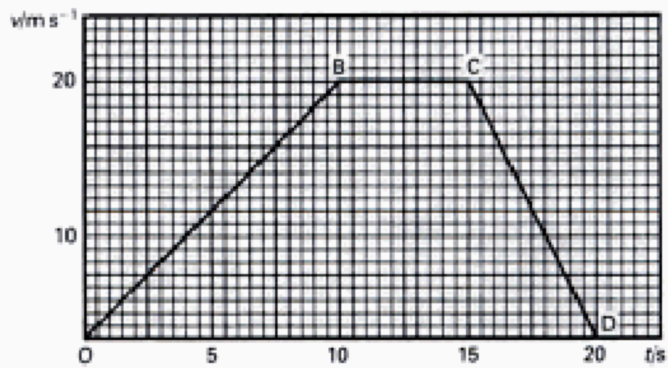
A58 The acceleration–time graph above is drawn for a body which starts from rest and moves in a straight line. The body is of mass 10 kg. Use the graph to find:



(a) the distance travelled in 15 s,
(b) the average force acting over the whole 15 s period. [L]

A59 A stone of mass 80 g is released at the top of a vertical cliff. After falling for 3 s, it reaches the foot of the cliff, and penetrates 9 cm into the ground. What is:
(a) the height of the cliff,
(b) the average force resisting penetration of the ground by the stone? [S]

A60 **(a)** A car of mass 1000 kg is initially at rest. It moves along a straight road for 20 s and then comes to rest again. The speed–time graph for the movement is:



(i) What is the total distance travelled?
(ii) What resultant force acts on the car during the part of the motion represented by CD?
(iii) What is the momentum of the car when it has reached its maximum speed? Use this momentum value to find the constant resultant accelerating force.

(iv) During the part of the motion represented by OB on the graph, the constant resultant force found in (iii) is acting on the moving car although it is moving through air. Sketch a graph to show how the driving force would have to vary with time to produce this constant acceleration. Explain the shape of your graph.

(b) If, when travelling at this maximum speed, the 1000 kg car had struck and remained attached to a stationary vehicle of mass 1500 kg, with what speed would the interlocked vehicles have travelled immediately after collision?

Calculate the kinetic energy of the car just prior to this collision and the kinetic energy of the interlocked vehicles just afterwards. Comment upon the values obtained.

Explain how certain design features in a modern car help to protect the driver of a car in such a collision. [L]

A61 A large cardboard box of mass 0.75 kg is pushed across a horizontal floor by a force of 4.5 N. The motion of the box is opposed by **(i)** a frictional force of 1.5 N between the box and the floor, and **(ii)** an air resistance force kv^2, where $k = 6.0 \times 10^{-2}\ \mathrm{kg\,m^{-1}}$ and v is the speed of the box in $\mathrm{m\,s^{-1}}$.

Sketch a diagram showing the directions of the forces which act on the moving box. Calculate maximum values for

(a) the acceleration of the box,

(b) its speed. [L]

A62 A stone is projected vertically upwards and eventually returns to the point of projection. Ignoring any effects due to air resistance draw sketch graphs to show the variation with time of the following properties of the stone: **(i)** velocity, **(ii)** kinetic energy, **(iii)** potential energy, **(iv)** momentum, **(v)** distance from point of projection, **(vi)** speed. [AEB, '82]

A63 **(a)** State Newton's laws of motion. Explain how the *newton* is defined from these laws.

(b) A rocket is propelled by the emission of hot gases. It may be stated that both the rocket and the emitted hot gases each gain kinetic energy and momentum during the firing of the rocket.

Discuss the significance of this statement in relation to the laws of conservation of energy and momentum, explaining the essential difference between these two quantities.

(c) A bird of mass 0.50 kg hovers by beating its wings of effective area $0.30\ \mathrm{m^2}$.

(i) What is the upward force of the air on the bird?

(ii) What is the downward force of the bird on the air as it beats its wings?

(iii) Estimate the velocity imparted to the air, which has a density of $1.3\ \mathrm{kg\,m^{-3}}$, by the beating of the wings.

Which of Newton's laws is applied in each of (i), (ii) and (iii) above? [L]

A64 A horizontal force of 2000 N is applied to a vehicle of mass 400 kg which is initially at rest on a horizontal surface. If the total force opposing motion is constant at 800 N, calculate:

(a) the acceleration of the vehicle,

(b) the kinetic energy of the vehicle 5 s after the force is first applied,

(c) the total power developed 5 s after the force is first applied. [AEB, '85]

A65 On a linear air-track the gliders float on a cushion of air and move with negligible friction. One such glider of mass 0.50 kg is at rest on a level track. A student fires an air rifle pellet of mass 1.5×10^{-3} kg at the glider along the line of the track. The pellet embeds itself in the glider which recoils with a velocity of $0.33\ \mathrm{m\,s^{-1}}$.

(a) State the principle you will use to calculate the velocity at which the pellet struck the glider.

Calculate the velocity at which the pellet struck.

(b) Another student repeats the experiment with the air-track inclined at an angle of 2° to the horizontal. Initially, the glider is at rest at the bottom of the track. After the impact the glider recoils with the same initial velocity but slows down and stops momentarily further along the track.

Explain *in words* how to calculate how far along the air track the glider moves before stopping instantaneously.

Calculate how far the glider moves along the air track before stopping momentarily. [O & C, '92]

A66 Two ski slopes are of identical length and vertical height. Slope A, Fig. 1 is concave whilst slope B, Fig. 2 is partly convex. Two skiers, of equal weight, start from rest at the top of each slope. Assume that the effects of friction on the skis and of air resistance on the moving skiers' motion are negligible.

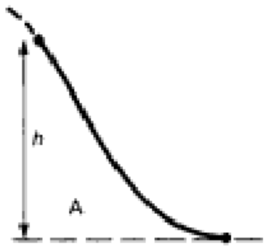


Fig. 1

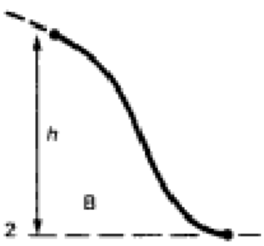


Fig. 2

(a) At the bottom of the ski run, will the skier on slope A be moving faster, at the same speed, or slower than the skier on slope B? Justify your answer.
(b) Will the skier on slope A take longer, the same time, or less time than the skier on slope B to complete the ski run? Justify your answer.
(c) A heavier skier joins the first skier on slope A. He also starts from rest at the top of the slope. Will he take longer, the same time, or less time than the first skier to complete the ski run? Justify your answer.
(d) Which of the three skiers will have most energy at the bottom of the ski run? Explain. [O & C, '90]

A67 **(a)** What do you understand by the *principle of conservation of energy*?
(b) Explain how the principle applies to: **(i)** an object falling from rest in vacuo, **(ii)** a man sliding from rest down a vertical pole, if there is a constant resistive force opposing the motion. Sketch graphs, using one set of axes for **(i)** and another set for **(ii)**, showing how each form of energy you consider varies with time, and point out the important features of the graphs.
(c) A motor car of mass 600 kg moves with constant speed up an inclined straight road which rises 1.0 m for every 40 m travelled along the road. When the brakes are applied with the power cut off, there is a constant resistive force and the car comes to rest from a speed of 72 km h^{-1} in a distance of 60 m. By using the principle of conservation of energy, calculate the resistive force and the deceleration of the car. [J]

A68 A vehicle has a mass of 600 kg. Its engine exerts a tractive force of 1500 N, but motion is resisted by a constant frictional force of 300 N. Calculate:
(a) the acceleration of the vehicle,
(b) its momentum 10 s after starting to move,
(c) its kinetic energy 15 s after starting to move. [S]

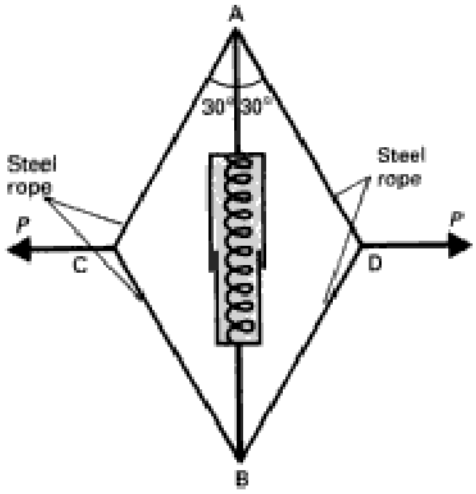
A69 A typical escalator in the London Underground rises at an angle of 30° to the horizontal. It lifts people through a vertical height of 15 m in 1.0 minute. Assuming all the passengers stand still whilst on the escalator, 75 people can step on at the bottom and off at the top in each minute. Take the average mass of a passenger to be 75 kg.
(a) Find the power needed to lift the passengers when the escalator is fully laden. For this calculation assume that any kinetic energy given to the passengers by the escalator is negligible.
(b) The frictional force in the escalator system is 1.4×10^4 N when the escalator is fully laden. Calculate the power needed to overcome the friction. Hence find the power input for the motor driving the fully laden escalator, given that the motor is only 70% efficient.
(c) When the passengers walk up the moving escalator, is more or less power required by the motor to maintain the escalator at the same speed? Explain your answer. [O & C, '90]

A70 A muscle exerciser consists of two steel ropes attached to the ends of a strong spring contained in a telescopic tube. When the ropes are pulled sideways in opposite directions, as shown in the simplified diagram (on p. 125), the spring is compressed.

The spring has an uncompressed length of 0.80 m. The force F (in N) required to compress the spring to a length x (in m) is calculated from the equation

$$F = 500\,(0.80 - x)$$

The ropes are pulled with equal and opposite forces, P, so that the spring is compressed to a length of 0.60 m and the ropes make an angle of 30° with the length of the spring.

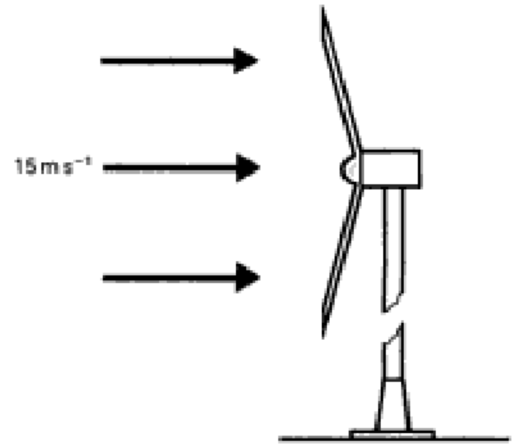


(a) Calculate
  (i) the force, F,
  (ii) the work done in compressing the spring.

(b) By considering the forces at A or B, calculate the tension in each rope.

(c) By considering the forces at C or D, calculate the force, P [J, '91]

A71 The blades of a large wind turbine, designed to generate electricity, sweep out an area of $1400\,m^2$ and rotate about a horizontal axis which points directly into a wind of speed $15\,m\,s^{-1}$, as illustrated in the diagram.



(a) Calculate the mass of air passing per second through the area swept out by the blades.
(Take the density of air to be $1.2\,kg\,m^{-3}$.)

(b) The mean speed of the air on the far side of the blades is reduced to $13\,m\,s^{-1}$. How much kinetic energy is lost by the air per second?

(c) How many turbines, operating with 70% efficiency, would be needed to equal the power output of a single conventional 1000 MW power station?

(d) Suggest *two* advantages, and *two* disadvantages, of wind turbines as a source of energy. [O, '92]

A72 The thrust F exerted on a rocket by the jet of expelled gases depends on the cross-sectional area, A, of the jet, the density, ρ, of the mixture of gases and the velocity, v, at which they are ejected. The following relationships have been suggested between these quantities, in each of which k is a dimensionless constant:

(a) $F = kA\rho v$
(b) $F = kA\rho v^2$
(c) $F = kA^2\rho v^2$.

Use the method of dimensions to show for each whether it is possible. [S]

CIRCULAR MOTION AND ROTATION (Chapter 6)

A73 What force is necessary to keep a mass of 0.8 kg revolving in a horizontal circle of radius 0.7 m with a period of 0.5 s? What is the direction of this force?
(Assume that $\pi^2 = 10$.) [L]

A74 Use Newton's laws of motion to explain why a body moving with uniform speed in a circle must experience a force towards the centre of the circle.

An aircraft of mass 1.0×10^4 kg is travelling at a constant speed of $0.2\,km\,s^{-1}$ in a horizontal circle of radius 1.5 km.

(a) What is the angular velocity of the aircraft?

(b) Show on a sketch the forces acting on the aircraft in the vertical plane containing the aircraft and the centre of the circle. Find the magnitude and direction of their resultant.

(c) Explain why a force is exerted on a passenger by the aircraft. In what direction does this force act? [C]

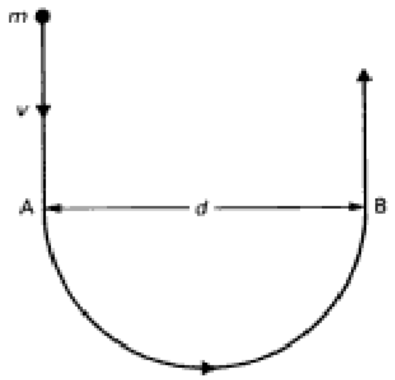
A75 A spaceman in training is rotated in a seat at the end of a horizontal rotating arm of length 5 m. If he can withstand accelerations up to $9g$, what is the maximum number of revolutions per second permissible? [L]

A76 A simple pendulum, suspended from a fixed point, consists of a light cord of length 500 mm and a bob of weight 2.0 N. The bob is made to move in a horizontal circular path. If the maximum tension which the cord can withstand is 5.0 N show whether or not it is possible for the radius of the path of the bob to be 300 mm. [L]

A77 Explain why there must be a force acting on a particle which is moving with uniform speed in a circular path. Write down an expression for its magnitude.

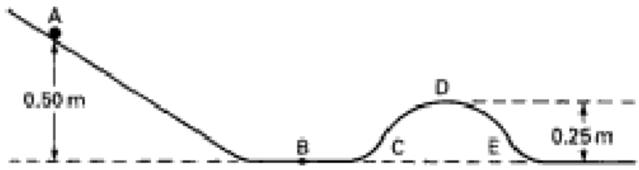
A conical pendulum consists of a small massive bob hung from a light string of length 1 m and rotating in a horizontal circle of radius 30 cm. With the help of a diagram indicate what forces are acting on the bob. How do they account for the motion of the bob? Deduce the speed of rotation in revolutions per minute. [J]

A78 A particle of mass m travels at constant speed, v, in a vacuum along a path consisting of two straight lines connected by a semicircle, AB, of diameter d as shown in the diagram.



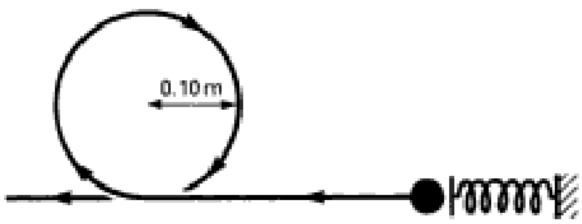
For the section of the path from A to B, find:
- **(a)** the time taken,
- **(b)** the change in the momentum of the particle,
- **(c)** the force acting on the particle at any point along the semicircular path AB,
- **(d)** the work done on the particle by this force. [J]

A79 The diagram shows a section of a curtain track in a vertical plane. The curved section, CDE, forms a circular arc of radius of curvature



0.75 m and the point D is 0.25 m higher than B. A ball-bearing of mass 0.060 kg is released from A, which is 0.50 m higher than B. Assume that rotational and frictional effects can be ignored and that the ball-bearing remains in contact with the track throughout the motion.
- **(a)** Calculate the speed of the ball-bearing **(i)** at B, **(ii)** at D.
- **(b)** Draw a diagram showing the forces acting on the ball-bearing when it is at D and calculate the reaction between the track and the ball-bearing at this point. [J]

A80 A compressed spring is used to propel a ball-bearing along a track which contains a circular loop of radius 0.10 m in a vertical plane. The spring obeys Hooke's law and requires a force of 0.20 N to compress it 1.0 mm.



- **(a)** The spring is compressed by 30 mm. Calculate the energy stored in the spring.
- **(b)** A ball-bearing of mass 0.025 kg is placed against the end of the spring which is then released. Calculate
  - **(i)** the speed with which the ball-bearing leaves the spring,
  - **(ii)** the speed of the ball at the top of the loop,
  - **(iii)** the force exerted on the ball by the track at the top of the loop.

  Assume that the effects of friction can be ignored. [J, '89]

A81 A special prototype model aeroplane of mass 400 g has a control wire 8 m long attached to its body. The other end of the control line is attached to a fixed point. When the aeroplane flies with its wings horizontal in a horizontal circle, making one revolution every 4 s, the control wire is elevated 30° above the horizontal. Draw a diagram showing the forces exerted on the plane and determine:

(a) the tension in the control wire,

(b) the lift on the plane.

(Assume that $\pi^2 = 10$.) [AEB, '79]

A82 A small mass of 5 g is attached to one end of a light inextensible string of length 20 cm and the other end of the string is fixed. The string is held taut and horizontal and the mass is released. When the string reaches the vertical position, what are the magnitudes of:

(a) the kinetic energy of the mass,

(b) the velocity of the mass,

(c) the acceleration of the mass,

(d) the tension in the string?

(Neglect air friction.) [J]

A83 A boy ties a string around a stone and then whirls the stone so that it moves in a horizontal circle at constant speed.

(a) Draw a diagram showing the forces acting on the stone, assuming that air resistance is negligible. Use your diagram to explain

(i) why the string cannot be horizontal,

(ii) the direction of the resultant force on the stone and

(iii) the effect that the resultant force has on the path of the stone.

(b) The mass of the stone is 0.15 kg and the length of the string between the stone and the boy's hand is 0.50 m. The period of rotation of the stone is 0.40 s. Calculate the tension in the string.

(c) The boy now whirls the stone in a vertical circle, but the string breaks when it is horizontal. At this instant, the stone is 1.0 m above the ground and rising at a speed of 15 m s^{-1}. Describe the subsequent motion of the stone until it hits the ground and calculate its maximum height. [O & C, '91]

A84 Derive an expression for the magnitude of the acceleration of a particle moving with speed v in a circle of radius r.

A particle of mass m moves in a circle in a vertical plane, being attached to a fixed point A by a string of length r. The motion of the mass is such that the string is just fully extended at the highest point. Determine:

(a) the minimum speed v at the highest point for this to happen,

(b) the speed V of the particle, and the tension in the string when the particle is at its lowest point.

What is the component of the acceleration of the particle along the tangent to the circle at the instant when the string makes an angle θ with the vertical?

If the particle was initially suspended at rest vertically below A, and was set in motion as described above by an impact with a particle of mass $2m$, determine the velocity u of this particle on the assumption that no energy is lost in the collision. [S]

A85 **(a)** In problems involving linear motion the following equations are often used:

(i) Force = mass × acceleration,

(ii) Kinetic energy = $\frac{1}{2}$ × mass × (velocity)^2,

(iii) Work = force × distance

Using words, write down the corresponding equations for rotational motion.

(b) A couple of torque 5 N m is applied to a flywheel initially at rest. Calculate its kinetic energy after it has completed 5 revolutions. Ignore friction. [W, '92]

A86 A gramophone record A is dropped on to a turntable B which is rotating freely at 10 revolutions per second. The mass of A is 0.25 kg and the mass of B is 0.50 kg. The radius of A is 0.05 m and the radius of B is 0.10 m. What is the final speed of rotation (in rev s^{-1}) of the record and turntable together? (The moment of inertia of a disc is given by $I = \frac{1}{2}MR^2$.) [W, '90]

A87 A swivel chair consists of a seat mounted on a screw-threaded column in such a way that when the seat is given a clockwise rotation it rises vertically. The seat, of moment of inertia about its rotation axis I and mass M, is given an initial clockwise rotation of angular velocity ω. How far does the seat rise, assuming no friction?

What change of angular momentum has occurred during the rise?

Explain the apparent violation of the law of conservation of angular momentum. [W]

A88 **(a)** For a rigid body rotating about a fixed axis, explain with the aid of a suitable diagram what is meant by *angular velocity*, *kinetic energy* and *moment of inertia*.

(b) In the design of a passenger bus, it is proposed to derive the motive power from the energy stored in a flywheel. The flywheel, which has a moment of inertia of $4.0 \times 10^2\,\text{kg m}^2$, is accelerated to its maximum rate of rotation of 3.0×10^3 revolutions per minute by electric motors at stations along the bus route.

(i) Calculate the maximum kinetic energy which can be stored in the flywheel.

(ii) If, at an average speed of 36 kilometres per hour, the power required by the bus is 20 kW, what will be the maximum possible distance between stations on the level? [J]

A89 A cylindrical rocket of diameter 2.0 m develops a spinning motion in space of period 2.0 s about the axis of the cylinder. To eliminate this spin two jet motors which are attached to the rocket at opposite ends of a diameter are fired until the spinning motion ceases. Each motor turns the rocket in the same direction and provides a constant thrust of $4.0 \times 10^3\,\text{N}$ in a direction tangential to the surface of the rocket and in a plane perpendicular to its axis. If the moment of inertia of the rocket about its cylindrical axis is $6.0 \times 10^5\,\text{kg m}^2$, calculate the number of revolutions made by the rocket during the firing and the time for which the motors are fired. [J]

A90 **(a)** **(i)** Explain what is meant by the *moment of inertia* of a body.

(ii) Why is there no unique value for the moment of inertia of a given body?

(iii) A rigid body rotates about an axis with an angular velocity ω. If the relevant moment of inertia of the body is I, show that its rotational kinetic energy is $\frac{1}{2}I\omega^2$.

(b) **(i)** A motor car is designed to run off the rotational kinetic energy stored in a flywheel in the car. The flywheel is to be accelerated up to some maximum rotational speed by electric motors placed at various stations along the route. If the flywheel has a moment of inertia of $300\,\text{kg m}^2$ and is accelerated to 4200 revolutions per minute at a station, calculate the kinetic energy stored in the flywheel. Assuming that at an average speed of $54\,\text{km h}^{-1}$ the power required by the car is 15 kW, what is the maximum possible distance between stations on the car's route?

(ii) What assumption did you make in the last calculation? Comment on the feasibility of the design.

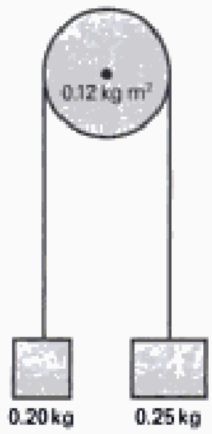
[W, '90]

A91 **(a)** A rigid body is rotating with angular velocity ω about a fixed axis O. Considering a small particle of the body of mass m, at distance r from the axis, state the linear velocity of the particle at any instant, the linear momentum of the particle at that instant, the angular momentum of the particle, and the kinetic energy of the particle.

Write down expressions for **(i)** the angular momentum about the axis O, **(ii)** the kinetic energy, of the whole body, regarded as an assemblage of individual particles, and hence explain the meaning and the importance of the idea of moment of inertia.

(b) Describe how you would determine experimentally the moment of inertia of a flywheel about its usual axis of rotation. An outline of the method only is required, and no formulae need be proved.

(c)



Masses 0.20 kg and 0.25 kg are suspended as in the figure on p. 128 from a light cord which passes over a wheel of radius 0.15 m and moment of inertia $0.12\,\mathrm{kg\,m^2}$. Initially, the two masses are held at the same horizontal level. Explain what happens when they are released from rest if the cord does not slip on the wheel. Assuming that the wheel rotates freely about its axis, calculate the angular velocity of the wheel and the speed of each mass when the vertical distance between the masses is 0.3 m. [O]

A92 The turntable of a record player rotates at a steady angular speed of $3.5\,\mathrm{rad\,s^{-1}}$. A record is dropped from rest on to the turntable. Initially, the record slips but eventually it moves with the same angular speed as the turntable.

(a) The angle the turntable turns thrrough while the record is slipping on its surface is 0.25 rad. Find the average angular acceleration of the record while it is attaining the steady speed of the turntable.

(b) The moment of inertia of the record about its axis of rotation is $1.1 \times 10^{-3}\,\mathrm{kg\,m^2}$. What additional torque must be applied by the turntable motor to maintain the constant angular speed of the turntable while the record is accelerating? [C]

A93 **(a)** Explain what is meant by *moment of inertia* and *angular momentum*. State the relationship between them.

(b) Explain the following observations:

(i) Flywheels are often large diameter wheels with heavy rims rather than disc type wheels of constant thickness.

(ii) A pirouetting skater rotates faster as she draws her arms closer to her body.

(c) In a single cylinder petrol engine, energy from the combustion process is supplied to the crankshaft (the axle of the engine) once every two rotations of the crankshaft, namely during each power stroke , and a flywheel is attached to the crankshaft to smooth the motion.

In such an engine each power stroke produces mechanical energy $E = 1\,\mathrm{kJ}$, and its flywheel has moment of inertia $I = 0.5\,\mathrm{kg\,m^2}$.

(i) If the engine starts from rest, determine the angular velocity after 20 revolutions. Neglect the effect of friction.

(ii) Show, that in the case of zero friction, the crankshaft angular acceleration α is given by

$$\alpha = \frac{E}{4\pi I}$$

(iii) In the practical case where there is friction, calculate the frictional torque when the engine is operating at its highest speed. [W, '92]

SIMPLE HARMONIC MOTION (Chapter 7)

A94 **(a)** Define simple harmonic motion (SHM) for a particle moving in a straight line.

(b) Use your definition to explain how SHM can be represented by the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$$

(c) A mechanical system is known to perform SHM. What quantity must be measured in order to determine ω for the system? [J]

A95 A body of mass 200 g is executing simple harmonic motion with an amplitude of 20 mm. The maximum force which acts upon it is 0.064 N. Calculate:

(a) its maximum velocity,

(b) its period of oscillation. [L]

A96 **(a)** State the conditions for an oscillatory motion to be considered *simple harmonic*.

(b) A body of mass 0.30 kg executes simple harmonic motion with a period of 2.5 s and an amplitude of $4.0 \times 10^{-2}\,\mathrm{m}$. Determine:

(i) the maximum velocity of the body,

(ii) the maximum acceleration of the body,

(iii) the energy associated with the motion. [S]

A97 A particle moves with simple harmonic motion in a straight line with amplitude 0.05 m and period 12 s. Find:

(a) the maximum speed,

(b) the maximum acceleration, of the particle.

Write down the values of the constants P and Q in the equation

$$x/\text{m} = P \sin [Q(t/\text{s})]$$

which describes its motion. [C]

A98 A sinusoidal voltage is applied to the Y plates of a cathode ray oscilloscope which has a calibrated time base. A stationary trace, with an amplitude of 4.0 cm and a wavelength of 1.5 cm, is obtained when the time base is set at $1.0\ \text{cm ms}^{-1}$. The time base is then switched off and the trace becomes a vertical line. Calculate the maximum speed of the spot of light on the end of the tube when producing the vertical line. [L]

A99 A small piece of cork in a ripple tank oscillates up and down as ripples pass it. If the ripples travel at $0.20\ \text{m s}^{-1}$, have a wavelength of 15 mm and an amplitude of 5.0 mm, what is the maximum velocity of the cork? [L]

A100 A body moving with simple harmonic motion has velocity v and acceleration a when the displacement from its mean position is x. Sketch graphs of a against x, and v against x. [L]

A101 The displacement y of a particle vibrating with simple harmonic motion of angular speed ω is given by

$$y = a \sin \omega t \quad \text{where } t \text{ is the time}$$

What does a represent?

Sketch a graph of the *velocity* of the particle as a function of time starting from $t = 0$ s.

A particle of mass 0.25 kg vibrates with a period of 2.0 s. If its greatest displacement is 0.4 m what is its maximum kinetic energy? [L]

A102 The displacement–time equation for a particle moving with simple harmonic motion is

$$x = a \sin (\omega t + \varepsilon)$$

(a) Explain what each of the symbols represents, illustrating your answer with a rough graph showing how x varies with t.

(b) Write down the velocity–time equation, and draw a corresponding graph showing how the velocity v varies with t.

(c) If m is the mass of the particle, the kinetic energy at displacement x is $\frac{1}{2} m\omega^2 (a^2 - x^2)$. Write down the expressions for the potential energy at displacement x, and the total energy.

(d) The total energy of an atom oscillating in a crystal lattice at temperature T is, on average, $3kT$, where k is the Boltzmann constant $1.38 \times 10^{-23}\ \text{J K}^{-1}$. Assuming that copper atoms, each of mass 1.06×10^{-25} kg, execute simple harmonic motion of amplitude 8×10^{-11} m at 300 K, calculate the corresponding frequency. [O]

A103 The bob of a simple pendulum moves simple harmonically with amplitude 8.0 cm and period 2.00 s. Its mass is 0.50 kg. The motion of the bob is undamped.

Calculate maximum values for

(a) the speed of the bob, and

(b) the kinetic energy of the bob. [L]

A104 The following statements refer to a body in simple harmonic motion along a straight line. Write each reference letter (A, B, etc.) on a new line and state whether the corresponding statement is correct or incorrect. If you consider a statement to be incorrect, make a short comment pointing out the error.

A. The displacement of the body must be small.

B. The kinetic energy of the body is constant.

C. The period is constant.

D. The amplitude varies sinusoidally with time.

E. At certain instants, the acceleration is zero.

F. The acceleration of the body can be greater than the acceleration due to gravity.

Show that the motion of a simple pendulum is simple harmonic, and obtain an expression for the period, stating any assumptions made.

How would you obtain experimentally the relationship between period and length? Explain how you would use your results to obtain the value of the acceleration due to gravity.

(If the acceleration of a body is related to its position x by a relationship of the type

$a = -kx$, you may assume that the subsequent motion is simple harmonic of period $2\pi/k^{1/2}$.)

Explain why the tension in the string of a simple pendulum is not constant as it swings. At what points does the tension have its maximum and minimum values? Consider whether these values are greater or less than that when the pendulum hangs stationary. [W]

A105 A light spring is suspended from a rigid support and its free end carries a mass of 0.40 kg which produces an extension of 0.060 m in the spring. The mass is then pulled down a further 0.060 m and released causing the mass to oscillate with simple harmonic motion.

(a) Potential energy is stored in two ways in this arrangement: explain briefly what they are.

(b) Calculate the kinetic energy of the mass as it passes through the mid-point of its motion. [L]

A106 **(a)** The displacement x, in m, from the equilibrium position of a particle moving with simple harmonic motion is given by

$$x = 0.05 \sin 6t$$

where t is the time, in s, measured from an instant when $x = 0$.

(i) State the amplitude of the oscillations.

(ii) Calculate the time period of the oscillations and the maximum acceleration of the particle.

(b) A mass hanging from a spring suspended vertically is displaced a small amount and released. By considering the forces on the mass at the instant when the mass is released, show that the motion is simple harmonic and derive an expression for the time period. Assume that the spring obeys Hooke's law. [J, '89]

A107 A small mass suspended from a light helical spring is drawn down 15 mm from its equilibrium position and released from rest. After 3 seconds the mass reaches this position once more. Find values for the constants a, ω and ε, in the equation $x = a \sin(\omega t + \varepsilon)$ which describes the motion of the mass. Here x measures the distance from the equilibrium position and t the elapsed time since release. [W]

A108 **(a)** Define *simple harmonic motion*.

(b) A light helical spring, for which the force necessary to produce unit extension is k, hangs vertically from a fixed support and carries a mass M at its lower end. Assuming that Hooke's law is obeyed and that there is no damping, show that if the mass is displaced in a vertical direction from its equilibrium position and released, the subsequent motion is simple harmonic. Derive an expression for the time period in terms of M and k.

(c) If $M = 0.30$ kg, $k = 30\ \text{N m}^{-1}$ and the initial displacement of the mass is 0.015 m, calculate:

(i) the maximum kinetic energy of the mass,

(ii) the maximum and minimum values of the tension in the spring during the motion.

(d) Sketch graphs showing how **(i)** the kinetic energy of the mass, **(ii)** the tension in the spring vary with displacement from the equilibrium position.

(e) If the same spring with the same mass attached were taken to the Moon, what would be the effect, if any, on the time period of the oscillations? Explain your answer. [J]

A109 **(a)** Define *simple harmonic motion*.

(b) The displacement of a body undergoing SHM is given by $y = A \sin \omega t$.

(i) Explain what A and ω represent.

(ii) Draw a graph showing how y varies with t.

(iii) Underneath this, and using the same scales for t, sketch graphs showing how the velocity v and the acceleration a vary with t.

(c) A mass m hangs on a string of length l from a rigid support. The mass is pulled aside, so that the string makes an angle θ with the vertical, and then released.

(i) Show that the mass executes SHM, stating any assumptions made.

(ii) Prove that the period T of this SHM is given by $T = 2\pi \sqrt{\dfrac{l}{g}}$.

(iii) A student times a simple pendulum to determine T. Does it matter how many oscillations are counted? Does it matter from where the counts are taken – the end or the middle of the swing? Give reasons.

(d) A piston in a car engine performs SHM. The piston has a mass of 0.50 kg and its amplitude of vibration is 45 mm. The revolution counter in the car reads 750 revolutions per minute. Calculate the maximum force on the piston. [W, '90]

A110 **(a)** A body of mass m is suspended from a vertical, light, helical spring of force constant k, as in Fig. 1. Write down an expression for the period T of vertical oscillations of m.

(b) Two such identical springs are now joined as in Fig. 2 and support the same mass m. In terms of T, what is the period of vertical oscillations in this case?

(c) The identical springs are now placed side by side as in Fig. 3, and m is supported symmetrically from them by means of a weightless bar. In terms of T, what is the period of vertical oscillations in this case?

Fig. 1

Fig. 2

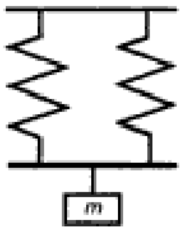


Fig. 3

[W, '91]

A111 **(a)** Define simple harmonic motion. Use your definition to explain what relationship must exist at any instant between the force acting on a body performing such a motion in a straight line and its distance from a fixed point. At what point(s) in the motion is **(i)** the velocity, **(ii)** the acceleration, a maximum?

(b) One end of a spring is attached to a fixed point and the other end carries a body of small mass m which produces a static extension a. Show that, if the body is displaced vertically through a further small distance, it will oscillate with simple harmonic motion. (The mass of the spring may be neglected.)

Given that the time period, T, of the body performing simple harmonic motion is given by the expression

$$T = 2\pi\sqrt{\frac{\text{Mass of the body}}{\text{Force on the body per unit displacement}}}$$

derive an expression for the period of oscillation of the body on the spring.

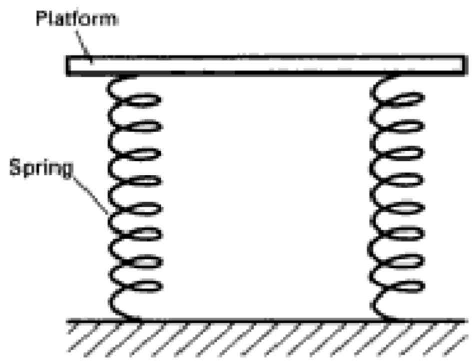
(c) Suggest how you would investigate experimentally whether the bob of a simple pendulum, when oscillating through a small angle, was executing simple harmonic motion. [L]

A112 A 100 g mass is suspended vertically from a light helical spring and the extension in equilibrium is found to be 10 cm. The mass is now pulled down a further 0.5 cm and is then released from rest. Stating any assumptions you make, show that the subsequent motion of the mass is simple harmonic motion. Calculate:

(a) the period of oscillation,

(b) the maximum kinetic energy of the mass. [J]

A113 A light platform is supported by two identical springs, each having spring constant $20\,\text{N m}^{-1}$, as shown in the diagram.



(a) Calculate the weight which must be placed on the centre of the platform in order to produce a displacement of 3.0 cm.

(b) The weight remains on the platform and the platform is depressed a further 1.0 cm and then released. **(i)** What is the frequency of oscillation of the platform? **(ii)** What is the maximum acceleration of the platform? [C, '91]

A114 Define *simple harmonic motion.*

An extension of 2.5 cm is produced when a mass is hung from the lower end of a light helical spring which is fixed at the top end and to which Hooke's law may be assumed to apply. If the mass is depressed slightly and then released, show that the vertical vibrations executed are simple harmonic and calculate their time period.

If the mass of the spring is taken into account, the oscillating mass M may be considered increased to $(M + m)$. Give reasons why m is less than the actual mass of the spring and describe an experiment in which a series of known masses is used to determine the value of m. [J]

A115 Define *simple harmonic motion* and state where the magnitude of the acceleration is **(a)** greatest, **(b)** least.

Some sand is sprinkled on a horizontal membrane which can be made to vibrate vertically with simple harmonic motion. When the amplitude is 0.10 cm, the sand just fails to make continuous contact with the membrane. Explain why this phenomenon occurs and calculate the frequency of vibration. [J]

A116 Define *simple harmonic motion*, and explain what is meant by the *amplitude* and *period* of such a motion.

Show that the vertical oscillations of a mass suspended by a light helical spring are simple harmonic, and obtain an expression for the period.

A small mass rests on a scale-pan supported by a spring; the period of vertical oscillations of the scale-pan and mass is 0.5 s. It is observed that when the amplitude of the oscillation exceeds a certain value, the mass leaves the scale-pan. At what point in the motion does the mass leave the scale-pan, and what is the minimum amplitude of the motion for this to happen? [S]

A117 **(a)** The displacemenmt y of a body moving with SHM is given by

$$y = A \sin \omega t.$$

(i) Sketch the variation of y with t.

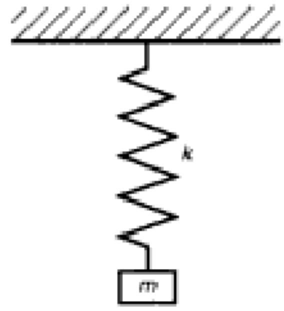
(ii) With reference to your sketch explain what is meant by A and ω.

(iii) Sketch on the same axes the variation of velocity v with time.

(iv) Copy and complete the expression for the velocity:

$$v = A\omega \sin (\omega t \text{ ....})$$

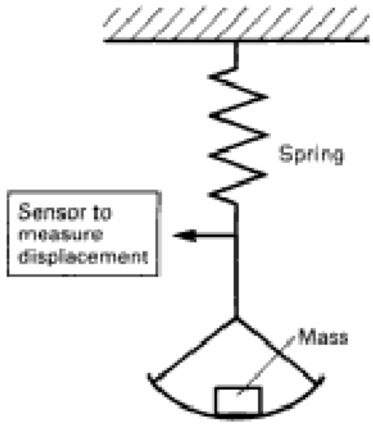
(b) A light spring of force constant k is attached to a solid support and a mass m is fixed to its lower end as shown below.



Prove that when displaced vertically and released, the mass moves with SHM of period

$$T = 2\pi \sqrt{\frac{m}{k}}.$$

(c) The following system may be used commercially to measure mass (or weight).



(i) If, at the sensor, the maximum measurable deflection is 10 mm and $k = 10^5\ \text{N m}^{-1}$, calculate the maximum measurable mass, M_{max}.

(ii) Calculate the frequency of oscillation when M_{max} is applied.

(iii) Suggest an improvement to the system to enable many measurements of mass to be made in rapid succession. [W, '92]

A118 A mass hangs from a light spring. The mass is pulled down 30 mm from its equilibrium position and then released from rest. The frequency of oscillation is 0.50 Hz.

(a) Calculate:

(i) the angular frequency, ω, of the oscillation

(ii) the magnitude of the acceleration at the instant it is released from rest.

(b) Sketch a graph of the acceleration of the mass against time during the first 4.0 s of its motion. Put a scale on each axis.

(c) After a few oscillations half of the mass becomes detached when it is at the lowest point of its motion. The act of detachment still leaves the remaining half instantaneously at rest.

Is the period of the subsequent oscillation the same, shorter or longer than the original period? Account for your answer. [O & C, '91]

A119 **(a)** **(i)** Define *simple harmonic motion.*

(ii) Show that the equation

$$y = a \sin(\omega t + \varepsilon)$$

represents such a motion and explain the meaning of the symbols y, a, ω and ε.

(iii) Draw with respect to a common time axis graphs showing the variation with time t of the displacement, velocity and kinetic energy of a heavy particle that is describing such a motion.

(b) When a metal cylinder of mass 0.2 kg is attached to the lower end of a light helical spring the upper end of which is fixed, the spring extends by 0.16 m. The metal cylinder is then pulled down a further 0.08 m.

(i) Find the force that must be exerted to keep it there, if Hooke's law is obeyed.

(ii) The cylinder is then released. Find the period of vertical oscillations, and the kinetic energy the cylinder possesses when it passes through its mean position. [O]

GRAVITATION (Chapter 8)

A120 Explorer 38, a radio-astronomy research satellite of mass 200 kg, circles the Earth in an orbit of average radius $3\,R/2$ where R is the radius of the Earth. Assuming the gravitational pull on a mass of 1 kg at the Earth's surface to be 10 N, calculate the pull on the satellite. [L]

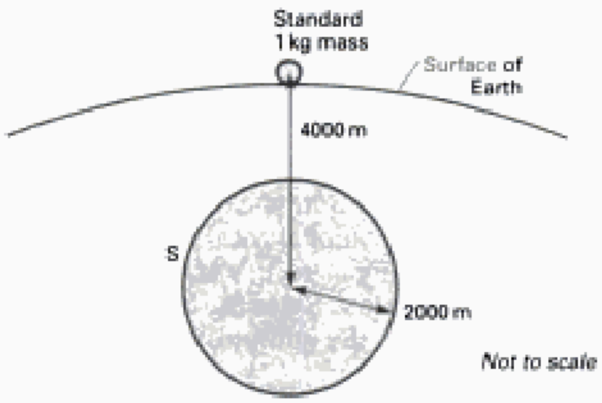
A121 A satellite of mass 66 kg is in orbit round the Earth at a distance of $5.7R$ above its surface, where R is the value of the mean radius of the Earth. If the gravitational field strength at the Earth's surface is $9.8\ \mathrm{N\,kg^{-1}}$, calculate the centripetal force acting on the satellite.

Assuming the Earth's mean radius to be 6400 km, calculate the period of the satellite in orbit in hours. [L]

A122 **(a)** Define *acceleration*. An object is thrown vertically upwards from the surface of the Earth. Air resistance can be neglected. Sketch labelled graphs on the same axes to show how **(i)** the velocity, **(ii)** the acceleration of the object vary with time. Mark on the graphs the time at which the object reaches maximum height and the time at which it returns to its original position.

(b) Modern gravity meters can measure g, the acceleration of free fall, to a high degree of accuracy. The principle on which they work is of measuring t, the time of fall of an object through a known distance h in a vacuum. Assuming that the object starts from rest, deduce the relation between g, t and h.

(c) State Newton's law of gravitation relating the force F between two point objects of masses m and M, their separation r and the gravitational constant G.



(d) The diagram shows a standard kilogram mass at the surface of the Earth and a spherical region S of radius 2000 m with its centre 4000 m from the surface of the Earth. The density of the rock in this region is $2800\ \mathrm{kg\,m^{-3}}$. What force does the matter in region S exert on the standard mass?

(e) If region S consisted of oil of density $900\ \mathrm{kg\,m^{-3}}$ instead of rock, what difference would there be in the force on the standard mass?

(f) Suggest how gravity meters may be used in oil prospecting. Find the uncertainty within which the acceleration of free fall needs to be measured if the meters are to detect the (rather large) quantity of oil stated in (e).
($G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$.) [C, '91]

A123 A communication satellite is placed in an orbit such that it remains directly above a fixed point on the Earth's surface at all times.

(a) What is the period of this satellite?

(b) Explain why the satellite must be in orbit above the equator.

(c) Show that the correct height for the orbit does not depend upon the mass of the satellite. [S]

A124 The gravitational force acting on an astronaut travelling in a space vehicle in low Earth orbit is only slightly less than if he were standing on Earth.

(a) Explain why the force is only slightly less.

(b) Explain why, when travelling in the space vehicle, the astronaut appears to be 'weightless'. [L]

A125 **(a)** State the Kepler law of planetary motion which relates period to orbit radius. Show that it is consistent with an inverse square law of force between massive bodies.

(b) When a space shuttle is in an orbit at a mean height of 0.33×10^6 m above the surface of the Earth, it requires 91 minutes to complete one orbit. Use this information to obtain a value for the mass of the Earth.

(c) Describe a laboratory experiment to measure the acceleration of free fall. Explain carefully how the value is obtained from the measurements made and comment upon the accuracy you would expect.

(d) Explain why an astronaut inside the shuttle of part (b) feels weightless even though the intensity of the Earth's gravitational field at that height is approximately $9\ \mathrm{N\,kg^{-1}}$.

(Mean Earth radius = 6.37×10^6 m; Universal Gravitational constant = $6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$.) [S]

A126 An artificial satellite travels in a circular orbit round the Earth. Explain why its speed would have to be greater for an orbit of small radius than for one of large radius. [L]

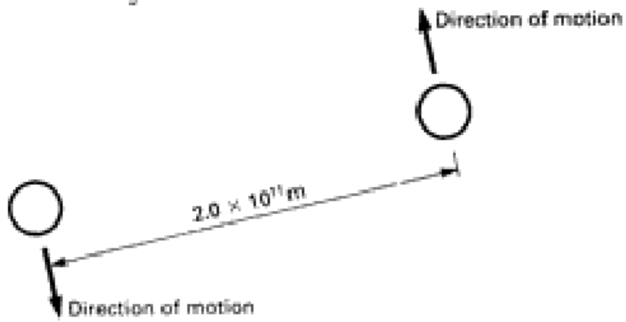
A127 A man is able to jump vertically 1.5 m on Earth. What height might he be expected to jump on a planet of which the density is one third that of the Earth but of which the radius is one half that of the Earth? [L]

A128 Assuming the Earth to be a sphere of radius 6×10^6 m, estimate the mass of the Earth, given that the acceleration of free fall is $10\ \mathrm{m\,s^{-2}}$ and that the gravitational constant G is $7 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$. [C(O)]

A129 The Moon-rover used by astronauts on the Moon breaks down. Explain whether or not the force required **(a)** to lift it, **(b)** to start it moving horizontally with a given acceleration would be more or less than on Earth. Frictional forces may be considered to be negligible.

While engaged in lifting the vehicle an astronaut lets drop simultaneously a spanner and a piece of paper. Describe and explain the fall of these two objects compared with what would be observed on Earth. [AEB, '79]

A130 The diagram shows a binary star system consisting of two stars each of mass 4.0×10^{30} kg separated by 2.0×10^{11} m. The stars rotate about the centre of mass of the system.



(a) (i) Copy the diagram and, on your diagram, label with a letter L a point where the gravitational field strength is zero. Explain why you have chosen this point.

(ii) Determine the gravitational potential at L.
($G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.)

(b) (i) Calculate the force on each star due to the other.

(ii) Calculate the linear speed of each star in the system.

(iii) Determine the period of rotation.
[AEB, '92]

A131 Kepler's third law of planetary motion, as simplified by taking the orbits to be circles round the Sun, states that if r denotes the radius of the orbit of a particular planet and T denotes the period in which that planet describes its orbit, then r^3/T^2 has the same value for all the planets.

The orbits of the Earth and of Jupiter are very nearly circular with radii of 150×10^9 m and 778×10^9 m respectively, while Jupiter's period round the Sun is 11.8 years.

(a) Show that these figures are consistent with Kepler's third law.

(b) Taking the value of the gravitational constant, G, to be 6.67×10^{-11} N m^2 kg^{-2}, estimate the mass of the Sun. [O*]

A132 (a) State Newton's law of gravitation and derive the dimensions of the gravitational constant G.

(b) If a planet is assumed to move around the sun in a circular orbit of radius r with periodic time T, derive an expression for T in terms of r and other relevant quantities. [J]

A133 Explain what is meant by the gravitational constant G, and derive its dimensions in terms of mass M, length L and time T.

Assuming that the period of rotation t of a planet in its orbit depends only on its distance d from the Sun, the mass M_s of the Sun and the gravitational constant G, show that t^2 is proportional to d^3/M_s. Use the following data on the Solar System to test, as far as possible, the validity of this result.

Planet	*Distance from Sun/km*	*Period/ days*
Mercury	0.53×10^8	88
Earth	1.49×10^8	365
Mars	2.28×10^8	687
Jupiter	7.78×10^8	4333
Uranus	28.7×10^8	30690

The distance of the Moon from the Earth is 3.8×10^5 km and its period of rotation is 27.3 days. Deduce the ratio of the mass of the Sun to that of the Earth. [O & C]

A134 Assuming that the Earth (mass m) describes a circular orbit of radius R at angular velocity ω round the Sun (radius r, mass M) due to gravitational attraction:

(a) write down the Earth's equation of motion,

(b) obtain the mean density of the Sun, given $\omega = 2.0 \times 10^{-7}$ rad s^{-1};
$R/r = 200$;
$G = 6.7 \times 10^{-11}$ kg^{-1} m^3 s^{-2};
volume of a sphere $= \frac{4}{3}\pi r^3$. [S]

A135 Explain how the mass, M, of the Sun can be calculated from a knowledge of the following:
R, distance from Earth to Sun,
r, distance from Earth to Moon,
T, orbital period of Earth,
t, orbital period of Moon,
m, the mass of the Earth. [L]

A136 Explain what is meant by the universal gravitational constant G. Derive the relationship between G and the acceleration of free fall, g, at the surface of the Earth (neglecting rotation of the Earth and assuming that it is spherical).

Explain why the rotation of the Earth about its axis affects the value of g at the equator.

Calculate the percentage change in g between the poles and the equator (again assuming that the Earth is spherical).

The orbit of the Moon is approximately a circle of radius 60 times the equatorial radius of the Earth. Calculate the time taken for the Moon to complete one orbit, neglecting the rotation of the Earth.

(Acceleration of free fall at the poles of the Earth = $9.8\,\mathrm{m\,s^{-2}}$. Equatorial radius of the Earth = $6.4 \times 10^6\,\mathrm{m}$. 1 day = 8.6×10^4 seconds.) [L]

A137 Describe the circumstances under which a body can be said to be *weightless*. [C]

A138 Show that $\mathrm{N\,kg^{-1}}$ is a valid unit for g, the acceleration due to gravity.

Draw a graph showing how g varies with distance from the Earth's centre. Start your graph from the Earth's surface and assume that $g = 10\,\mathrm{m\,s^{-2}}$ at the surface. Take as your unit along your distance axis the Earth's radius ($6.4 \times 10^6\,\mathrm{m}$) and extend the axis to six radii.

Estimate from your graph the loss in potential energy as a body of mass 1 kg falls from $2.56 \times 10^7\,\mathrm{m}$ to $1.92 \times 10^7\,\mathrm{m}$ from the Earth's centre.

Determine (*not* from your graph) the distance from the Earth to the Moon, and the value of the Earth's g at the Moon. (You may assume that 1 lunar month = 28 days.) [W]

A139 Distinguish between the gravitational constant G and the acceleration due to gravity g.

Assuming that the Earth is a uniform homogeneous sphere of radius R and density Δ obtain expressions for the acceleration due to gravity:

(a) at a pole of the Earth,
(b) at a height h above the Earth at the pole,
(c) at a point on the equator. [J]

A140 **(a)** **(i)** Define gravitational field strength.
(ii) Show that gravitational field strength is equal to g (the acceleration due to gravity).
(b) Explain carefully the distinction between weight and mass.
(c) How are weight and mass each measured? (*One sentence on each is expected.*) [W, '90]

A141 **(a)** **(i)** Explain what is meant by *gravitational potential* and *gravitational potential energy*.
(ii) Use your explanations to show that the difference in potential energy between a point on the Earth's surface and one at a height h above it is, to a close approximation, equal to mgh where m is the mass of the body under consideration and g is the gravitational field strength at the Earth's surface.
(b) The base of a mountain is at sea level where the gravitational field strength is $9.810\,\mathrm{N\,kg^{-1}}$. The value of the gravitational field strength at the top of the mountain is $9.790\,\mathrm{N\,kg^{-1}}$. Calculate the height of the mountain above sea level.
(c) Outline a method of measuring the gravitational field strength to the accuracy required in **(b)** above.
(Radius of the Earth = 6000 km.) [O & C, '92]

A142 **(a)** Define *gravitational field strength* and *gravitational potential*, stating the relationship between them. Explain what is meant by the term *uniform field* and discuss to what extent the gravitational field of the Earth can be considered to be uniform by considering two points on the surface **(i)** separated by a distance of about 10 km, **(ii)** at opposite ends of a diameter. Assume that the Earth is a homogeneous sphere.
(b) Write down an expression for the gravitational potential at the surface of the Earth in terms of its mass M, radius R and the gravitational constant G. Sketch a graph showing the variation of potential with position along a line passing through the centre of the Earth and point out the important features of the graph. (Only consider points external to the surface and in one direction only.)
(c) Derive an expression for the escape velocity, v, at the surface of a planet in terms of the radius, r, of the planet and the acceleration of free fall, g_p, at the surface of the planet. [J]

A143 What are the gravitational potentials at a point on the Earth's surface due to **(a)** the Earth, **(b)** the Sun?

(Mass of Earth = 6.0×10^{24} kg; radius of Earth = 6.4×10^6 m; mass of Sun = 2.0×10^{30} kg; radius of Earth's orbit = 1.5×10^{11} m; $G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.) [C]

A144 **(a)** **(i)** State Newton's law of gravitation. Give the meaning of any symbol you use.

(ii) Define *gravitational field strength*.

(iii) Use your answers to **(i)** and **(ii)** to show that the magnitude of the gravitational field strength at the Earth's surface is

$$\frac{GM}{R^2}$$

where M is the mass of the Earth, R is the radius of the Earth and G is the gravitational constant.

(b) Define *gravitational potential*. Use the data below to show that its value at the Earth's surface is approximately -63 MJ kg^{-1}.

(c) A communications satellite occupies an orbit such that its period of revolution about the earth is 24 hr. Explain the significance of this period and show that the radius, R_0, of the orbit is given by

$$R_0 = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

where T is the period of revolution and G and M have the same meanings as in (a) (iii).

(d) Calculate the least kinetic energy which must be given to a mass of 2000 kg at the Earth's surface for the mass to reach a point a distance R_0 from the centre of the Earth. Ignore the effect of the Earth's rotation.
($G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}, $M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m.) [J, '89]

A145 **(a)** Define *gravitational potential* at a point.

(b) As a spacecraft falls towards the Earth, it loses gravitational potential energy. What becomes of the lost potential energy

(i) when the spacecraft is falling freely towards the Earth well away from the Earth's atmosphere,

(ii) when the spacecraft is falling through the Earth's atmosphere at constant speed?

(c) **(i)** Calculate the gravitational potential difference between a point on the Earth's surface and a point 1600 km above the Earth's surface.

(ii) Calculate the minimum energy required to project a spacecraft of mass 2.0×10^6 kg from the surface of the Earth so that it escapes completely from the influence of the Earth's gravitational field.

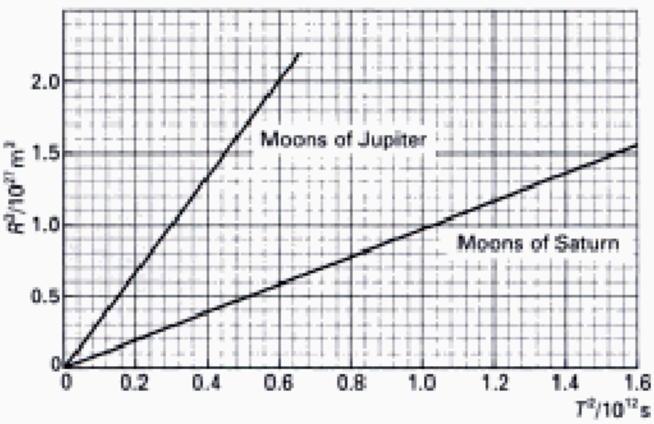
(Radius of Earth=6400 km; Universal Gravitational constant = 6.7×10^{-11} N m^2 kg^{-2}; mass of the Earth = 6.0×10^{24} kg.) [AEB, '87]

A146 What do you understand by the term *gravitational field*; define *gravitational field strength*.

Show that the radius R of a satellite's circular orbit about a planet of mass M is related to its period as follows:

$$R^3 = \frac{GM}{4\pi^2} T^2$$

where G is the universal gravitational constant.



The diagram shows two graphs of R^3 against T^2; one is for the moons of Jupiter and the other is for the moons of Saturn. R is the mean distance of a moon from a planet's centre and T is its period.

The orbits are assumed to be circular.

The mass of Jupiter is 1.90×10^{27} kg.

(a) Why are the lines straight?

(b) Find a value for the mass of Saturn.

(c) Find a value for the universal gravitational constant G. [L*]

A147 Explain what is meant by the statement: *The gravitational potential at one Earth's radius above the Earth's surface is* $-31.3 \times 10^6\,\mathrm{J\,kg^{-1}}$.

(a) The table below gives the gravitational potential V_g at distances from the centre of the Earth in units of Earth radii. The radius of the Earth $R = 6.38 \times 10^6$ m.

V_g/MJ kg^{-1}	−62.7	−31.3	−20.9	−15.7
Distance/R	1.0	2.0	3.0	4.0

(i) Use these data to determine the gravitational potential at distances from the Earth's centre of 10×10^6 m and 15×10^6 m. Indicate how you determined the values.

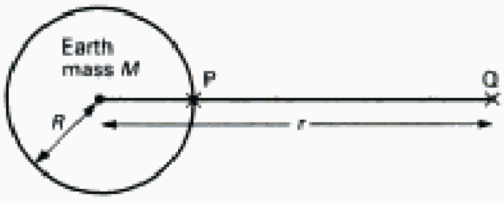
(ii) A spacecraft of mass 4.0×10^4 kg has its motors switched off. It slows down as it moves away from 10×10^6 m above the Earth's centre to 15×10^6 m. Find the loss of kinetic energy of the craft and the average force acting on the craft.

(iii) A slow-moving meteorite is captured by the Earth's gravitational field. Determine the speed with which it will crash into the surface on the assumption that it is not slowed by air resistance.

(b) The Space Shuttle, with its engines shut down, is moving in the same circular orbit above the Earth and at the same speed as a satellite that it is trying to capture. The two craft are separated by a distance of a few kilometres. The Shuttle can catch up the satellite by using its engines in reverse for a few seconds to slow it down. The Shuttle falls into a lower orbit and passes the satellite. Using its engines to accelerate for a few seconds it returns to the original orbit just in front of the satellite. Use your knowledge of gravitational forces and uniform motion in a circular orbit to explain the physics of this procedure.

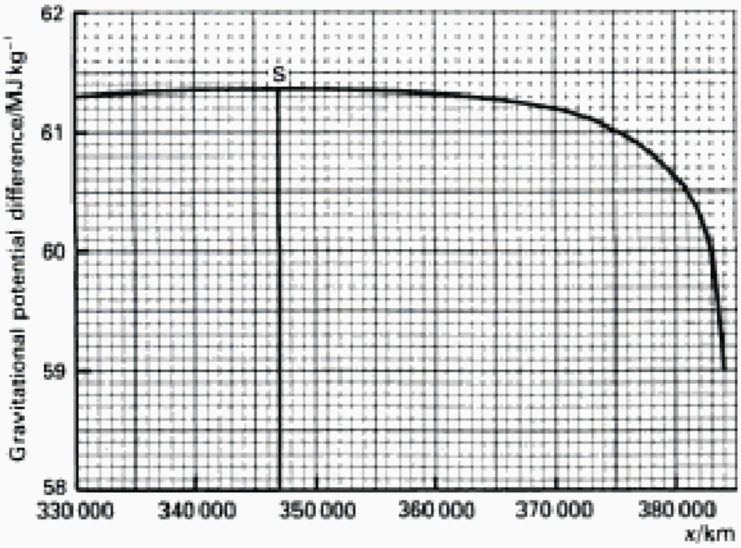
[O & C, '90]

A148 **(a)** Write down an expression for the gravitational potential difference between a point P on the Earth's surface and a distant point Q as shown below.



Show that if r is only slightly greater than R, the gravitational potential difference becomes $g(r - R)$ where g is the gravitational field strength on the Earth's surface.

(b) The graph shows how the gravitational potential difference between a point on the Earth's surface and a distant point, distance x from the Earth's surface, changes near to the Moon's surface. The Moon's surface is 384 000 km from the Earth's surface.



The graph shows the gravitational potential difference first increasing, then achieving a maximum value and finally decreasing to a smaller value on the Moon's surface.

(i) Use the graph to determine the amount of potential energy released as a mass of 200 kg falls to the surface of the Moon from a height of 14 000 km. At what speed will it hit the surface?

(ii) What feature of the graph justifies the assumption that the potential energy of a body measured with respect to the Moon's surface is proportional to its height above that surface? Obtain from the graph the height to which this assumption is true.

(iii) The net force acting on a body moving from the Earth's surface to the surface of the Moon is the resultant of two components, one due to the attraction of the body towards the Earth and the other due to its attraction towards the Moon.

Explain how the net force changes in going from the Earth's surface to the point S, shown on the graph, where the gravitational potential difference is a maximum.

What is the value of the net force at the point where the gravitational potential difference is a maximum? Give a reason for your answer.

Explain why the gravitational potential difference is a maximum at this point. [L]

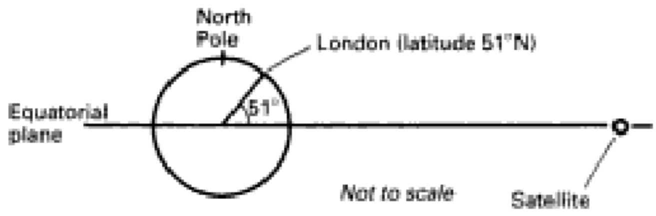
A149 **(a)** **(i)** Explain why a force is required for a mass to travel at a constant speed in a circular path. State the direction of this force and give an equation for its magnitude, defining any terms used.

(ii) State how this force is provided in the case of a satellite orbitting the Earth.

(iii) Show that the speed of a satellite in orbit close to the Earth is given by $(gR)^{\frac{1}{2}}$ where g is the acceleration of free fall and R is the radius of the Earth.

(iv) Calculate the speed of the satellite and the period of the orbit given that $g = 9.8\,\text{m}\,\text{s}^{-2}$ and $R = 6.4 \times 10^3\,\text{km}$.

(b) The most useful communication satellites are those in geostationary orbits. A satellite in geostationary orbit remains above the same point on the Earth's



surface at all times. This is only possible when the satellite is in an orbit in the equatorial plane. Only 'line of sight' communication is possible in satellite communications. This means that communication can only occur provided there is no obstruction between the transmitter, the satellite and the receiver.

(i) The relationship between the period T and radius R of an orbit is

$$T^2 = kR^3$$

where k is a constant.

Using your answer to (a) (iv) determine the radius of the orbit for a geostationary satellite.

(ii) Estimate the delay between the transmission and reception of a signal using the satellite. Show how you arrive at your answer.

(iii) Calculate the most northerly latitude for which satellite communication is possible.

(iv) State with reasons how many satellites are needed to provide communication between all places on the equator and indicate on a diagram how this can be achieved.

(v) State and explain the advantages of communicating using geostationary satellites compared with those whose position relative to the Earth's surface is continually changing. [AEB, '89]

SECTION B

STRUCTURAL PROPERTIES OF MATTER

9

SOLIDS AND LIQUIDS

9.1 INTRODUCTION

The kinetic theory accounts for all three states of matter (solid, liquid and gas) by assuming that matter is made up of molecules which are in continual motion. This motion exists at all temperatures above absolute zero, and the kinetic energy associated with it is often referred to as **thermal energy**. The molecules* exert forces of attraction on each other, and so they also possess potential energy. The forces are due to the electrostatic interactions of the electrons and nuclei of the molecules. The force between a pair of molecules depends upon the spatial distribution of the electrons and the separation of the molecules. At very small separations the net force must become repulsive, for if the attractive force were to exist right down to zero separation, all matter would collapse in on itself. The force must be negligible at large separations in order to account for the properties of gases. The kinetic energy, on the other hand, depends only on temperature; in fact **temperature is the outward manifestation of kinetic energy**. It is the relative magnitude of the kinetic and potential energies which determines whether a substance is in the solid, the liquid or the gaseous state.

9.2 INTERMOLECULAR FORCE AND POTENTIAL ENERGY

Consider two isolated molecules whose separation is such that they are exerting attractive forces on each other. If one of the molecules were to be removed to infinity, work would have to be done on it in order to overcome the attractive force, and therefore its potential energy would increase. However, it is convenient to regard the potential energy of each molecule as being zero when their separation is infinite (because at such a separation they have no influence on each other), and therefore when two molecules are attracting each other their potential energy is negative.

9.3 TO SHOW THAT $F = -dE/dr$

Consider two molecules exerting forces of attraction on each other (Fig. 9.1). If the force F on A moves it a small distance δr (so that F can be considered constant) to the right, then the work done δW on A is given by

$$\delta W = F\,\delta r \qquad [9.1]$$

*We shall not distinguish between atoms and molecules.

Fig. 9.1
Mutually attracting molecules

If δE is the resulting change in the potential energy of A, then

$$\delta E = -\delta W \qquad [9.2]$$

The minus sign is present because as A moves towards B, under the influence of the attractive force, its potential energy decreases. By equations [9.1] and [9.2],

$$\delta E = -F\,\delta r$$

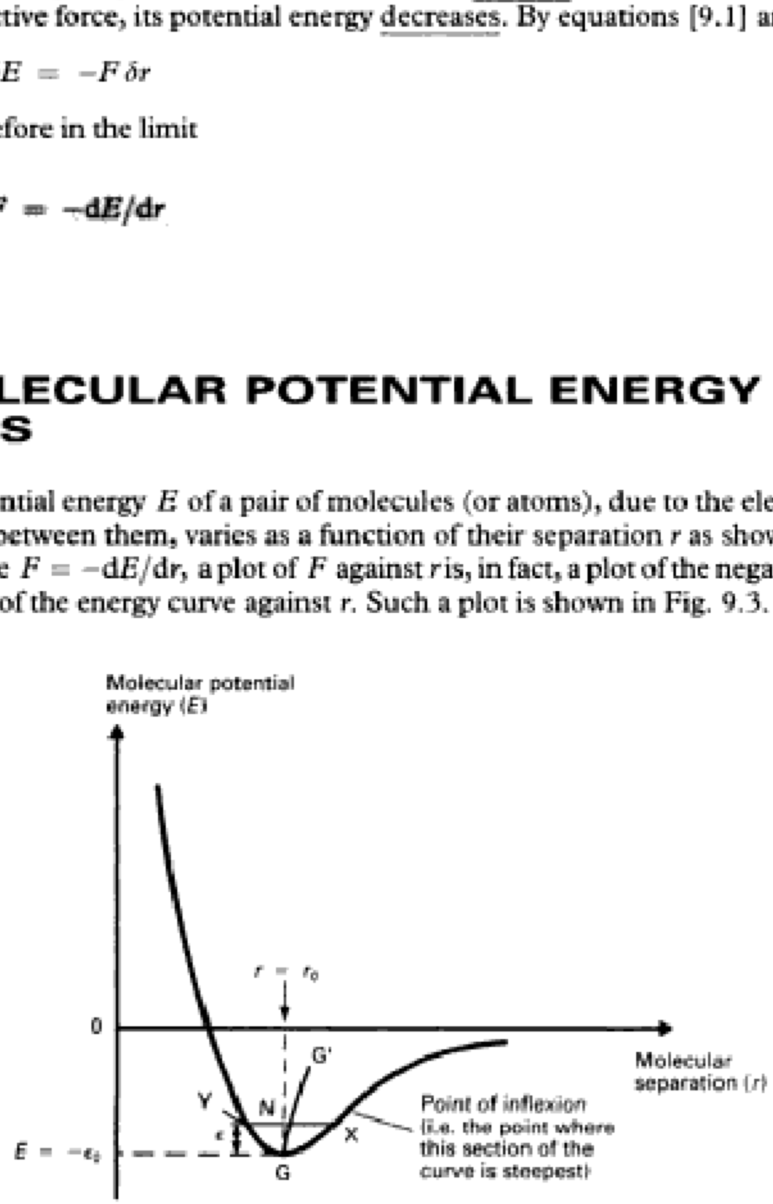
and therefore in the limit

$$\boldsymbol{F = -\mathrm{d}E/\mathrm{d}r}$$

9.4 THE INTERMOLECULAR POTENTIAL ENERGY AND FORCE CURVES

The potential energy E of a pair of molecules (or atoms), due to the electrostatic force F between them, varies as a function of their separation r as shown in Fig. 9.2. Since $F = -\mathrm{d}E/\mathrm{d}r$, a plot of F against r is, in fact, a plot of the negative of the gradient of the energy curve against r. Such a plot is shown in Fig. 9.3.

Fig. 9.2
The potential energy of a pair of molecules as a function of their separation



When $r = r_0$ there is no net force between the molecules and their potential energy (Fig. 9.2) has its minimum value. Thus if two molecules have a separation of r_0 they are at their equilibrium separation. Any increase or decrease in their separation would require energy, since work would have to be done against the net attractive or the net repulsive force respectively. The equilibrium is stable because an increase in r leads to an attractive force which restores r to r_0; similarly a decrease in r produces a repulsive force which again restores r to r_0. (The value of r_0 depends on the particular solid but it is often $\sim 3 \times 10^{-10}$ m.)

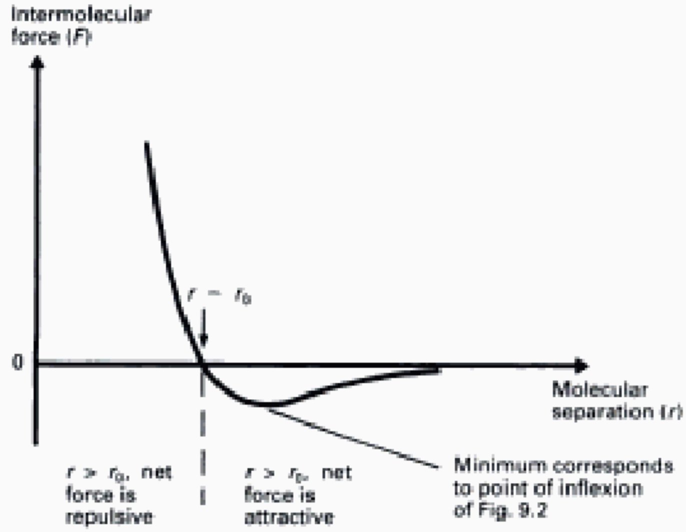


Fig. 9.3
The force between a pair of molecules as a function of their separation

9.5 SOLIDS

Solids have fixed shapes and fixed volumes. Consider a solid at absolute zero; the molecules would have no kinetic energy and therefore would be stationary at their equilibrium separation r_0. At higher temperatures the molecules would possess kinetic energy and could use this to oppose the intermolecular forces. Suppose a pair of molecules has an amount of kinetic energy ε. By exchanging this kinetic energy for potential energy, they would be able to increase their separation such that their situation became that represented by the point X on the energy curve (Fig. 9.2); or decrease it to Y. At X the molecules would feel an attractive force which would restore them to their equilibrium separation r_0. On reaching r_0 they would once again have kinetic energy ε but would now be moving toward each other; they would therefore decrease their separation to the state represented by Y. At Y their directions of motion would again reverse. Thus, **the molecules of a solid at temperatures above absolute zero oscillate about their equilibrium positions**. Because their kinetic energy is low compared with their potential energy ($\varepsilon < 0.1\varepsilon_0$), the molecules of solids can merely vibrate about fixed positions. They are therefore locked into a geometrically ordered array, and as a consequence a solid has both a fixed volume and a fixed shape.

At temperatures above absolute zero the mean separation is not necessarily r_0. In Fig. 9.2 XN is greater than YN, and therefore the mid-point of XY, the point on which the oscillation is centred, corresponds to a separation which is greater than r_0. As the temperature increases from absolute zero, the mid-point moves from G towards G′. Thus, the mean separation of the two molecules (and consequently of all the pairs of molecules within the solid) increases with temperature, i.e. the curve shown represents the normal situation, that of a solid which expands on heating.

The linear expansivity α of a solid is defined by

$$\alpha = \frac{\delta L}{L\,\delta\theta} \qquad [9.3]$$

where

δL = the increase in length brought about by a small increase in temperature $\delta\theta$

L = the original length of the specimen.

The value of α depends on the temperature at which it is measured. However, for temperature increases of less than about 100°C the variation is slight and equation [9.3] can be replaced by the more useful expression

$$L_1 = L_0 + \alpha L_0(\theta_1 - \theta_0) \quad [9.4]$$

where

α = the mean linear expansivity of the solid in the temperature range θ_0 to θ_1 (unit = $°C^{-1}$ or K^{-1})

L_0 = the length of the specimen at θ_0

L_1 = the length of the specimen at θ_1.

9.6 LIQUIDS

A liquid has a fixed volume but no fixed shape. The molecules of liquids, like those of solids, vibrate. In liquids though, each molecule has a particular set of nearest neighbours for only a short time. This occurs because the molecules of liquids have greater average kinetic energies than those of solids. (Note that the molecules of liquids, like those of solids and gases, have a range of kinetic energies, and the energy of any particular molecule is constantly changing due to intermolecular collisions.) The increased kinetic energy results in larger amplitudes of vibration, and therefore there is more likelihood of a molecule being able to pass through the gaps between the molecules surrounding it. There is, therefore, a continual molecular migration superimposed upon the vibrational motion, and this accounts for the ability of a liquid to adopt the shape of its container. The molecules, however, are close together and a change in volume would require that the intermolecular forces were overcome – liquids, therefore, have fixed volumes.

9.7 BROWNIAN MOTION

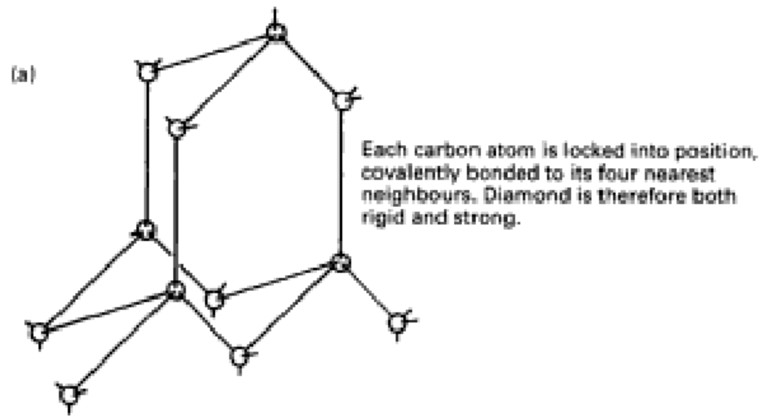
This was first observed in 1827 by Robert Brown, a Scottish botanist, while using a microscope to look at a suspension of pollen grains in water. He noticed that the pollen grains were in a state of continual motion. The motion was both random and jerky. Brownian motion can be observed when small particles of any kind are suspended in a fluid (e.g. smoke particles suspended in air). The motion can be made more pronounced by:

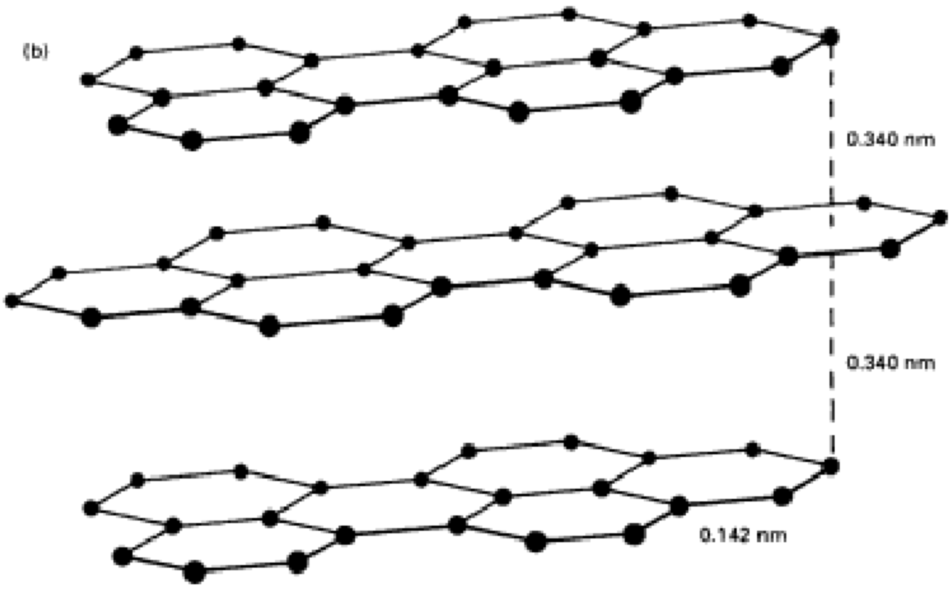
(i) increasing the temperature of the fluid, and/or

(ii) decreasing the size of the suspended particles.

Brownian motion is now regarded as strong evidence that fluids are composed of molecules in a state of unceasing random motion. For example, we consider that a smoke particle suspended in air is constantly being bombarded by air molecules. At any one time, though, if the air molecules move randomly, the smoke particle is likely to receive a bigger impact on one side than on the opposite side. Because the smoke particle is small, this statistical imbalance will be significant and therefore the particle will speed up or slow down and/or

Fig. 9.4
The structure of
(a) diamond, (b) graphite





Each carbon atom is covalently bonded to the three nearest atoms in its own layer. Only three of the four valence electrons are used for bonding; the fourth is free to move throughout the entire layer. This gives graphite high electrical conductivity. The bonding between adjacent layers is due only to weak van der Waals forces. The layers can therefore slide past each other with relative ease, accounting for the softness of graphite and its tendency to flake.

Fig. 9.5
Ideal crystals of
(a) ammonium alum,
(b) quartz

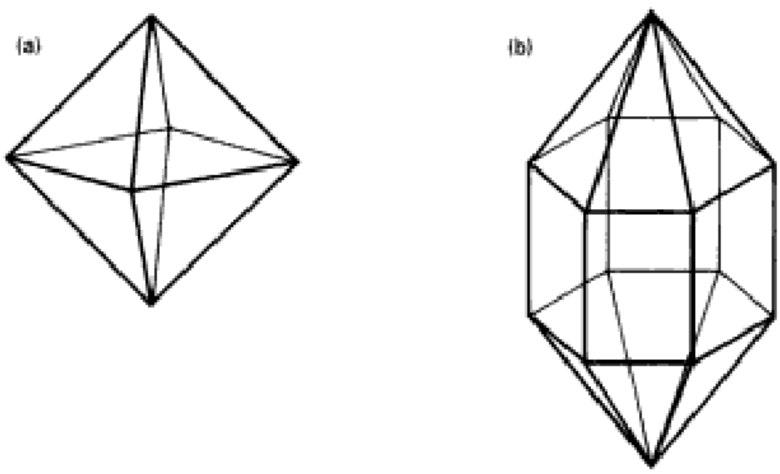
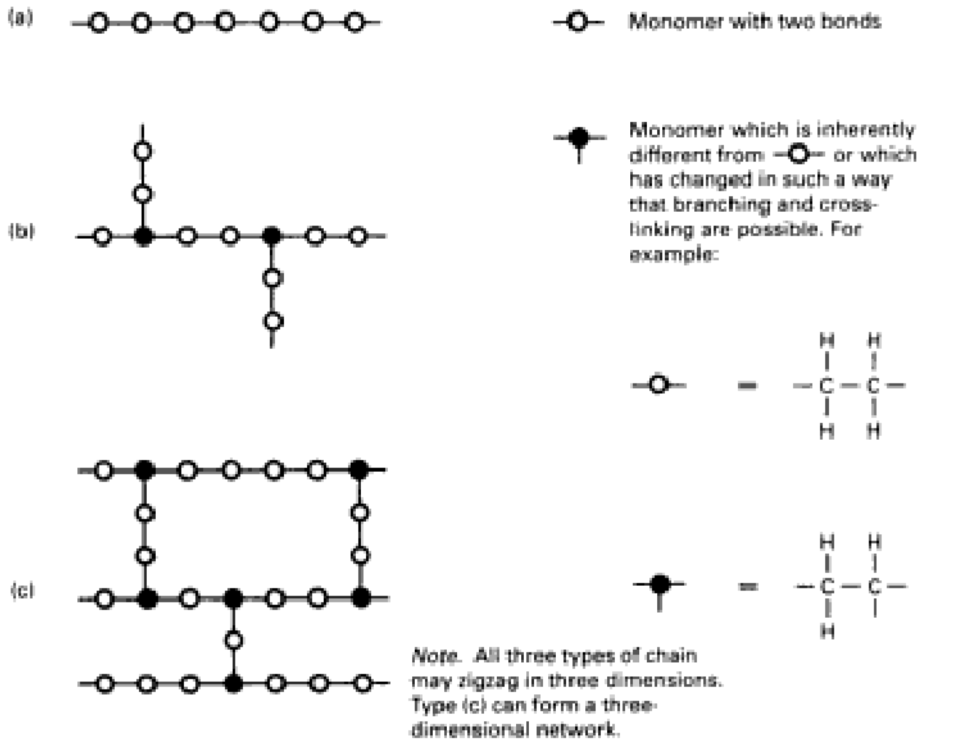


Fig. 9.9
Different types of long-chain molecule: (a) linear, (b) branched chain, (c) cross-linked chains



Many polymers may be classified as being either thermoplastics or thermosetting plastics. Some polymers are described as elastomers, some as fibres. These four classifications are discussed in the sections that follow and examples are given in Table 9.1.

Thermoplastics

These soften and become more flexible on heating; they regain their previous rigidity on cooling. They can be moulded while warm and retain their moulded form when they cool. There are usually only weak forces (e.g. van der Waals forces) between the chains. Heating overcomes these, and the chains can then slide past each other so that the material takes up the shape of the mould. Since the bonds are weak, the amount of heat required is not so great that the polymer decomposes. The bonds reform and restore the rigidity on cooling.

Thermosetting Plastics

These are cross-linked polymers and are more brittle and more rigid than the thermoplastics. They do not soften on heating and can withstand higher temperatures than thermoplastics because more energy is needed to break the relatively strong bonds between the chains. If the temperature is increased to the extent that the bonds break, the material decomposes.

Thermosetting plastics are moulded before polymerization is complete. They are then heated to produce further cross-linking, so setting the material, irreversibly, in its moulded form*.

*With some materials, epoxy resins for example, polymerization can be completed at room temperature.

Fibres

These are linear chain polymers in which the chains have been aligned along the length of the fibre, and in which there are reasonably strong bonds between the chains (hydrogen bonding in the case of nylons, dipole–dipole bonding in the case of Terylene). Synthetic fibres are thermoplastic materials; many of them can be used in their non-fibre forms (e.g. nylon). Cellulose is a natural polymeric fibre.

Crystallinity in Polymers

In some polymers there are regions in which the chains are close together and parallel to each other. There is therefore a degree of long-range order in these regions, and they are said to be **crystalline**. At the other extreme are the so-called **amorphous polymers** in which the chains criss-cross in a random way like tangled strands of spaghetti. Linear chain polymers may be either crystalline or amorphous. Crystallinity tends not to occur in polymers with highly branched chains because the chains cannot pack sufficiently closely. Highly cross-linked polymers are completely amorphous.

We have seen that increased rigidity can be produced by increasing the amount of cross-linking between the chains. It can also be produced by creating crystalline regions in the polymer. The rigidity is due to the forces between individual atoms in adjacent chains in the crystalline regions. Although these forces are usually weak (e.g. van der Waals forces), the side-by-side arrangement of the chains means that there are large numbers of these 'bonds', making it difficult for the chains to slide past each other.

The greater the crystallinity, the higher the melting point and the higher the density. The effects of crystallinity are illustrated by the two forms of polythene* (see Table 9.2).

Table 9.2
A typical low-density and a typical high-density polythene compared

	Low-density polythene	*High-density polythene*
Crystallinity	50%	76%
Density	920 kg m^{-3}	960 kg m^{-3}
Melting point	110 °C	135 °C
Tensile strength at yield	12×10^6 Pa	31×10^6 Pa
Chain type	Branched	Linear

(Data kindly supplied by BP Chemicals Limited)

General Properties of Polymers

The main bonds in polymers are covalent; this accounts for their low thermal and electrical conductivities. Polymers are less dense than both metals and ceramics. They are usually resistant to water and acids but may be attacked by organic solvents. Production costs for plastics are much less than those for metals – a polythene bucket is much cheaper than a metal one.

Plastics often have other materials incorporated with them. The purpose of these additives may be to increase flexibility, increase strength, improve weathering properties, provide better insulation characteristics, add colour or simply to

*The two forms are produced by employing different conditions during polymerization.

reduce cost. For example, glass fibres can be added to epoxy and polyester resins to increase strength. The addition of mica to some thermosetting plastics makes them even better electrical insulators. Lead compounds can be added to PVC to prevent it decomposing in strong sunlight.

9.11 CRYSTAL STRUCTURES

Early crystallographers suspected that the external regularity of crystals was due to their atoms being arranged in regular three-dimensional arrays. This was confirmed in 1912 when von Laue and his students, Friedrich and Knipping, showed that crystals could be used to diffract X-rays, and therefore must be acting as three-dimensional diffraction gratings. The diffraction pattern they obtained was too complicated for them to use it to determine the structure of the crystal that had produced it, but one year later, using a simplified version of the technique, W.H. and W.L. Bragg succeeded. X-ray diffraction has since proved to be the single most important method of determining crystal structures.

The atoms (or ions or molecules) in a crystal are arranged in such a way that the total potential energy of the structure is as small as possible, in which case the structure is as stable as possible. The way in which this is achieved depends on the type of crystal concerned. Some examples are discussed below.

Metal Crystals

The valence electrons of metals are free to move throughout the whole of the metal, and therefore metals can be regarded as an array of positive ions in a 'sea' of electrons. For the purpose of this discussion we may consider the ions to be incompressible, equal-sized spheres. There is no directional bonding (as there is, for example, in diamond). In these circumstances the most stable arrangement is that in which the spheres occupy the minimum possible volume. This arrangement is known as **close packing**. The spheres are arranged in layers, where each sphere is surrounded by a hexagonal ring of six others in contact with it (Fig. 9.10). Fig. 9.11 shows the way in which two of these layers must fit together in order to fulfil the requirement of close packing. There are two types of hollows in layer B – those marked by crosses and those marked by dots. There are therefore two ways in which a third layer may be added to the first two.

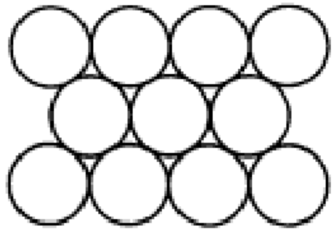


Fig. 9.10 Spheres packing together to occupy the minimum space

If the spheres of the third layer occupy the hollows marked by crosses, they are directly above the spheres in layer A. When this is so, the fourth layer is always a repeat of layer B, and the overall sequence is ABAB, etc. – a structure known as **hexagonal close packing** (Fig. 9.12).

If the spheres of the third layer occupy the hollows marked by dots, the layer is different from both A and B; we shall call it layer C. In this situation the overall sequence is always ABCABC, etc. – a structure known as **cubic close packing**.

Fig. 9.11
Close-packed layers

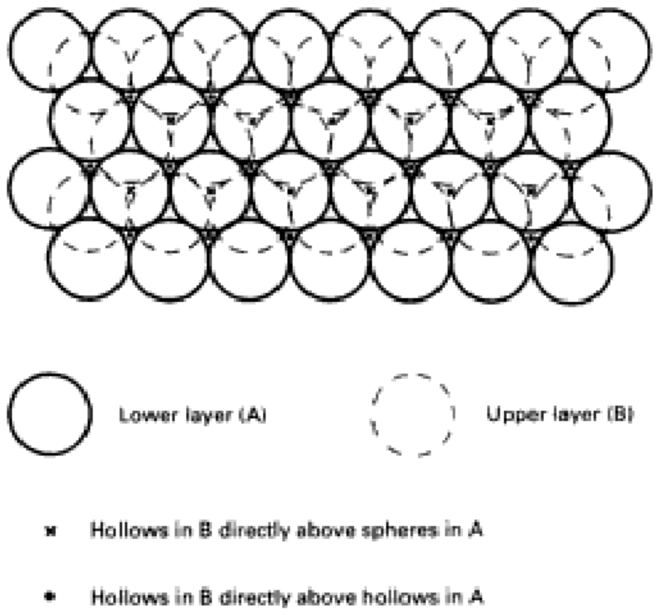
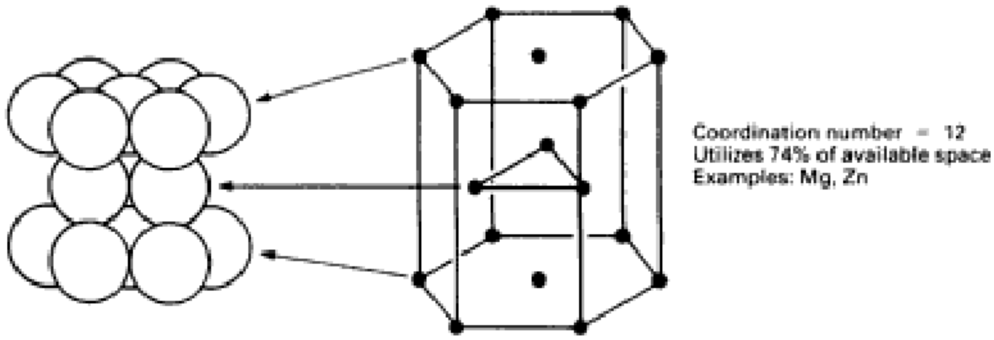


Fig. 9.12
Unit cell of hexagonal close-packed structure



The unit cell contains ions from four layers and is known as a **face-centred** cube (Fig. 9.13) because there is an ion at the centre of each face. Fig. 9.13(a) shows the relationship between the layers and the unit cell.

A more open structure than the two discussed so far is that known as **body-centred cubic** (Fig. 9.14). The least stable metals (e.g. lithium, sodium, potassium) tend to crystallize in this form. Close packing cannot occur because the thermal vibrations of the ions are able to overcome the relatively weak cohesive forces in these metals.

Fig. 9.13
Unit cell of cubic close-packed (face-centred cubic) structure

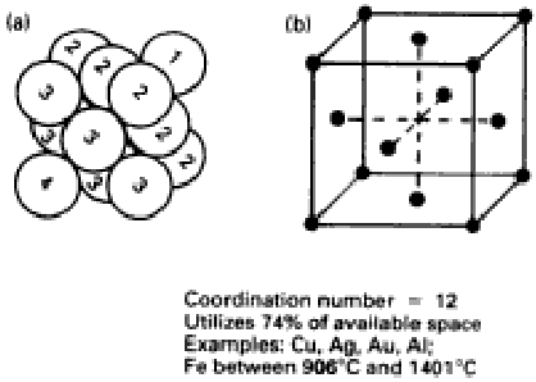
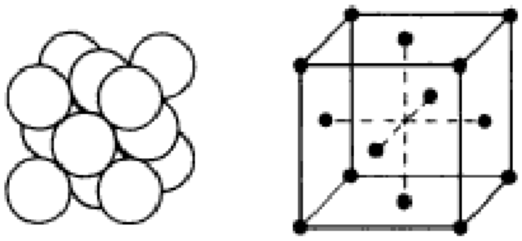


Fig. 9.14
Unit cell of body-centred cubic structure



Coordination number = 8
Utilizes 68% of available space
Examples: Li, Na, K; Fe below 906°C

Ionic Crystals

As with metals, the ions tend to pack closely together because the bonding is non-directional. We may still regard the ions as incompressible spheres, but we need to take account of there being both positively charged and negatively charged ions present. The arrangement shown in Fig. 9.15(a) is more stable than that in Fig. 9.15(b) because in Fig. 9.15(b) there are much stronger repulsive forces between the negative ions. Consideration must also be given to the fact that in practice the two types of ion are normally of different sizes. Fig. 9.16 illustrates the effect of the relative size of the central ion and those around it. Each arrangement makes maximum use of the available space, and each does so without allowing the negative ions to be in contact with each other.

Fig. 9.15
(a) Stable, and
(b) unstable arrangements of negative ions around a positive ion

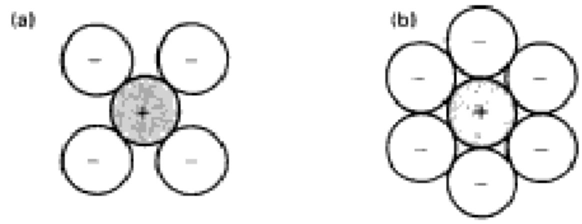


Fig. 9.16
The effect of the relative size (r_+/r_-) of the central ion on those around it

(a) $0.73 < r_+/r_- < 1.00$
Example: CsCl

(b) $0.41 < r_+/r_- < 0.73$
Example: NaCl

Caesium chloride crystallizes in the form of Fig. 9.16(a); the extended structure is shown in Fig. 9.17. There are eight Cl^- ions around each Cs^+ ion, and eight Cs^+ ions around each Cl^- – the so-called 8 : 8 coordination. Though the lattice resembles that shown in Fig. 9.14, it is not known as body-centred cubic because there are two types of ion present.

Sodium chloride crystallizes in the form of Fig. 9.16(b); the extended structure is shown in Fig. 9.18. It can be regarded as two interpenetrating face-centred cubic structures. Each Na^+ ion has six Cl^- ions as its nearest neighbours, and each Cl^- ion has six Na^+ ions as nearest neighbours – 6 : 6 coordination.

There are two crystalline forms of zinc sulphide – zinc blende and wurtzite. The radius ratio of Zn^{2+} to S^{2-} is 0.48, and therefore zinc sulphide would be expected to have the sodium chloride structure. However, both crystalline forms of zinc

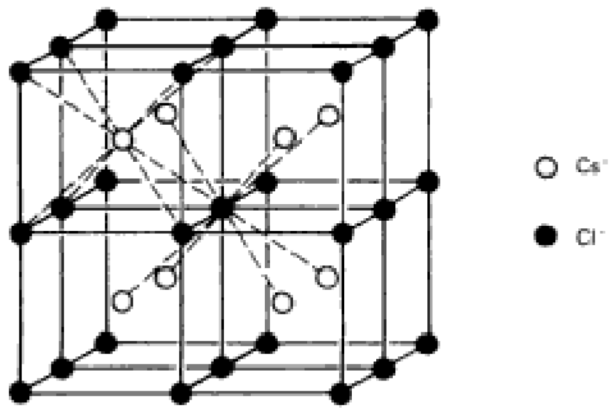


Fig. 9.17
Caesium chloride lattice showing 8:8 coordination

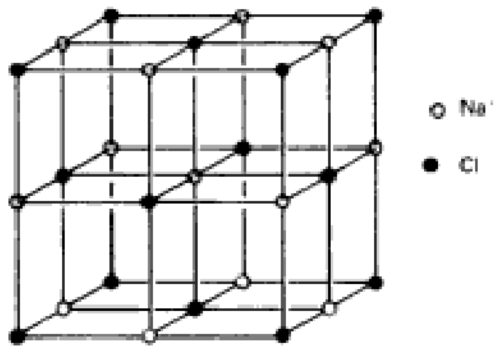


Fig. 9.18
Sodium chloride lattice

sulphide are such that each Zn^{2+} ion is surrounded tetrahedrally by four S^{2-} ions and vice versa. The reason for this is thought to be that the Zn—S bond has a degree of covalent character which imposes directional constraints on the crystal form adopted.

Diamond Structure

Diamond is one of the two crystalline forms of carbon (see section 9.10). Each carbon atom is covalently bonded to four others. Covalent bonds are highly directional, and we can no longer think in terms of spheres being packed together as closely as possible. The four bonds on each carbon atom point towards the vertices of a regular tetrahedron (Fig. 9.19(a)). Fig. 9.19(b) shows how the tetrahedron can be orientated so that its vertices are at four corners of a cube. The extended structure of diamond is shown in Fig. 9.4(a).

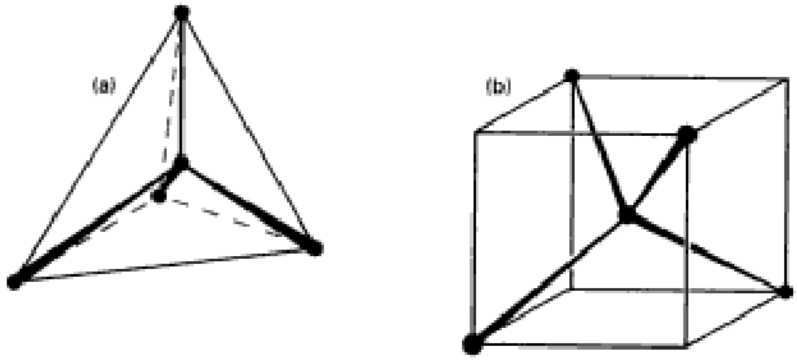


Fig. 9.19
Tetrahedral structure in diamond

9.12 MEASUREMENT OF LINEAR EXPANSIVITY

Fig. 9.20 shows an apparatus for determining the linear expansivity of a material in the form of a rod about 50 cm long. The length, L, of the rod is measured at room temperature. The rod is then placed in the apparatus with one end against the frame, and the micrometer is screwed up until it is in contact with the other end. The micrometer reading, and the temperature, θ_1, are noted. The micrometer is now screwed back to allow the rod room to expand, and steam is passed through the jacket. When the thermometer reading has stopped increasing, the temperature, θ_2, is recorded. The micrometer is brought back into contact with the rod and the reading is noted. The difference, ΔL, between the two micrometer readings is the amount by which the length of the rod has increased. The linear expansivity, α, is calculated from

$$\alpha = \frac{\Delta L}{L(\theta_2 - \theta_1)}$$

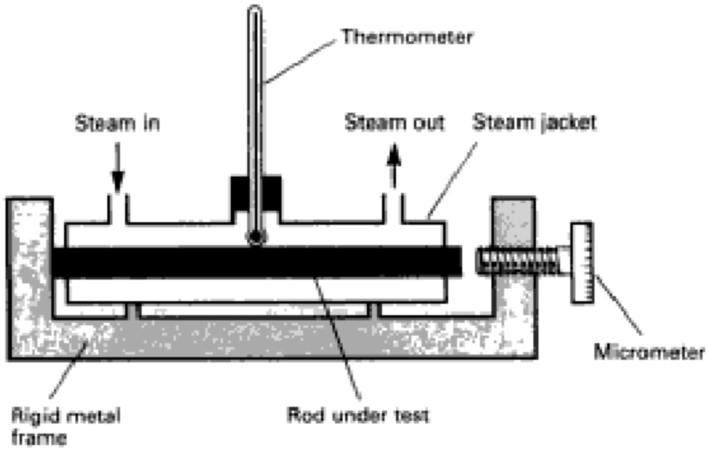


Fig. 9.20 Apparatus for determining linear expansivity

9.13 THE OIL FILM EXPERIMENT

If a small drop of olive oil is placed on the surface of some clean water, the oil spreads to form a large circular film. If it is assumed that the oil spreads until the film is only one molecule thick (i.e. a monomolecular layer), an estimate of the size of an oil molecule can be made by determining the thickness of the film.

It is important that the surface of the water is clean. To this end, water is poured into a large shallow tray until it is overflowing. The surface is then cleaned by drawing two waxed rods across it from the centre outwards. Lycopodium powder is now sprinkled onto the surface so that when the film is formed its edges may be seen easily. A small, spherical drop of oil is obtained on a V-shaped fine wire by dipping it into the oil. The diameter of the drop is measured by holding it in front of a millimetre scale and viewing it through a magnifying glass, or by using a travelling microscope. The drop is then touched onto the water surface. The oil spreads and pushes the lycopodium powder outwards to leave a clear film of oil whose diameter can be measured.

$$\text{Volume of oil drop} = \text{Volume of film}$$

$$\therefore \quad \tfrac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \pi\left(\frac{D}{2}\right)^2 t$$

where

d = diameter of oil drop

D = diameter of film

t = thickness of film

Rearranging gives

$$t = \frac{2}{3}\frac{d^3}{D^2}$$

hence t.

A drop with a diameter of 0.5 mm produces a film with a diameter of about 200 mm, and gives a value for t of approximately 2 nm. Oil molecules are long and thin, and 'stand on end' on water. It follows that this figure of 2 nm represents the length of the molecule.

CONSOLIDATION

Kinetic energy of molecules depends on temperature.

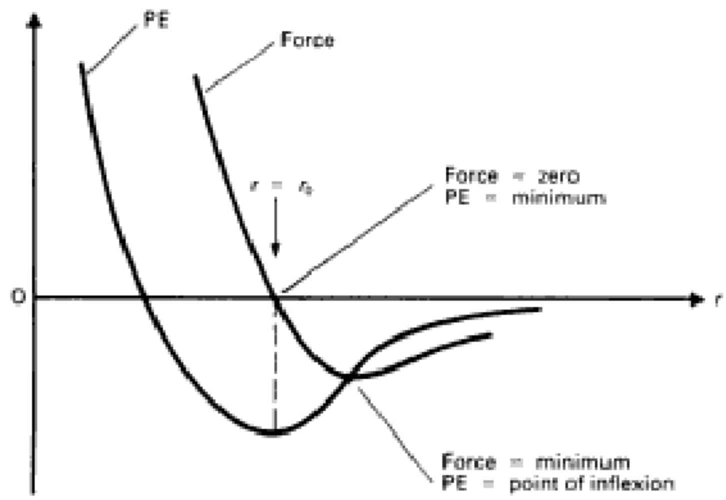
Potential energy of each of two molecules is taken to be zero at infinite separation because they can have no influence on each other at infinite separation. The PE is negative at the equilibrium separation because work has to be done (and therefore the PE has to be increased) to separate the molecules to infinity.

Intermolecular PE and Force Curves

Minimum on PE curve corresponds to zero on force curve.

Point of inflexion on PE curve corresponds to minimum on force curve.

Force = − (gradient of PE curve)



Solids

Fixed volume, fixed shape.

Molecules vibrate about fixed positions.

$$L_1 = L_0 + \alpha L_0(\theta_1 - \theta_0)$$

Liquids

Fixed volume, no fixed shape.

Molecules vibrate about non-fixed positions.

10

FLUIDS AT REST

10.1 INTRODUCTION

This chapter is concerned with fluids. A fluid is a substance that can flow; it follows that **both liquids and gases are fluids**.

An important concept in connection with fluids is that of pressure. The pressure in a fluid depends on its density, and we shall begin the chapter by discussing density.

10.2 DENSITY

The density of a substance is defined by

$$\rho = \frac{m}{V} \qquad [10.1]$$

where

ρ = density of substance ($kg\,m^{-3}$)

m = mass of substance (kg)

V = volume of substance (m^3)

The relative density of a substance is defined by

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water (at 4°C)}} \qquad [10.2]$$

Relative density has no units.

The specific volume of a substance is the reciprocal of its density, i.e. it is the volume of unit mass of the substance. Unit = $m^3\,kg^{-1}$. (Note the use of the word 'specific' to denote unit mass, as it does in specific heat capacity, etc.)

Methods of determining densities by experiment are summarized in section 10.14.

10.3 PRESSURE

The pressure acting on a surface is defined as the force per unit area acting at right angles to the surface, i.e.

$$p = \frac{F}{A} \qquad [10.3]$$

where

p = pressure on surface (SI unit = **the pascal** (Pa). 1 Pa = 1 N m^{-2}.)

F = the force acting at right angles to the surface (N)

A = the area over which the force is acting (m^2).

Note The SI unit of pressure is the pascal. Other units in common use are the atmosphere (atm), the millimetre of mercury (mmHg) and the bar. None of these is an SI unit. Standard atmospheric pressure is 1.01×10^5 Pa (3 sig. fig.) and in these various other units it is 1 atm (exactly), 760 mmHg and 1.01 bar.

10.4 PRESSURE IN FLUIDS

(a) The pressure in a fluid increases with depth. All points at the same depth in the fluid are at the same pressure.

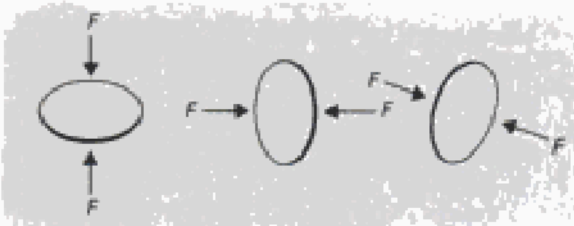
(b) Any surface in a fluid experiences a force due to the pressure of the fluid.

(i) The force is perpendicular to the surface no matter what the orientation of the surface.

(ii) The magnitude of the force is independent of the orientation of the surface.

This final statement is illustrated in Fig. 10.1 and is often stated as **'pressure acts equally in all directions'**.

Fig. 10.1
'Pressure acts equally in all directions'



$F = p\delta A$ in each case where p = pressure in fluid and δA = area of surface. (The surfaces are assumed to be small so that variation of pressure with depth may be ignored.)

Note Though the force associated with the pressure at a point is a vector quantity, the pressure itself is a scalar, i.e. pressure has no direction. (The statement that pressure acts equally in all directions can be misleading in this respect!) Consider the pressure at a point in a fluid.

We cannot assign a direction to the pressure – all we can do is assign a direction to the force that the pressure creates on some surface placed in the fluid, and this depends on the orientation of the surface.

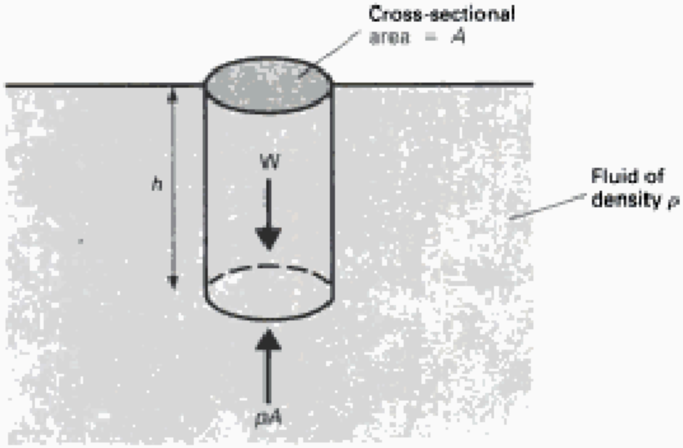


Fig. 10.2
To calculate pressure as a function of depth

Pressure Variation with Depth

Consider a cylindrical region of cross-sectional area A and height h in a fluid of density ρ (Fig. 10.2). The top of the cylinder is at the surface of the fluid, and the (vertical) forces acting on it are its weight, W, and an upward directed force of pA due to the pressure, p, at the bottom of the cylinder. The cylinder is in equilibrium and therefore

$$pA = W$$
$$= \text{mass of cylinder} \times g$$
$$= \text{volume of cylinder} \times \rho g$$
$$= hA\rho g$$

$$\therefore \quad p = h\rho g \qquad [10.4]$$

where p is the pressure due to the fluid at a depth h below the surface.

Notes

(i) Equation [10.4] is not valid in the case of gases when h is large. The density of a gas decreases with height, and the equation has been derived on the assumption that the density is constant. The equation is a reasonable approximation when h is small. However, the densities of gases are low, and when h is small the pressure variation with depth is also small and is usually ignored.

(ii) We have derived equation [10.4] by considering a cylindrical region within the fluid. The same result would have been obtained whatever shape the region had been taken to be.

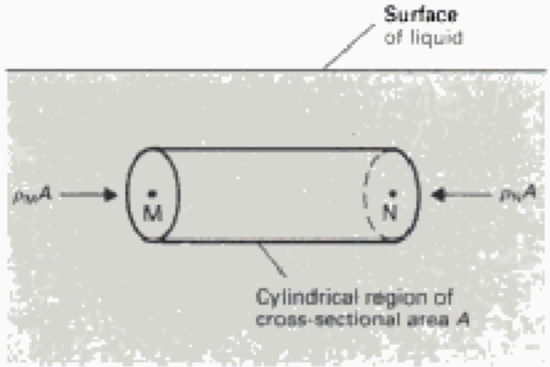
(iii) A little thought should convince the reader that the difference in pressure, Δp, between two points separated by a vertical distance h in a fluid of density ρ is given by

$$\Delta p = h\rho g$$

10.5 WHY THE SURFACE OF A LIQUID IS HORIZONTAL

Consider two points, M and N, on the same horizontal level in a stationary liquid. Consider also a cylindrical region of cross-sectional area A whose end faces are centred on M and N (Fig. 10.3).

Fig. 10.3
To show that all points on the same level are at the same pressure



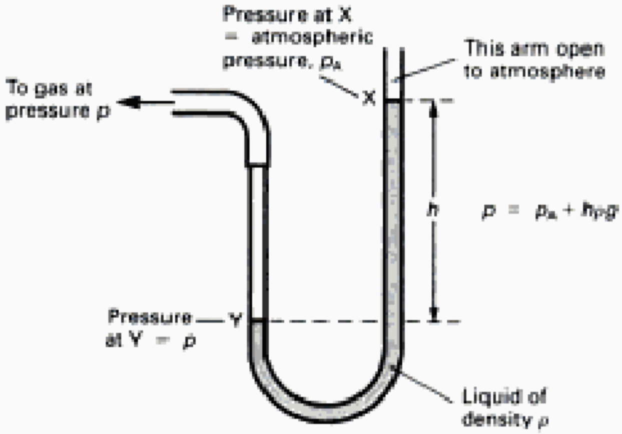
The horizontal forces acting on the cylinder are $p_M A$ and $p_N A$ as shown, where p_M and p_N are the fluid pressures at M and N respectively. The cylinder is in equilibrium and therefore $p_M A = p_N A$. It follows that $p_M = p_N$ and therefore that **all points on the same horizontal level are at the same pressure**.

All points in the liquid surface must be at atmospheric pressure, p_A, and it follows from equation [10.4], therefore, that they must all be at the same height above MN. Since MN is horizontal, the surface must also be horizontal.

10.6 THE U-TUBE MANOMETER

This consists of a U-shaped tube containing a liquid. It is used to measure pressure. The pressure to be measured (that of a gas, say) is applied to one arm of the manometer; the other arm is open to the atmosphere (Fig. 10.4).

Fig. 10.4
The (open) U-tube manometer



The liquid surface at Y is a vertical distance h below that at X. Therefore, by equation [10.4],

$$p = p_A + h\rho g$$

hence p.

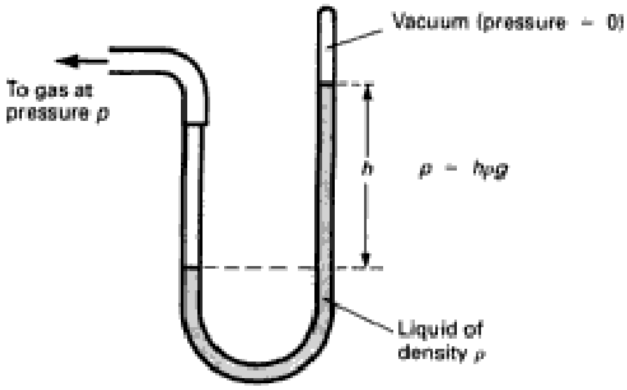
Notes

(i) Manometers can be used to measure pressures both above and below atmospheric pressure.

(ii) Mercury is used as the manometer liquid unless the pressure being measured is close to atmospheric pressure, in which case a liquid of lower density (e.g. oil or water) is more suitable.

(iii) The pressure registered by the manometer, $h\rho g$, is known as the **gauge** pressure. The actual pressure, $p_A + h\rho g$, is called the **absolute** pressure. The manometer shown in Fig. 10.5, in which the arm on the right is closed and evacuated, registers absolute pressure directly.

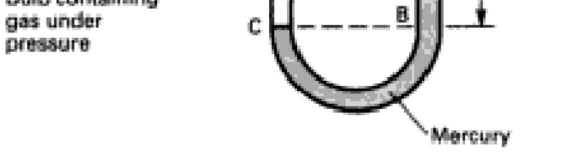
Fig. 10.5
U-tube manometer for absolute pressure



EXAMPLE 10.1

Refer to Fig. 10.6. Calculate the pressure of the gas in the bulb. (Atmospheric pressure = 1.01×10^5 Pa, density of mercury = $1.36 \times 10^4\,\mathrm{kg\,m^{-3}}$, $g = 9.81\,\mathrm{m\,s^{-2}}$.)

Fig. 10.6
Diagram for Example 10.1



Solution

$$\text{Pressure at A} = \text{atmospheric pressure} = 1.01 \times 10^5\,\text{Pa}$$

Since pressure increases with depth,

$$\begin{aligned}\text{Pressure at B} &= 1.01 \times 10^5 + 0.120 \times 1.36 \times 10^4 \times 9.81 \\ &= 1.01 \times 10^5 + 0.16 \times 10^5 \\ &= 1.17 \times 10^5\end{aligned}$$

Since C is at the same level as B,

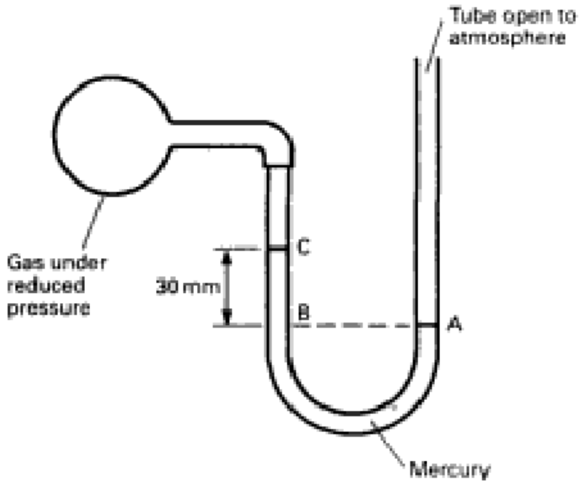
$$\text{Pressure at C} = 1.17 \times 10^5$$

i.e. $\text{Pressure of gas} = 1.17 \times 10^5\,\text{Pa}$

EXAMPLE 10.2

Refer to Fig. 10.7. Calculate the pressure of the gas in the bulb. (Atmospheric pressure = 760 mmHg.)

Fig. 10.7
Diagram for Example 10.2



Solution

Pressure at A = 760 mmHg

∴ Pressure at B = 760 mmHg

∴ Pressure at C = 760 − 30 = 730 mmHg

i.e. Pressure of gas = 730 mmHg

QUESTIONS 10A

Where necessary and unless otherwise stated use the following data: atmospheric pressure = 760 mmHg, density of mercury = $1.36 \times 10^4\ \text{kg m}^{-3}$, $g = 9.81\ \text{m s}^{-2}$.

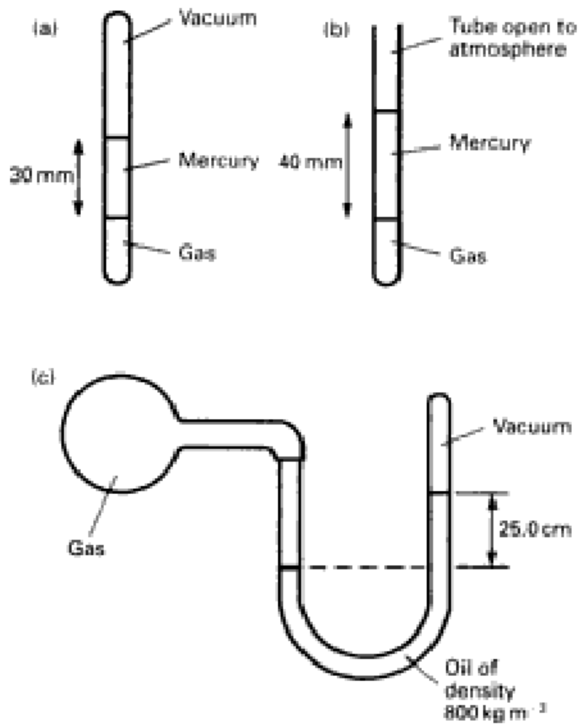
1. An open U-tube manometer containing an oil of density $897\ \text{kg m}^{-3}$ is used to measure the pressure of a gas. The oil level in the open tube is 25.0 cm higher than that in the limb connected to the gas. Calculate **(a)** the gauge pressure, **(b)** the absolute pressure of the gas. (Atmospheric pressure = 9.98×10^4 Pa.)

2. What is the atmospheric pressure (in pascals) on a day when a mercury barometer is reading 772 mmHg?

3. A beaker of cross-sectional area $60\ \text{cm}^2$ contains $600\ \text{cm}^3$ of mercury. Find the pressure on the inner surface of the base of the beaker.

4. The pressure on the upper surface of a submerged submarine is 1.20×10^6 Pa; the pressure on the base of the hull is 1.40×10^6 Pa. Calculate the height of the submarine. (Density of seawater = $1.04 \times 10^3\ \text{kg m}^{-3}$.)

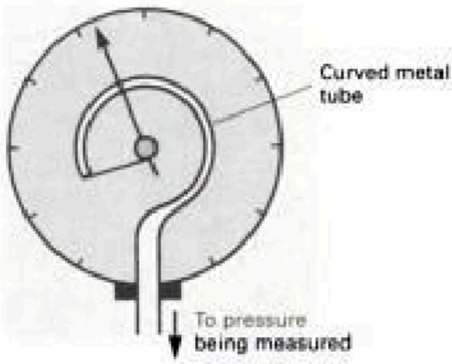
5. Find the pressure of the enclosed gas in each of the following situations.



10.7 THE BOURDON GAUGE

The Bourdon gauge (Fig. 10.8) has a curved metal tube which is closed at one end and of elliptical cross-section. The closed end is linked to a pointer. If the pressure in the tube increases, the tube straightens slightly and moves the pointer over the scale.

Fig. 10.8
The Bourdon gauge



The gauge detects the difference in pressure between the inside and outside of the curved tube, i.e. it detects the difference between the pressure being measured and the atmospheric pressure prevailing at the time. In this sense, then, it behaves like the open tube manometer shown in Fig. 10.4.

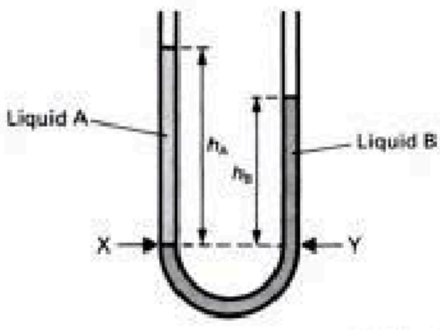
The scale is calibrated in some suitable unit of pressure (e.g. Pa, mmHg, atm). Some gauges are calibrated in such a way that when they are open to the atmosphere the scale reading is zero, i.e. the pointer 'starts' from zero. The absolute (actual) pressure is obtained by adding the value of the prevailing atmospheric pressure to the scale reading. Others have the pointer offset so that it 'starts' at a reading of one atmosphere (or its equivalent, e.g. 760 mmHg or 1.0×10^5 Pa) and gives a reading which is (approximately) equal to the actual value of the pressure being measured. The reading is approximate because the gauge cannot take account of variations in atmospheric pressure.

Bourdon gauges with an extensive variety of pressure ranges are available, and they can be used to measure pressures below atmospheric pressure as well as above it. Some gauges are used to measure (actual) pressures of as little as one millimetre of mercury, whilst others have ranges extending up to a few thousand atmospheres.

10.8 BALANCING COLUMNS

Fig. 10.9 shows a U-tube containing two immiscible liquids (i.e. liquids that do not mix with each other, such as paraffin and water).

Fig. 10.9
Balancing columns



The pressure, p_X, at X is equal to atmospheric pressure, p, plus the pressure exerted by the head, h_A, of liquid A, i.e.

$$p_X = p + h_A \rho_A g$$

where ρ_A is the density of liquid A. Similarly, the pressure, p_Y, at Y is given by

$$p_Y = p + h_B \rho_B g$$

where ρ_B is the density of liquid B. Since X and Y are at the same level in liquid B, $p_X = p_Y$, and therefore

$$p + h_A \rho_A g = p + h_B \rho_B g$$

$$\therefore \quad h_A \rho_A g = h_B \rho_B g$$

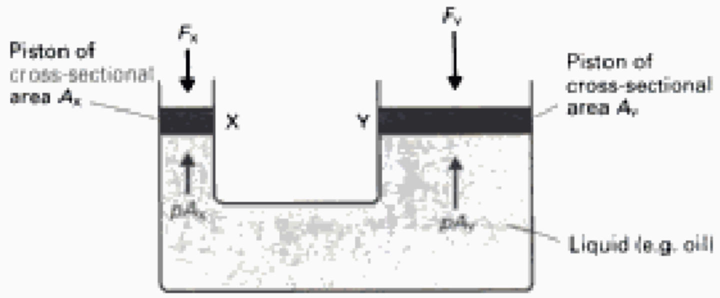
$$\text{i.e.} \quad \frac{\rho_A}{\rho_B} = \frac{h_B}{h_A}$$

The ratio of the densities of the liquids can therefore be found by measuring h_A and h_B. If liquid B is water, h_B/h_A is the relative density of liquid A.

10.9 THE HYDRAULIC JACK. PASCAL'S PRINCIPLE

Fig. 10.10 illustrates the operating principle of a hydraulic jack in which a downward directed force, F_X, is being used to balance a much larger force, F_Y. The pressure in the liquid at both X and Y is p. The liquid therefore exerts upward directed forces of pA_X and pA_Y on the pistons at X and Y respectively.

Fig. 10.10
The hydraulic jack



It follows that

$$F_X = pA_X \quad \text{and} \quad F_Y = pA_Y$$

Eliminating p between these equations gives

$$\frac{F_X}{F_Y} = \frac{A_X}{A_Y}$$

Thus, the ratio of the forces is equal to the ratio of the areas of the respective pistons. Bearing in mind that the areas of the pistons are proportional to the squares of their diameters, it follows that if the diameter of the piston at Y is ten times that of X, then a moderate effort of 100 N, say, at X could move a load of 10 000 N at Y. However, (assuming that the liquid is total incompressible) the effort has to move one hundred times further than the load is moved. Hydraulic braking systems and hydraulic presses work in a similar fashion.

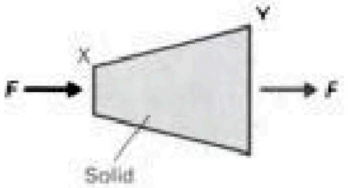
Suppose that the pressure at X and Y in the absence of the forces F_X and F_Y is p_0. When the forces are applied the pressure at both X and Y increases by $(p - p_0)$. In fact, the pressure at every point of the liquid increases by $(p - p_0)$. This is an illustration of **Pascal's principle**, which can be stated as:

Any pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid and to the walls of its container regardless of its shape.

Notes (i) Although Pascal's principle applies to both liquids and gases, a gas cannot be used as the working fluid in a hydraulic jack because gases are compressible. Most of the effort would go into compressing the gas rather than into moving the load.

(ii) Pascal's principle illustrates an important difference between fluids and solids, namely that **a fluid transmits pressure (unchanged), whereas a solid transmits force (unchanged)**. Consider Fig. 10.11. If a force, F, is applied to the (smaller) left-hand face, X, of the solid, the solid (assuming that it does not move) exerts the same force on anything in contact with its right-hand face, Y, even though the faces are not the same size. Thus, when the force at X increases, the force at Y increases by the same amount. However, the increase in pressure at X is greater than that at Y.

Fig. 10.11
Solids transmit force

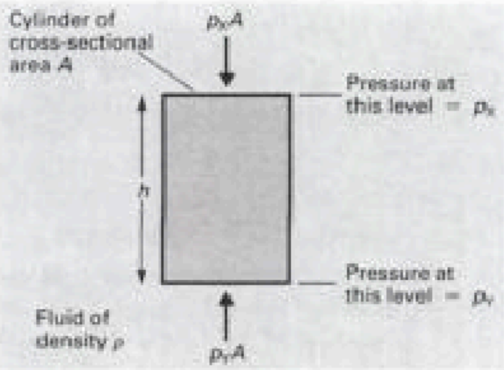


10.10 ARCHIMEDES' PRINCIPLE

A body immersed in a fluid (totally or partially) experiences an upthrust (i.e. an apparent loss of weight) which is equal to the weight of fluid displaced.

The principle is easily deduced. Consider a cylinder of height h and cross-sectional area A a distance h_0 below the surface of a fluid of density ρ (Fig. 10.12).

Fig. 10.12
To deduce Archimedes' principle



$$\text{Volume of fluid displaced} = \text{Volume of cylinder}$$
$$= Ah$$
$$\therefore \quad \text{Mass of fluid displaced} = Ah\rho$$
$$\therefore \quad \text{Weight of fluid displaced} = Ah\rho g \qquad [10.5]$$

The fluid exerts forces of $p_X A$ and $p_Y A$ on the top and bottom faces of the cylinder. The upthrust (i.e. the resultant upward force due to the fluid) is therefore given by

$$\begin{aligned}\text{Upthrust} &= p_Y A - p_X A \\ &= (h + h_0)\rho g A - h_0 \rho g A \qquad \text{(by equation [10.4])} \\ &= h\rho g A\end{aligned}$$

Therefore, by equation [10.5],

$$\text{Upthrust} = \text{Weight of fluid displaced}$$

which is Archimedes' principle. (Note, the result clearly does not depend on the fact that we have considered a cylinder.) Archimedes' principle can be verified by experiment (see section 10.12).

If a body is more dense than the fluid in which it is immersed, then its weight is greater than the weight of the fluid it displaces. By Archimedes' principle, therefore, its weight is greater than the upthrust, and it falls down through the fluid unless it is supported in some way. A body which is less dense than the fluid around it, on the other hand, experiences a net upward force and rises up through the fluid.

When a body floats the upthrust on it must be equal to its weight for it moves neither up nor down. It follows from Archimedes' principle, therefore, that the weight of the body is equal to the weight of the fluid displaced. This is known as the **principle of flotation**. As we have just seen, it is a special case of Archimedes' principle. It can be stated as:

A floating body displaces its own weight of fluid.

The principle of flotation, like Archimedes' principle itself, applies to both partially immersed bodies (e.g. ships) and totally immersed bodies (e.g. submarines and airships).

EXAMPLE 10.3

An object is weighed with a spring balance, first in air and then whilst totally immersed in water. The readings on the balance are 0.48 N and 0.36 N respectively. Calculate the density of the object. (Density of water = $1.0 \times 10^3\,\text{kg m}^{-3}$.)

Solution

The object has the same volume as the water it displaces and therefore

$$\frac{\text{Density of object}}{\text{Density of water}} = \frac{\text{Mass of object}}{\text{Mass of water displaced}}$$

$$= \frac{\text{Weight of object}}{\text{Weight of water displaced}}$$

By Archimedes' principle, weight of water displaced = upthrust, and therefore

$$\frac{\text{Density of object}}{\text{Density of water}} = \frac{\text{Weight of object}}{\text{Upthrust in water}} \qquad [10.6]$$

$$\therefore \quad \text{Density of object} = \frac{0.48}{0.12} \times 1.0 \times 10^3$$
$$= 4.0 \times 10^3\,\text{kg}\,\text{m}^{-3}$$

10.11 MEASUREMENT OF DENSITY USING ARCHIMEDES' PRINCIPLE

Solids

Weigh the solid in air and in water, and then use equation [10.6].

Liquids

The density of a liquid can be found by determining the upthrust on some suitable object when it is immersed in the liquid and then when it is immersed in water.

By analogy with equation [10.6]

$$\frac{\text{Density of object}}{\text{Density of liquid}} = \frac{\text{Weight of object}}{\text{Upthrust in liquid}} \qquad [10.7]$$

Dividing equation [10.6] by equation [10.7] gives

$$\frac{\text{Density of liquid}}{\text{Density of water}} = \frac{\text{Upthrust in liquid}}{\text{Upthrust in water}}$$

from which the density of the liquid can be found.

10.12 VERIFICATION OF ARCHIMEDES' PRINCIPLE BY EXPERIMENT

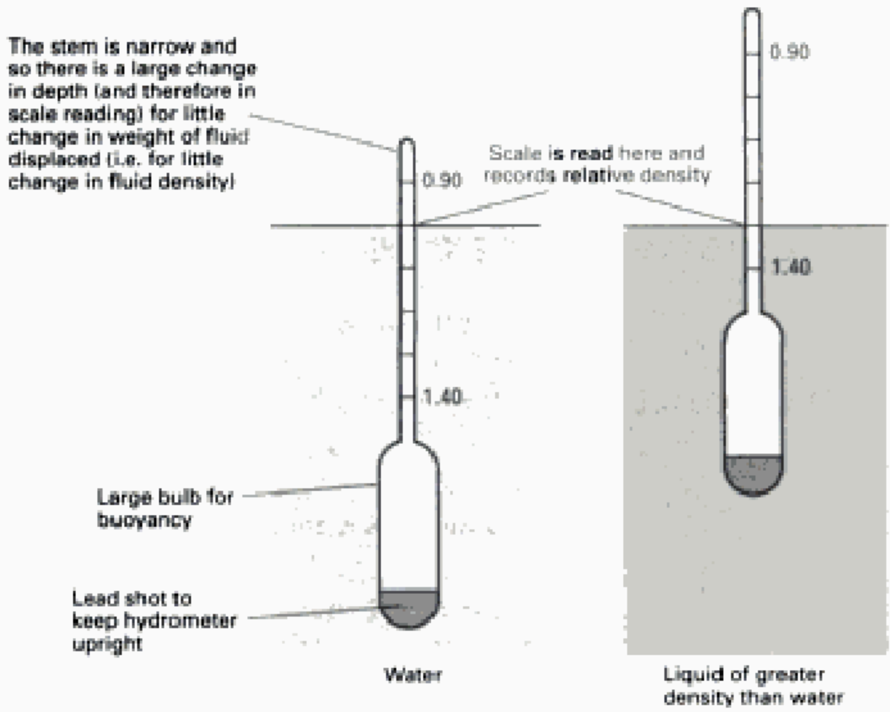
Suspend a glass stopper from a spring balance to obtain the weight of the stopper in air. Gently lower the stopper into a displacement can filled to the spout with water. The difference between the two spring balance readings is the upthrust on the stopper. Collect the water that runs out of the can in a previously weighed beaker. Weigh the beaker with the water in it to find the weight of the water displaced by the stopper. If the weight of the water is equal to the upthrust, Archimedes' principle has been verified.

10.13 THE HYDROMETER

The hydrometer provides a quick method of measuring the relative densities of liquids.

In accordance with the principle of flotation, whenever the hydrometer floats in a liquid the weight of the liquid it displaces is equal to its own weight. It follows that it sinks further into water, say, than it does into a liquid of higher density (Fig. 10.13).

Fig. 10.13
The hydrometer



10.14 SUMMARY OF METHODS OF DETERMINING DENSITIES AND RELATIVE DENSITIES

(i) Measure the mass (m) and volume (V) and then use $\rho = m/V$ to obtain the density.

The mass should be found by using a beam balance or top-pan balance rather than a spring balance – a spring balance measures weight. The volume of a solid may be found by measuring its dimensions or by a displacement method. The volume of a liquid may be found by using a measuring cylinder, pipette or burette. The volume of a gas may be found by enclosing it in a container of known volume.

(ii) The method of 'balancing columns' can be used for liquids. (See section 10.8.)

(iii) The hydrometer can be used for liquids. (See section 10.13.)

(iv) The method based on Archimedes' principle can be used for liquids and solids. (See section 10.11.)

(v) Relative density bottle can be used for liquids and fine powders. (See GCSE texts.)

10.15 SURFACE TENSION

A steel needle can be caused to float on water even though steel is more dense than water. A liquid spilled on to a surface that it does not wet tends to form into small drops, rather than spread into a continuous film. These are two examples of phenomena which suggest that the surface of a liquid behaves like an elastic skin in a state of tension. This is indeed the case and can be understood by a consideration of the effects of intermolecular forces.

10.16 MOLECULAR EXPLANATION OF SURFACE TENSION EFFECTS

A molecule in the surface of a liquid is subject to intermolecular forces from below, but not from above (providing the effects of the molecules of the vapour are ignored). Thus, if the coordination number of the molecules of the interior is n, then that of a surface molecule will be $n/2$. Therefore, if a molecule of the interior has a potential energy of (say) -0.4 eV, then a surface molecule, being involved in only half as many bonds, will have a potential energy of -0.2 eV. Thus, **the potential energy of a molecule in the surface exceeds that of one in the interior**.

All systems arrange themselves in such a way that they have the minimum possible potential energy. In order that the potential energy associated with the intermolecular forces (the surface tension forces) can be a minimum, the number of molecules which reside in the surface has to be a minimum. Therefore:

(i) liquids have the smallest possible surface area, and

(ii) the average separation of the molecules in the surface of a liquid is greater than that of molecules in the interior.

The requirement that the surface area is a minimum means that a liquid subject to surface tension forces only, will assume the shape of a sphere. (This is because a sphere is the shape which allows a given volume of material to have the smallest possible surface area.) Liquids are normally subject to gravitational forces in addition to surface tension forces, in which case the adopted shape is that which minimizes the total potential energy. Small drops of liquid are nearly spherical, and become more so as the drop decreases in size. This is because the ratio of the surface area (which is proportional to r^2) to the weight (which is proportional to r^3) and therefore of surface tension force to gravitational force, increases as r decreases. Soap bubbles are almost perfect spheres because they have large surface areas and negligible masses. The effect of gravity can be eliminated by using two immiscible liquids of the same density (Fig. 10.14). The phenylamine (aniline) and water are at such a temperature that their densities are equal, in which case the upthrust on each phenylamine globule is exactly equal to its weight, and therefore the globule is not subject to any net gravitational force. Drops of liquid which are falling freely under gravity are also spherical. This is because every part of the drop is being accelerated to the same extent and the acceleration cannot, therefore, affect the shape of the drop.

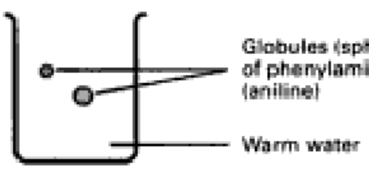


Fig. 10.14
Eliminating the effects of gravity

The molecules in the interior of a liquid are, of course, at their equilibrium separation, and therefore the attractive forces of their neighbours are balanced by the repulsive forces. This is not true of the surface molecules, the separation of these is greater than the equilibrium separation (requirement (ii)), and therefore they exert a net attractive force on each other. Thus, at any point in the surface of a liquid there is a net force away from that point due to the attractions of the molecules around it. **The surface therefore behaves like an elastic skin in a state of tension**.

10.17 SURFACE TENSION AND FREE SURFACE ENERGY

The surface tension γ of a liquid is defined as the force per unit length acting in the surface and perpendicular to one side of an imaginary line drawn in the surface. (Unit = $N\,m^{-1}$.)

Free surface energy σ is defined as the work done in isothermally creating unit area of new surface. (Unit = $J\,m^{-2} = N\,m^{-1}$.)

Whenever the surface area of a given volume of liquid is increased, work has to be done against the surface tension forces. Alternatively, one may think of the work being necessary to provide the extra energy needed to have an increased number of molecules in the surface.

Consider stretching a thin film of liquid on a horizontal frame (Fig. 10.15). Since the film has both an upper and lower surface, the force F on AB due to surface tension is given by

$$F = 2L\gamma$$

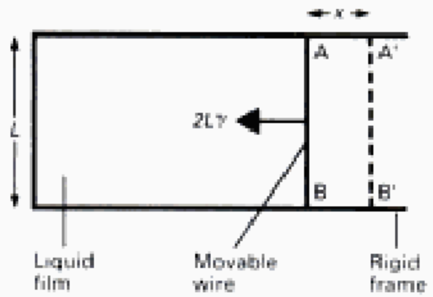


Fig. 10.15
A thin film of liquid being stretched

If AB is moved a distance x to A′B′, then work has to be done against this force. The surface tension, γ, is independent of the area of the film (because as the size of the surface increases more molecules enter it and by so doing maintain the average molecular separation), but decreases with increasing temperature (because this decreases the binding energy). Thus, provided AB is moved isothermally to A′B′, the force on AB will be constant, and therefore since

$$\text{Work done} = \text{Force} \times \text{distance}$$

$$\text{Work done} = 2L\gamma x$$

The increase in surface area is $2Lx$ (upper and lower surfaces), and therefore the work done per unit area increase (the free surface energy σ) is given by

$$\sigma = \frac{2L\gamma x}{2Lx}$$

i.e. $\sigma = \gamma$

Thus, the free surface energy σ is equal to the surface tension γ. This provides a second definition of γ:

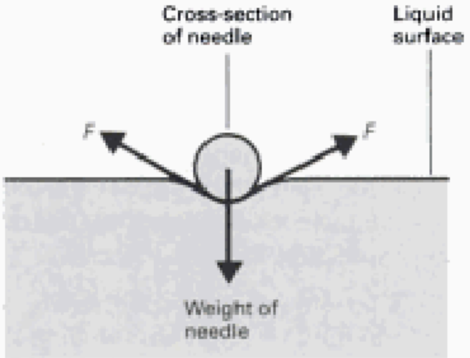
The surface tension γ is the work done in isothermally increasing the surface area of the liquid by unit area. (Unit $= \mathrm{J\,m^{-2}} = \mathrm{N\,m^{-1}}$.)

10.18 SOME SURFACE TENSION PHENOMENA

Floating Needle

The needle (Fig. 10.16) creates a depression in the liquid surface so that the surface tension forces $\boldsymbol{F}$ (which act in the surface) now have an upward directed component which is capable of supporting the weight of the needle.

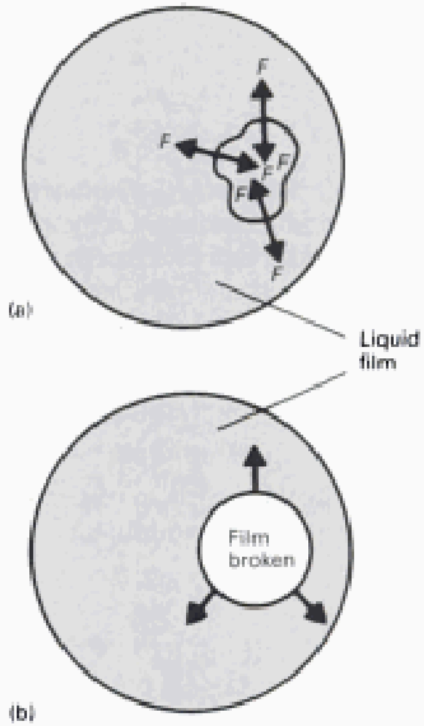
Fig. 10.16
A needle supported by surface tension forces



Thread on a Soap Film

In Fig. 10.17(a) there are equal and opposite forces on each side of the thread and therefore it stays where it has been placed. If the film is broken in the region bounded by the thread (Fig. 10.17(b)), there are forces on the outside of the thread only. The thread is therefore pulled into a circle (the shape with the maximum area for a given perimeter) and therefore the liquid film has the minimum possible area.

Fig. 10.17
To show that a liquid attains the minimum possible surface area



Camphor Boat

The camphor (Fig. 10.18) sublimes and interacts with the water at the back of the boat, reducing the surface tension there, so that F' is less than F. There is therefore a net forward force which drags the boat through the water.

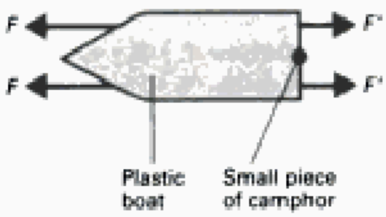


Fig. 10.18 Camphor boat

10.19 ANGLE OF CONTACT

The surface of a liquid is usually curved where it is in contact with a solid. The particular form that this curvature takes is determined by the relative strengths of what are called the cohesive and adhesive forces.

The cohesive force is the attractive force exerted on a liquid molecule by the neighbouring liquid molecules.

The adhesive force is the attractive force exerted on a liquid molecule by the molecules in the surface of the solid.

Consider a liquid in a container with vertical sides. If the adhesive force is large compared with the cohesive force, the liquid tends to stick to the wall and so has a concave meniscus (Fig. 10.19(a)). On the other hand, if the adhesive force is small compared with the cohesive force, the liquid surface is pulled away from the wall and the meniscus is convex (Fig. 10.19(b)). Whether the meniscus is concave or convex depends on the liquid concerned and on the solid with which it is in contact. For example, water has a concave meniscus when in contact with glass and a convex meniscus when in contact with wax; mercury has a convex meniscus with (clean) glass.

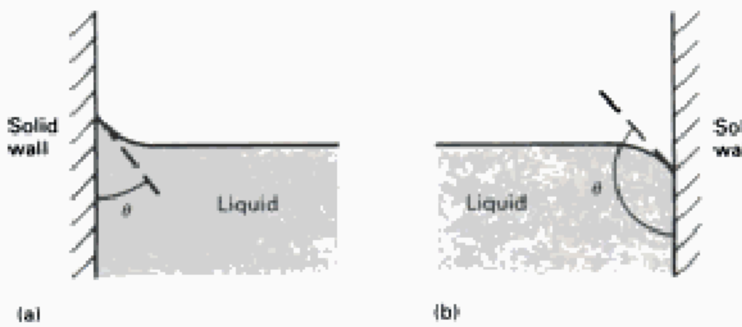


Fig. 10.19 (a) Concave, and (b) convex menisci

The angle of contact θ is defined as the angle between the solid surface and the tangent plane to the liquid surface at the point where it touches the solid; the angle is measured through the liquid. It can be seen from Fig. 10.19 that the meniscus is concave when θ is less than 90° and is convex when θ is greater than 90°. A liquid is said to 'wet' a surface with which its angle of contact is less than 90°. The angle of contact between water and clean glass is zero, that between mercury and clean glass is 137°. Thus water 'wets' clean glass, mercury does not.

The zero angle of contact between water and clean glass is due to the adhesive force between water and glass being very much larger than the cohesive force between the water molecules themselves. This explains why water tends to spread into a thin continuous film when splashed on a horizontal clean glass surface. Mercury, on the other hand, forms into little drops; water on the roof of a freshly waxed car behaves in a similar fashion.

The addition of a detergent to a liquid lowers its surface tension and reduces the contact angle. Water-proofing agents have the opposite effect.

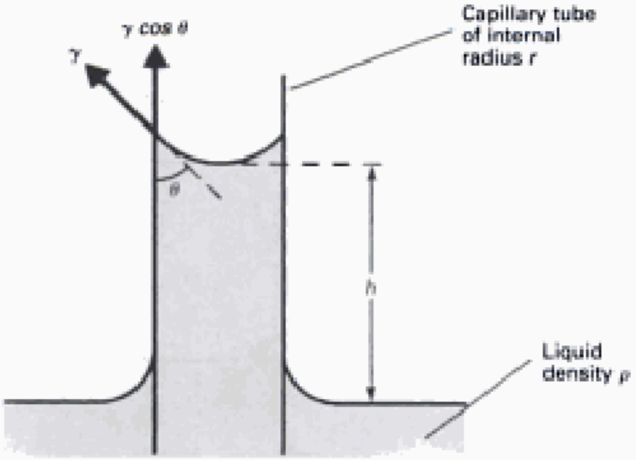
10.20 CAPILLARY RISE. MEASUREMENT OF γ

Water in a capillary tube rises above the level of the water outside. The effect is known as capillary rise and is most marked with narrow tubes. The ability of blotting paper to soak up ink is due to the same effect; the spaces between the fibres act as fine capillary tubes. A liquid whose angle of contact is greater than 90° suffers capillary depression. Both capillary rise and capillary depression are caused by surface tension and provide a means by which the surface tension γ of a liquid may be measured.

Suppose that a capillary tube is held vertically in a liquid which has a concave meniscus (Fig. 10.20). Surface tension forces cause the liquid to exert a downward directed force on the walls of the tube. In accordance with Newton's third law, the tube exerts an equal and opposite force on the liquid and it rises in the tube. At equilibrium the weight of the liquid which has been lifted up is equal to the vertical component of the force exerted by the tube. The mass of the raised liquid is the product of its density ρ and its volume $\pi r^2 h$, and therefore its weight is

$$\rho \pi r^2 hg$$

Fig. 10.20
Liquid in a capillary tube (not to scale)



The length of the liquid surface in contact with the tube is equal to the circumference $2\pi r$ of the tube, and therefore the vertical component of the force exerted by the tube is

$$2\pi r\gamma \cos\theta$$

Therefore at equilibrium

$$2\pi r\gamma \cos\theta = \rho \pi r^2 hg$$

i.e.
$$\gamma = \frac{\rho r h g}{2\cos\theta} \qquad [10.8]$$

from which γ can be determined.

Notes (i) The weight of the small quantity of liquid in the meniscus has been ignored in deriving equation [10.8].

(ii) h and r are normally measured with a travelling microscope. The tube should be broken at the level of the meniscus in order to measure r.

(iii) θ and ρ are found from tables or measured in separate experiments.

(iv) Equation [10.8] also holds for capillary depression.

10.21 PRESSURE DIFFERENCE ACROSS A SPHERICAL INTERFACE

The pressure inside a soap bubble is greater than the pressure of the air outside the bubble. If this were not so, the combined effect of the external pressure and the surface tension forces in the soap film would cause the bubble to collapse. Similarly, the pressure inside an air bubble in a liquid exceeds the pressure in the liquid, and the pressure inside a mercury drop is greater than that outside it.

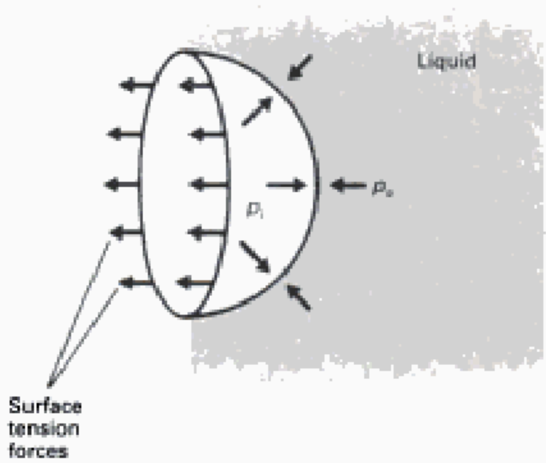
In order to derive an expression for the excess pressure inside an air bubble in a liquid we shall consider the forces acting on one half of such a bubble (Fig. 10.21). Suppose that the radius of the bubble is r and that the surface tension of the liquid is γ. The half not shown exerts a surface tension force around the rim of the half we are considering. This force is directed to the left and, since the length of the rim is $2\pi r$, is of magnitude $2\pi r\gamma$. The resultant force due to the pressure p_o outside is also to the left and is acting perpendicular to an area πr^2 (the area of the flat face of the hemisphere) and is therefore of magnitude $p_o\pi r^2$ since p_o is the force per unit area. The resultant force due to the internal pressure p_i is to the right and its magnitude is $p_i\pi r^2$. The hemisphere is in equilibrium under the action of these forces, and therefore

$$p_i\pi r^2 = p_o\pi r^2 + 2\pi r\gamma$$

i.e.

$$p_i - p_o = \frac{2\pi r\gamma}{\pi r^2}$$

Fig. 10.21
Forces on air bubble in a liquid



Writing the excess pressure $p_i - p_o$ as Δp gives

$$\Delta p = \frac{2\gamma}{r} \quad \text{(for air bubbles and spherical drops)} \quad [10.9]$$

Note The smaller the bubble, the greater the excess pressure.

The excess pressure inside a spherical drop of mercury is given by the same expression if γ and r are taken to represent the surface tension of mercury and the radius of the drop respectively.

A soap film has two surfaces and therefore the excess pressure Δp inside a soap bubble is given by

$$\Delta p = \frac{4\gamma}{r} \quad \text{(for a soap bubble)}$$

where γ is the surface tension of the soap solution and r is the radius of the bubble. (**Note**. This assumes that the inner and outer surfaces have the same radius of curvature – a reasonable approximation.)

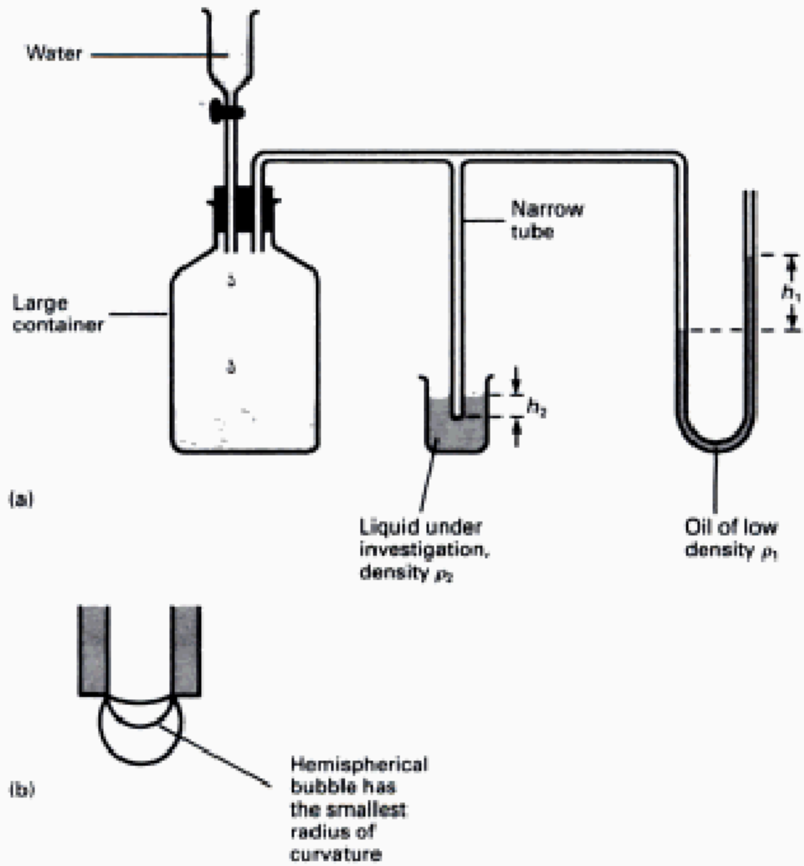
10.22 EXPERIMENTAL DETERMINATION OF γ BY JAEGER'S METHOD

The apparatus is shown in Fig. 10.22(a). When the tap is opened water drips into the large container and increases the pressure in the system. An air bubble starts to form at the lower end of the narrow tube. As more water drips into the container the bubble grows, and as it does so its radius of curvature decreases (see Fig. 10.22(b)).

Suppose that when the radius of the bubble is r the head of the liquid in the manometer is h_1, in which case the pressure inside the bubble is

$$h_1 \rho_1 g + A$$

Fig. 10.22
Apparatus for the determination of γ



where ρ_1 is the density of the manometer liquid, g is the acceleration due to gravity and A is the atmospheric pressure. The pressure outside the bubble is

$$h_2\rho_2 g + A$$

where h_2 is the depth at which the bubble is formed and ρ_2 is the density of the liquid whose surface tension is being measured. The excess pressure in the bubble is therefore

$$h_1\rho_1 g - h_2\rho_2 g$$

The excess pressure in a bubble of this type is given by equation [10.9] as $2\gamma/r$, and therefore

$$\frac{2\gamma}{r} = h_1\rho_1 g - h_2\rho_2 g \qquad [10.10]$$

where γ is the surface tension of the liquid under test. The only variables in this equation are r and h_1, and therefore it follows that h_1 will have its maximum value when r has its minimum value. The bubble has its smallest possible radius of curvature when it is hemispherical, for if it were to grow any larger, its radius would increase. It follows that when h_1 has its maximum value the bubble is hemispherical and its radius of curvature is equal to the internal radius of the tube.

In practice the bubble becomes unstable and breaks away from the end of the tube as soon as its size increases beyond the stage where the bubble is hemispherical. When this happens the pressure in the system falls to atmospheric and another bubble begins to form as more water drips from the funnel. The tap is set so that bubbles form slowly (about one per second). Once a suitable rate has been achieved the maximum value of h_1 is recorded. A travelling microscope can be used to measure r (the internal radius of the lower end of the narrow tube) and h_2. The values of, ρ_1, ρ_2 and g can be obtained from tables and used in equation [10.10], together with the measured values of h_1, h_2 and r, to calculate γ.

Note When a bubble breaks away its radius is not exactly equal to that of the tube. This limits the accuracy to which absolute determinations of γ may be made by Jaeger's method. It does, however, provide a reliable means of investigating the temperature dependence of surface tension. Providing bubbles are formed at the same rate for each measurement, the relative values of γ are very accurate.

CONSOLIDATION

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water (at 4°C)}}$$

$$\text{Specific volume} = \frac{\text{Volume}}{\text{Mass}}$$

The pressure on a surface is defined as the force per unit area acting at right angles to the surface. Pressure is a scalar.

Pressure in Fluids

(a) Increases with depth ($p = h\rho g$)

(b) 'Acts equally in all directions' (Strictly, it is the force due to the pressure that acts equally in all directions.)

Archimedes' Principle A body immersed in a fluid (totally or partially) experiences an upthrust (i.e. an apparent loss of weight) which is equal to the weight of the fluid displaced.

The Principle of Flotation A floating body displaces its own weight of fluid.

The surface tension of a liquid is defined as the force per unit length acting in the surface and perpendicular to one side of an imaginary line drawn in the surface. (Unit = $N\,m^{-1}$.)

Free surface energy is defined as the work done in isothermally creating unit area of new surface. (Unit = $J\,m^{-2} = N\,m^{-1}$.)

Surface tension = Free surface energy

A liquid is said to 'wet' (i.e. stick to) a surface with which its angle of contact is less than 90°.

Solids transmit force; fluids transmit pressure.

11

ELASTICITY

11.1 DEFINITIONS

If forces are applied to a material in such a way as to deform it, then the material is said to be being stressed. As a result of the stress the material becomes strained. Initially we shall be concerned only with solids, and with stress which results in an increase in length (tensile stress) or a decrease in length (compressive stress).

Stress Force per unit area of cross-section.
Unit $= \text{N m}^{-2} =$ pascal (Pa)

Strain $\dfrac{\text{Change in length}}{\text{Original length}}$ (pure number)

Elasticity A material is said to be elastic if it returns to its original size and shape when the load which has been deforming it is removed.

Hooke's law Up to some maximum load (known as the **limit of proportionality**) the extension of a wire (or spring) is proportional to the applied load.

Elastic limit This is the maximum load which a body can experience and still regain its original size and shape once the load has been removed. (The elastic limit sometimes coincides with the limit of proportionality.)

Yield point If the stress is increased beyond the elastic limit, a point is reached at which there is a marked increase in extension. This is the yield point. The internal structure of the material has changed – the crystal planes have (effectively)* slid across each other. The material is said to be showing **plastic** behaviour. Few materials exhibit a yield point – mild steel is one that does.

Strength This relates to the maximum force which can be applied to a material without it breaking.

Breaking stress This is also called **ultimate tensile strength** and is the maximum stress which can be applied to a material.

Stiffness This relates to the resistance which a material offers to having its size and/or shape changed.

Ductility A ductile material is one which can be permanently stretched.

*See section 11.12.

Brittleness A brittle material cannot be permanently stretched; it breaks soon after the elastic limit has been reached. Brittle materials are often very strong in compression.

The stress-strain curve of a (hypothetical) elastic material is shown in Fig. 11.1. The corresponding load–extension curve is slightly different because as the length of the specimen increases its cross-sectional area decreases. The curve shown represents a ductile material; for a brittle material section EB is very short.

If the stress is removed at a point such as C, which is beyond the elastic limit, the body has a permanent strain equal to OO′.

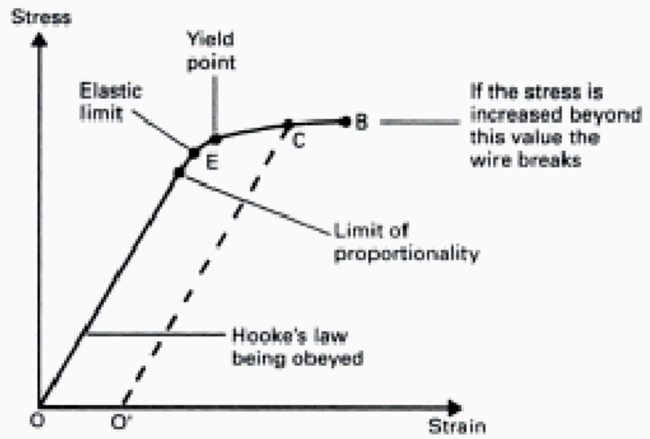


Fig. 11.1
Stress–strain curve for an elastic material

11.2 YOUNG'S MODULUS

Provided the stress is not so high that the limit of proportionality has been exceeded, the ratio stress/strain is a constant for a given material and is known as **Young's modulus**. Thus

$$E = \frac{\text{Tensile (or compressive) stress}}{\text{Tensile (or compressive) strain}} \quad [11.1]$$

where

E = Young's modulus ($N\ m^{-2}$ = Pa).

Young's modulus is clearly a measure of a material's resistance to changes in length. For example:

E (natural rubber)* $= 1 \times 10^6\ N\ m^{-2}$

E (mild steel) $= 2 \times 10^{11}\ N\ m^{-2}$

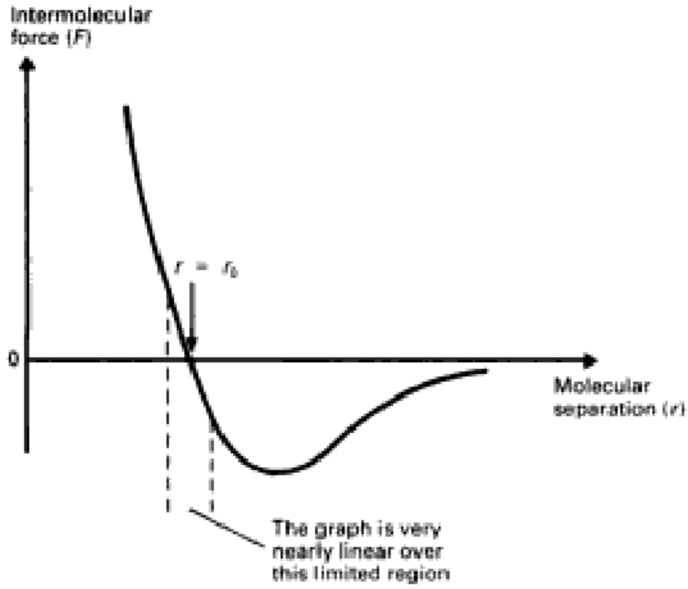
Note Bending a beam involves both tensile and compressive stress – the outer surface is stretched, the inner surface is compressed.

11.3 MOLECULAR EXPLANATION OF HOOKE'S LAW

Consider a plot of intermolecular force, F, against intermolecular separation, r, for a solid (Fig. 11.2). When the stress is zero the mean separation of the molecules is r_0. A tensile stress acts in opposition to the attractive forces between the molecules,

*This is an average value. Rubber does not obey Hooke's law and therefore the ratio stress/strain depends on the stress applied.

Fig. 11.2
Molecular explanation of Hooke's law



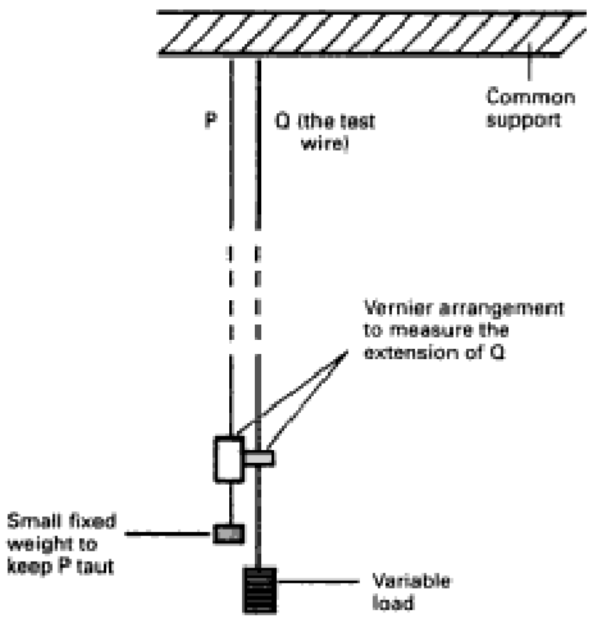
and is therefore capable of increasing their separation. For values of r close to r_0 the graph can be considered to be linear, and therefore, providing the stress is not so large as to take r out of this region, equal increases in tensile stress will produce equal increases in extension (Hooke's law). Note that Hooke's law applies also to compressive stress.

The work done in stretching a wire is stored as elastic potential energy (see section 11.5). On a molecular level this corresponds to the increased potential energy of the molecules which results from their increased separation.

11.4 EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS

Consider the experimental arrangement shown in Fig. 11.3. When Q is loaded there is a tendency for its support to sag. The errors that would result if this were to happen are avoided by carrying the reference scale on a second wire, P, suspended

Fig. 11.3
Apparatus for investigating the extension of a wire



from the same beam as Q. Both P and Q are made from the same material and are of the same length; errors due to expansion, as a result of temperature changes during the experiment, are therefore avoided. The test wire is loaded (typically up to 100 N in 5 N steps), and the resulting extension is measured as a function of the load. The wires are as long as is convenient (typically 2 m) and thin in order to obtain as large an extension as possible; even so a vernier arrangement is needed to measure the extension (typically 1 mm). If the test wire is free of kinks at the start and the limit of proportionality is not exceeded, the measurement can be used to produce a plot similar to that in Fig. 11.4.

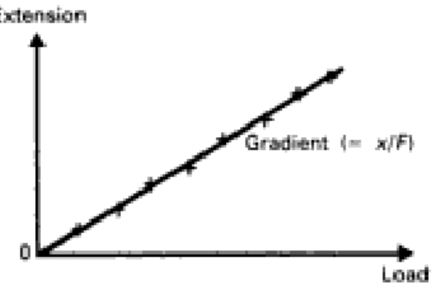


Fig. 11.4 Typical results of extending a wire

From equation [11.1]

$$E = \frac{\text{Stress}}{\text{Strain}}$$

i.e. $$E = \frac{F/A}{x/L} = \frac{L}{A\,(x/F)}$$

where

F = applied load (N)

A = area of cross-section of wire (m^2)

x = extension (m)

L = original length (m).

Bearing in mind that x/F is the gradient of the graph, we have

$$E = \frac{L}{A \times \text{gradient}}$$

The gradient is measured from the graph, often as the mean of the results obtained with an increasing load and a decreasing load. L can be measured with an extending ruler or metre rule. A is obtained by determining the diameter of the wire at several places with a micrometer.

11.5 THE WORK DONE IN STRETCHING A WIRE (STRAIN ENERGY)

Consider a wire whose extension is x when the force on it is F. If the extension is increased by δx, where δx is so small that F can be considered constant, then (by equation [5.1]) the work done, δW, is given by

$$\delta W = F\delta x$$

The total work done in increasing the extension from 0 to x, i.e. the **elastic potential energy** stored in the wire (the **strain energy**) when its extension is x, is given by W, where

$$W = \int_0^x F \mathrm{d}x \qquad [11.2]$$

If the wire obeys Hooke's law, we may put

$$F = kx$$

where k is a constant, and therefore by equation [11.2]

$$W = \int_0^x kx \, \mathrm{d}x$$

i.e.

$$W = \tfrac{1}{2}kx^2 \qquad [11.3]$$

Alternatively, since $F = kx$, substituting for k gives

$$W = \tfrac{1}{2}Fx \qquad [11.4]$$

A wire of length L and cross-sectional area A has a volume of AL and therefore by equation [11.4]

$$\text{Strain energy per unit volume} = \frac{\frac{1}{2}Fx}{AL}$$

$$= \frac{1}{2}\frac{F}{A} \times \frac{x}{L}$$

i.e.

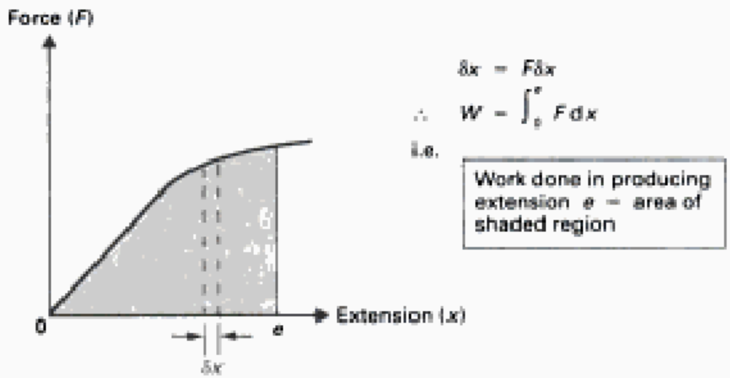
$$\text{Strain energy per unit volume} = \tfrac{1}{2}\,\text{stress} \times \text{strain} \qquad [11.5]$$

Notes (i) For a wire of a material with Young's modulus E it follows from equation [11.1] that $F = (EA/L)x$, i.e. $k = EA/L$ and therefore by equation [11.3]

$$W = \frac{EAx^2}{2L} \qquad [11.6]$$

(ii) Equations [11.3] to [11.6] apply only as long as Hooke's law is obeyed. If the extension is so great that the limit of proportionality is exceeded or the wire does not obey Hooke's law anyway, the work done can be found from a graph of force against extension (Fig. 11.5). **The strain energy per unit volume is the area under a graph of stress against strain.**

Fig. 11.5
Work done in stretching a wire



EXAMPLE 11.1

A steel wire, AB, of length 0.60 m and cross-sectional area $1.5 \times 10^{-6}\,m^2$ is attached at B to a copper wire, BC, of length 0.39 m and cross-sectional area $3.0 \times 10^{-6}\,m^2$. The combination is suspended vertically from a fixed point at A, and supports a weight of 250 N at C. Find the extension of each section of the wire. (Young's modulus of steel = 2.0×10^{11} Pa, Young's modulus of copper = 1.3×10^{11} Pa.)

Solution

Each section of the wire is subject to the full force of 250 N. Let x_1 = extension of AB; let x_2 = extension of BC.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Therefore for steel

$$2.0 \times 10^{11} = \frac{250/1.5 \times 10^{-6}}{x_1/0.60} = \frac{1.00 \times 10^{8}}{x_1}$$

$$\therefore \quad x_1 = \frac{1.00 \times 10^{8}}{2.0 \times 10^{11}} = 5.0 \times 10^{-4}\,\text{m} = 0.50\,\text{mm}$$

For copper

$$1.3 \times 10^{11} = \frac{250/3.0 \times 10^{-6}}{x_2/0.39} = \frac{3.25 \times 10^{7}}{x_2}$$

$$\therefore \quad x_2 = \frac{3.25 \times 10^{7}}{1.3 \times 10^{11}} = 2.5 \times 10^{-4}\,\text{m} = 0.25\,\text{mm}$$

EXAMPLE 11.2

A steel rod of length 0.60 m and cross-sectional area $2.5 \times 10^{-5}\,m^2$ at 100 °C is clamped so that when it cools it is unable to contract. Find the tension in the rod when it has cooled to 20 °C. (Young's modulus of steel = 2.0×10^{11} Pa, linear expansivity of steel = $1.6 \times 10^{-7}\,°C^{-1}$.)

Solution

It follows from equation [9.4] that if the rod were allowed to contract, its length would decrease by

$$0.60 \times 1.6 \times 10^{-7}\,(100 - 20) = 7.68 \times 10^{-6}\,\text{m}$$

The extension of the clamped rod at 20 °C is therefore 7.68×10^{-6} m.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \quad 2.0 \times 10^{11} = \frac{\text{Stress}}{7.68 \times 10^{-6}/0.60}$$

i.e. Stress = 2.56×10^{6} Pa

$$\text{Stress} = \frac{\text{Tension}}{\text{Cross-sectional area}}$$

$$\therefore \quad 2.56 \times 10^{6} = \frac{\text{Tension}}{2.5 \times 10^{-5}}$$

i.e. $\text{Tension} = 64\,\text{N}$

QUESTIONS 11A

1. An aluminium wire of length 0.35 m and radius 0.20 mm is stretched by 1.4 mm. Young's modulus of aluminium is 7.0×10^{10} Pa.
   (a) Find the strain in the wire.
   (b) Find the stress in the wire.
   (c) Find the cross-sectional area of the wire.
   (d) Find the tension in the wire.

11.6 BULK MODULUS AND SHEAR MODULUS

So far we have been concerned only with stress which results in a change in length. Two other types of stress will now be considered. The associated moduli of elasticity are called the **bulk modulus** and the **shear modulus**. The latter is sometimes referred to as the **rigidity modulus**.

Shear (Rigidity) Modulus

A shear stress is one which changes the shape of a body; the strain which results is called a **shear strain**. Fig. 11.6 illustrates a solid block WXYZ whose lower face is fixed. A force F acts on the block tangential to its upper face. The force provides a shear stress which distorts the block so that its new shape is WX′Y′Z. **The shear modulus** G **is defined by**

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} \qquad (\text{Unit} = \text{N m}^{-2} = \text{Pa})$$

where

$$\text{Shear stress} = \text{Tangential force per unit area} = F/A$$

and

$$\text{Shear strain} = \text{Tangent of angle of shear} = \tan\alpha = \Delta x/y$$

i.e.

$$G = \frac{F/A}{\Delta x/y} \qquad [11.7]$$

(**Note**: twisting a wire involves shear stress.)

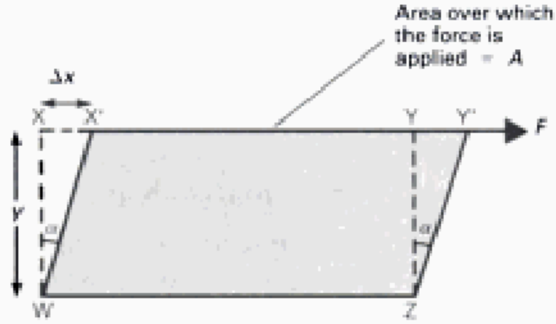


Fig. 11.6
Block subjected to a shear stress

Bulk Modulus

This refers to situations in which the volume of a substance is changed by the application of an external stress. Unlike the shear modulus and Young's modulus, which refer to solids only, bulk moduli are possessed by solids, liquids and gases.

In Fig. 11.7 the application of a force ΔF, which is everywhere normal to the surface of a spherical body, has changed its volume by ΔV. The **bulk modulus** K is defined by

$$K = \frac{\text{Bulk stress}}{\text{Bulk strain}} \qquad (\text{Unit} = \text{N m}^{-2} = \text{Pa})$$

where

$$\text{Bulk stress} = \text{Increased force per unit area} = \Delta F/A$$

and

$$\text{Bulk strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

i.e. $$K = -\frac{F/A}{\Delta V/V}$$

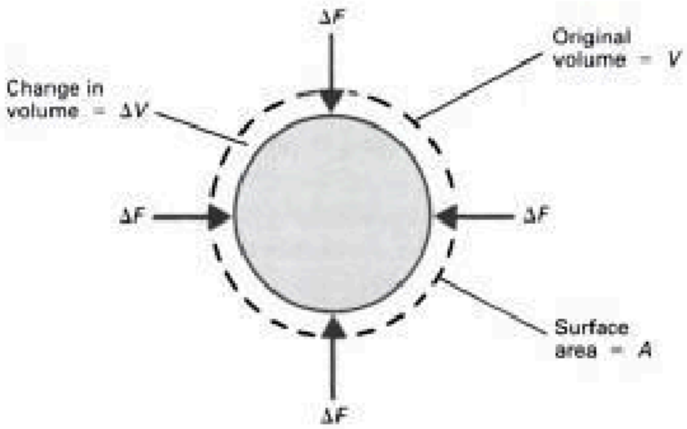


Fig. 11.7 Sphere subjected to a radial stress

Note When ΔF is positive ΔV is negative, and therefore it has been necessary to include the minus sign in order to make K a positive constant.

$\Delta F/A$ is the change in pressure Δp, and therefore

$$K = -\frac{\Delta p}{\Delta V/V}$$

which in the limit as $\Delta p \to 0$ becomes

$$K = -V\frac{\mathrm{d}p}{\mathrm{d}V}$$

Notes (i) **The compressibility** κ of a substance is given by $\kappa = 1/K$.

(ii) The three elastic moduli have the same order of magnitude for any one material, and apply only in the region where the ratio of stress to strain is constant.

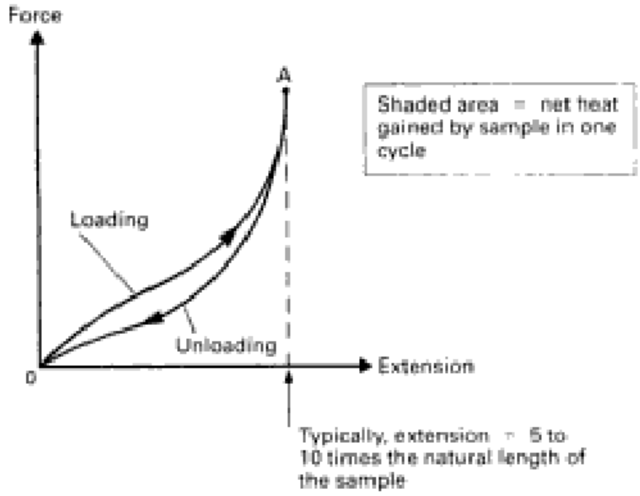
11.7 PLASTICITY

A **perfectly plastic** material is one which shows no tendency to return to its original size and shape when the load which has been deforming it is removed (plasticine is a good example). In this sense a perfectly plastic material is the opposite of an elastic material. The application of a load to a plastic material causes **dislocations** (i.e. gaps in the crystal lattice – see section 11.12) to move. This produces the same effect as planes of atoms sliding past each other.

11.8 ELASTIC HYSTERESIS

Fig. 11.8 shows the force–extension curve of a sample of rubber for both loading and unloading. The extension due to any given force is greater during unloading than during loading, i.e. the unloading extension lags behind the loading extension. The effect is called **elastic hysteresis,** and the region enclosed by the two curves is called a **hysteresis loop**. Metals also exhibit hysteresis, but to a much smaller extent.

Fig. 11.8
The effect of loading and unloading a sample of rubber



When rubber is stretched it becomes warmer. When the stress is released its temperature falls but it remains a little warmer than it was initially. The net increase in the heat content of the sample during the cycle is equal to the area of the hysteresis loop.

11.9 SOME PROPERTIES OF RUBBER

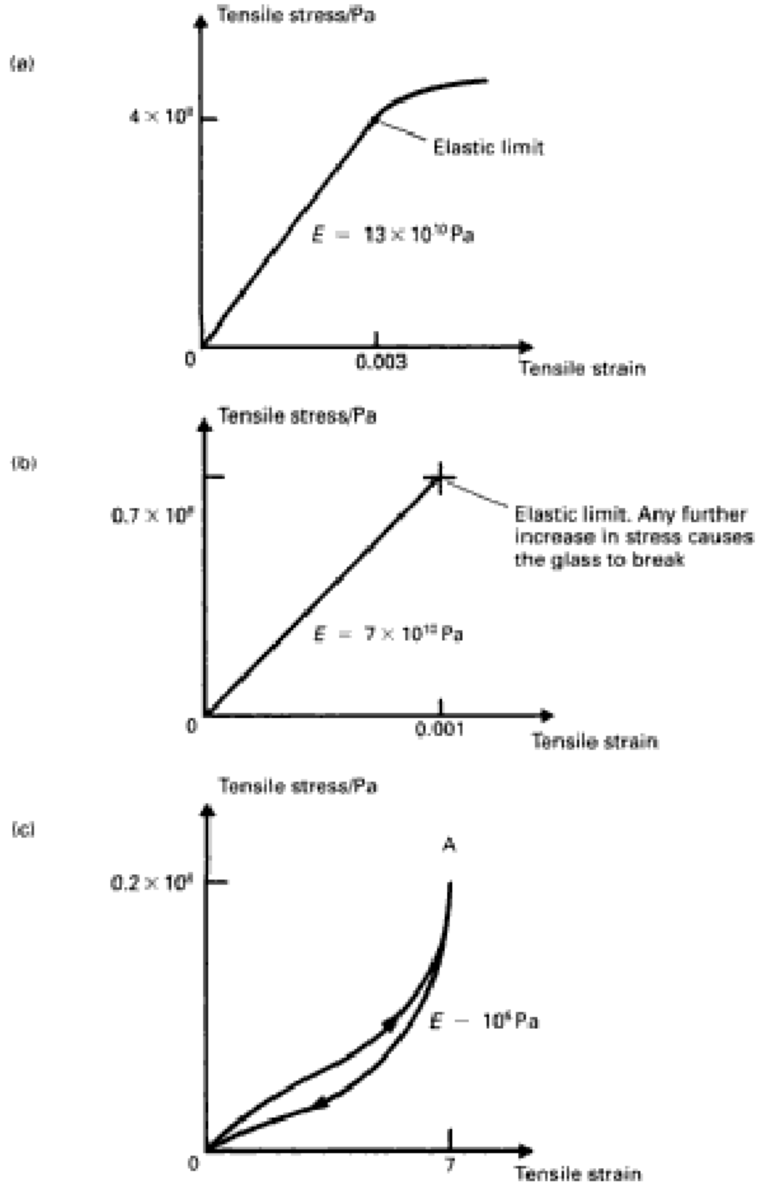
(i) Samples of some types of rubber can be stretched as much as 10 times their natural lengths and still regain their original sizes when the stresses are removed. A typical metal, on the other hand, can be subjected to only about 1/10 000 of this extension before its elastic limit is exceeded.

(ii) Rubber does not obey Hooke's law, i.e. the value of the ratio stress/strain depends on the particular stress at which it is measured – see Fig. 11.8. The sample stretches easily at first, but has become very stiff (steep slope) by the time the extension corresponding to point A has been reached. At A the extension is such that the long-chain molecules of the rubber (see 'Elastomers' in section 9.10) have become fully straightened out. Any further extension can be achieved only by stretching the bonds between the carbon atoms in the chains.

(iii) Some types of rubber have particularly large hysteresis loops, and so are useful as vibration absorbers. If a block of such a rubber is placed between a piece of vibrating machinery and the floor, much of the energy of the mechanical vibration is converted to heat energy in the rubber, and so is not transmitted to the floor. The rubber used in the manufacture of tyres has a small hysteresis loop, for it is clearly desirable that as little heat as possible is generated in a tyre.

(iv) Heating a stretched* rubber band causes it to contract. The higher temperature produces increased lateral bombardment of the long-chain molecules causing them to kink and so shorten.

Fig. 11.9
Stress–strain curves of typical samples of (a) copper, (b) glass, (c) rubber



*An unstressed sample of rubber, on the other hand, behaves quite normally and expands on heating.

(v) For most materials the value of Young's modulus decreases with increasing temperature. In the case of rubber, though, the ratio stress/strain increases with increasing temperature. This is because the increased lateral bombardment of the long-chain molecules which occurs at higher temperatures makes it more difficult to straighten them.

(vi) If a rubber band is stretched rapidly, its temperature increases. This behaviour is opposite to that of metals and most other materials. Stretching the rubber produces greater alignment of its long-chain molecules. This increased amount of order is akin to crystallization in the sense that as a result of it the rubber is in a lower energy state than previously. The energy released in attaining this state heats the sample.

The stress/strain curve for rubber is compared with those of copper and glass in Fig. 11.9.

11.10 FATIGUE

If a material is repeatedly stressed and unstressed (or stressed first in one direction and then in another), it becomes weaker, i.e. the strain produced by a given amount of stress increases. If the repeated stressing is continued, the material may fracture even though the maximum stress applied in any of the stress cycles could have been sustained indefinitely if it had been applied steadily. The failure of a material under these circumstances is called **fatigue failure** or **fatigue fracture**. It has been estimated that about 90% of the failures which occur in aircraft components are due to fatigue.

Mild steel and many other ferrous metals can safely undergo an infinite number of stress cycles, provided that the maximum stress is kept below a particular value known as the **fatigue limit**. There is no such limit for non-ferrous materials. In such cases the maximum loading is kept below that which would cause failure within the time for which the component is required to last.

Fatigue fractures usually start in the surface at points of high stress, e.g. at sharp corners and around rivet holes. It is believed that each time the material is stressed a small amount of plastic strain is produced. Since it is plastic strain, the effects of repeated stressings are cumulative and eventually produce fracture.

11.11 CREEP

The term **creep** is used to describe the gradual increase in strain which occurs when a material is subjected to stress for a long period of time. Unlike fatigue it occurs even when the stress is constant. It is most marked at elevated temperatures and may be so severe that the material eventually fractures. The greater the stress, the more quickly this happens. The turbine blades in jet engines are particularly susceptible to creep because they are under high stress and are at high temperatures. Soft metals (e.g. lead) and most plastics show considerable creep even at room temperature. Fig. 11.10 shows a typical creep curve. Note the accelerated rate of creep just before fracture.

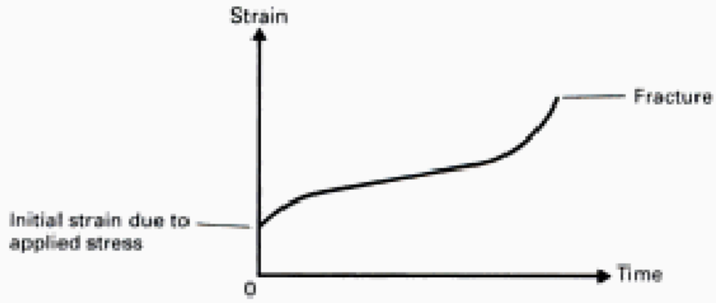


Fig. 11.10
Typical creep curve

11.12 DISLOCATIONS

The ductility of metals (i.e. their ability to undergo plastic strain) might be thought to be due to the various crystal planes which make up the structure slipping over each other to take up new positions (Fig. 11.11). According to this idea every atom in plane X has had to break a bond with an atom in plane Y, and then form a new one with a different atom in plane Y. However, calculations reveal that this process

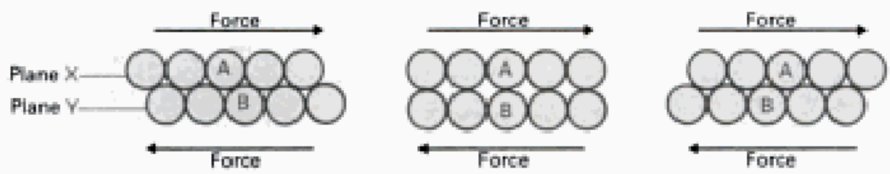


Fig. 11.11
Sliding of crystal planes

would require stresses which are about a hundred times greater than those which are needed to produce plastic strain in practice. Thus real metals are not as strong as this simple model would suggest. An explanation was offered in 1934 by G.I. Taylor who put forward the idea of **dislocations**. One type, an **edge dislocation**, is shown in Fig. 11.12. It takes the form of an incomplete plane of atoms (AB in Fig. 11.12(a)). Forces applied in the manner shown move atoms B and N closer together and eventually a bond forms between them at the expense of that between M and N (Fig. 11.12(b)).

If the stress is maintained, M and Z bond together leaving plane XY incomplete (Fig. 11.12(c)). In this way, then, the dislocation moves from left to right through the crystal. The end result is the same as it would have been if rows 1 and 2 had slipped over row 3. However, it has been achieved much more easily for only one bond has been broken at a time, whereas the wholesale movement of the planes would require a large number of bonds to be broken at the same time. The process is commonly likened to the movement of a ruck in a carpet. A large force is required to drag a heavy carpet over a floor. However, if there is a ruck in the carpet, it can be moved by the almost effortless process of pushing the ruck from one side to the other (Fig. 11.13).

11.13 THE STRENGTHENING OF METALS

It follows from what has been said in section 11.12 that metals can be made stronger by impeding the movement of dislocations. This can be done in a number of ways.

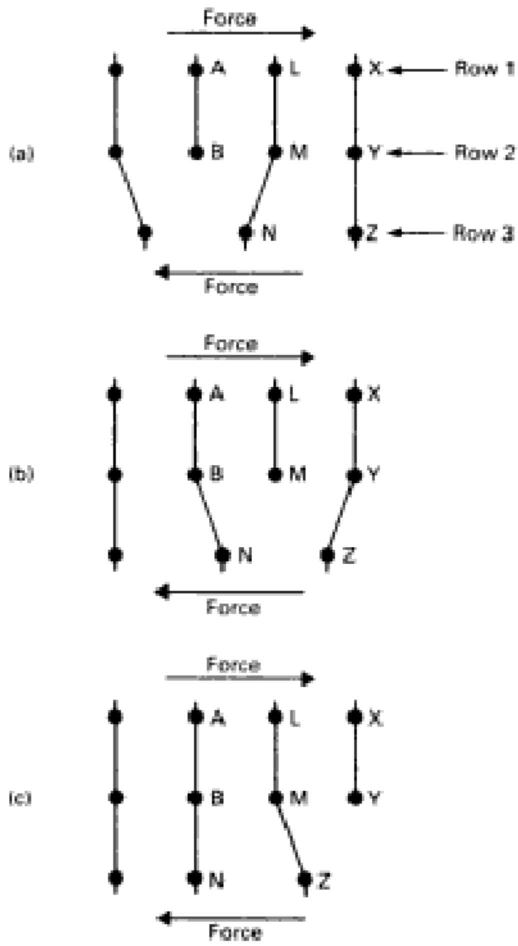


Fig. 11.12
Movement of a dislocation

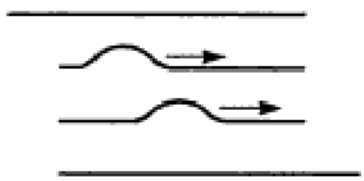


Fig. 11.13
Sliding a carpet by moving a ruck

(i) Increasing the number of dislocations, because the increased number makes it more likely that the various dislocations will obstruct each other in a 'log jam' effect. The number of dislocations can be increased by plastically deforming the metal repeatedly – this is the process known as **work hardening**.

(ii) Introducing 'foreign' atoms (e.g. carbon atoms in steel) into the structure. These disturb the regularity of the lattice and by so doing hinder the movement of dislocations.

(iii) Dislocations have difficulty in moving across grain boundaries and therefore samples in which the grain size is small (and which therefore have many grain boundaries) tend to be strong.

A metal in which there were no dislocations would, of course, be extremely strong. To date, though, such perfect crystals have been made only on a very small scale. They are known as 'whiskers' and are typically only a few micrometers thick, though a few millimetres long.

12

FLUID FLOW

12.1 TERMINOLOGY

Fluids

Both liquids and gases are fluids.

Viscosity

If a fluid is viscous then it offers a resistance to the motion through it of any solid body – or what amounts to the same thing, to its own motion past a solid body. In both these circumstances (except where the fluid is a gas of very low density) the layer of fluid in immediate contact with the solid surface is stationary with respect to that surface, and therefore the motion causes adjacent layers of fluid to move past each other. There exists a kind of internal friction which offers a resistance to the motion of one layer of fluid past another, and it is this that is the origin of the viscous force. In liquids the internal friction is due to intermolecular forces of attraction. In gases the viscous force arises as a result of the interchange of molecules that takes place between the different layers of the flowing gas. Thus, whenever molecules move from a fast-flowing layer into a more slowly moving layer, they increase the average speed of the molecules of that layer. It is as if the faster layer is dragging the slower layer along with it. At the same time, the random molecular motion means that molecules from the slower-moving layer move into the faster-moving layer, and therefore the average molecular speed of the faster-moving layer is reduced. Thus, the presence of an adjacent slow-moving layer slows down the fast-moving layer.

Steady Flow

If the flow of a fluid is steady (also known as **streamline flow, orderly flow** and **uniform flow**), then all the fluid particles that pass any given point follow the same path at the same speed (i.e. they have the same velocity). Thus, in steady flow no aspect of the flow pattern changes with time.

Turbulent Flow

This is also known as **disorderly flow**. In this type of flow the speed and direction of the fluid particles passing any point vary with time.

Line of Flow

The path followed by a particle of the fluid is called the line of flow of the particle.

Streamline

A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particle at that point. Streamlines never cross.

For a fluid undergoing steady flow all the fluid particles that pass any given point follow the same path, i.e. all the particles passing any given point have the same line of flow. It follows that **in steady flow the streamlines coincide with the lines of flow.**

Laminar Flow*

This is a special case of steady flow in which the velocities of all the particles on any given streamline are the same, though the particles of different streamlines may move at different speeds (Fig. 12.1). As an example of laminar flow, consider a liquid flowing in an open channel of uniform cross-section. If the fluid is viscous, it flows as a series of parallel layers (laminae). The layer in contact with the base of the channel is at rest, and the speed of each layer is greater than the speeds of those below it. If the channel is wide, the drag effects of the side walls can be ignored, and therefore the velocities of all the particles within each layer are the same (Fig. 12.2).

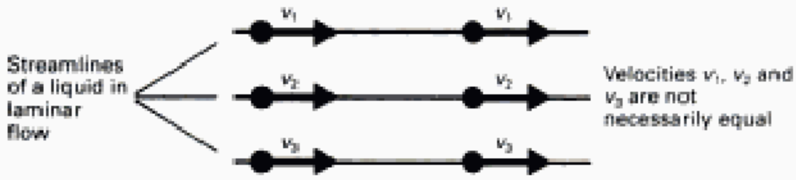


Fig. 12.1
Streamlines of a liquid in laminar flow

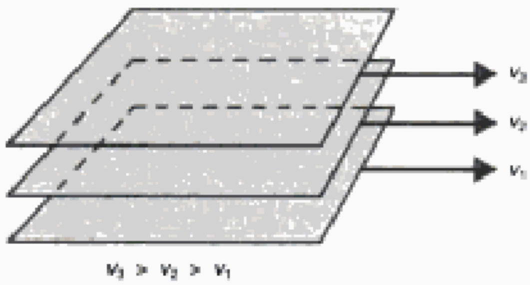


Fig. 12.2
To illustrate laminar flow

As a second example, consider a viscous fluid flowing in a pipe of uniform circular cross-section. In this case the fluid flows as a series of concentric cylinders. All the particles of fluid within such a cylinder flow at the same speed. The speed of the cylinder adjacent to the wall of the pipe is zero, and the speeds increase towards the centre.

Tube of Flow

This is a tubular region of a flowing fluid whose boundaries are defined by a set of streamlines.

Incompressible Fluid

This is a fluid in which changes in pressure produce no change in the density of the fluid. Liquids can be considered to be incompressible; gases subject only to small pressure differences can also be taken to be incompressible.

*The term 'laminar flow' is often used loosely as being synonymous with the less restricting term 'steady flow'.

12.2 THE EQUATION OF CONTINUITY

If a fluid is undergoing steady flow, then the mass of fluid which enters one end of a tube of flow must be equal to the mass that leaves at the other end during the same time. This must be so because in steady flow no fluid can leave the tube of flow through the side walls of the tube (streamlines do not cross each other), and therefore there would be a change in the mass within the tube if it were not so. A change in mass would mean a change in the number of fluid particles within the tube, in which case there would exist fluid particles where none had previously existed (or no particles where there had been some). This cannot happen under conditions of steady flow because the velocity at any point has to be unvarying.

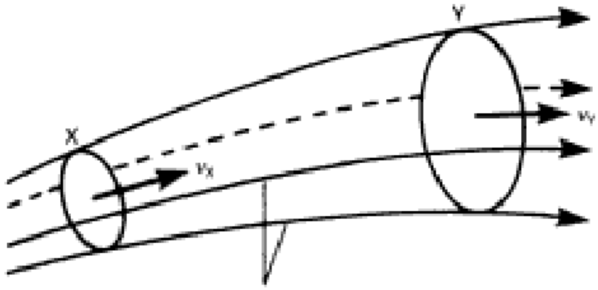
Consider a fluid undergoing steady flow, and consider a section XY of a tube of flow within the fluid (Fig. 12.3). Let

A_X and A_Y be the cross-sectional areas of the tube of flow at X and Y respectively,

ρ_X and ρ_Y be the densities of the fluid at X and Y respectively,

v_X and v_Y be the velocities of the fluid particles at X and Y respectively.

Fig. 12.3
Section of tube of flow



In a time interval Δt the fluid at X will move forward a distance $v_X\Delta t$. Therefore a volume $A_X v_X \Delta t$ will enter the tube at X. The mass of fluid entering at X in time Δt will therefore be

$$\rho_X A_X v_X \Delta t$$

Similarly, the mass leaving at Y in the same time will be

$$\rho_Y A_Y v_Y \Delta t$$

Since the mass entering at X is equal to the mass leaving at Y,

$$\rho_X A_X v_X \Delta t = \rho_Y A_Y v_Y \Delta t$$

i.e.

$$\boldsymbol{\rho_X A_X v_X = \rho_Y A_Y v_Y} \qquad [12.1]$$

Equation [12.1] is known as the **equation of continuity**. For an incompressible fluid $\rho_X = \rho_Y$, and therefore the equation takes the form

$$\boldsymbol{A_X v_X = A_Y v_Y} \qquad [12.2]$$

$A_X v_X$ is known as the **flow rate** (or **volume flux**) of the fluid at X.

12.3 BERNOULLI'S EQUATION

This states that **for an incompressible, non-viscous fluid undergoing steady flow, the pressure plus the kinetic energy per unit volume plus the potential energy per unit volume is constant at all points on a streamline.**

i.e. $$p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{A constant}$$

where

p = the pressure within the fluid

ρ = the density of the fluid

v = the velocity of the fluid

g = the acceleration due to gravity, and

h = the height of the fluid (above some arbitrary reference line).

Proof Consider a tube of flow within a non-viscous, incompressible fluid undergoing steady flow (Fig. 12.4). Let

p_X and p_Y = pressures at X and Y

v_X and v_Y = velocities at X and Y

A_X and A_Y = areas of cross-section at X and Y

h_X and h_Y = average heights at X and Y.

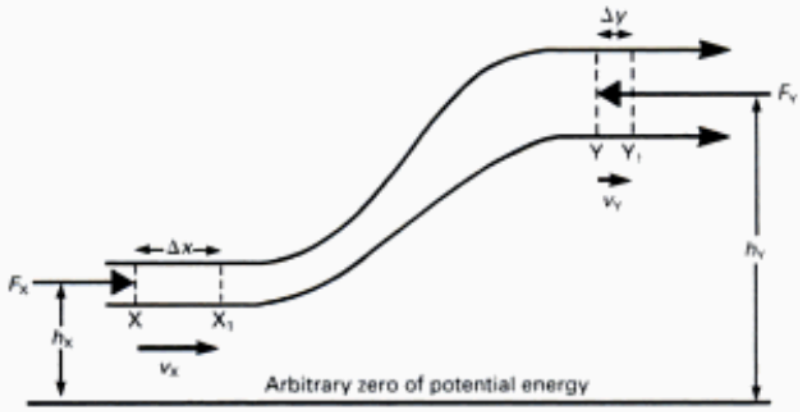


Fig. 12.4 Derivation of Bernoulli's equation

Let X_1 be close to X so that each of the parameters listed above has the same value at X_1 as at X. Let Y_1 be close to Y with a similar consequence. Since the fluid is incompressible, the density will be the same at all points; let this be ρ.

Consider the section of fluid which is between X and Y, moving to occupy the region between X_1 and Y_1. The fluid moves in this direction because the force F_X is greater than the force F_Y. The force F_X moves a distance Δx, and the fluid moves a distance Δy against the force F_Y.

The net work done on the fluid is therefore given by

$$\text{Work done on fluid} = F_X \Delta x - F_Y \Delta y$$

Since the fluid is undergoing steady flow, the mass of fluid that was originally between X and X_1 is equal to the mass which is now between Y and Y_1. Let this mass be m. Thus a mass m which originally had velocity v_X and average height h_X has been replaced by an equal mass with velocity v_Y and average height h_Y. Therefore,

$$\text{Gain in kinetic energy} = \tfrac{1}{2}mv_Y^2 - \tfrac{1}{2}mv_X^2$$
$$\text{Gain in potential energy} = mgh_Y - mgh_X$$

None of the work done on the fluid has been used to overcome internal friction because the fluid is non-viscous, and therefore by the principle of conservation of energy,

$$\text{Work done} = \text{Gain in KE} + \text{Gain in PE}$$

$$\therefore \quad F_X\Delta x - F_Y\Delta y = \tfrac{1}{2}mv_Y^2 - \tfrac{1}{2}mv_X^2 + mgh_Y - mgh_X$$

$$\text{i.e.} \quad p_X A_X \Delta x - p_Y A_Y \Delta y = \tfrac{1}{2}mv_Y^2 - \tfrac{1}{2}mv_X^2 + mgh_Y - mgh_X$$

But, $A_X\Delta x$ = Volume between X and X_1 $= m/\rho$, and similarly $A_Y\Delta y = m/\rho$. Therefore,

$$p_X\frac{m}{\rho} - p_Y\frac{m}{\rho} = \tfrac{1}{2}mv_Y^2 - \tfrac{1}{2}mv_X^2 + mgh_Y - mgh_X$$

$$\text{Thus} \quad p_X - p_Y = \tfrac{1}{2}\rho v_Y^2 - \tfrac{1}{2}\rho v_X^2 + \rho gh_Y - \rho gh_X$$

$$\text{i.e.} \quad p_X + \tfrac{1}{2}\rho v_X^2 + \rho gh_X = p_Y + \tfrac{1}{2}\rho v_Y^2 + \rho gh_Y$$

Since X and Y were arbitrarily chosen points we may write

$$p + \tfrac{1}{2}\rho v^2 + \rho gh = \text{A constant}$$

In practice, Bernoulli's equation cannot apply exactly – real fluids are viscous and gases are easily compressed. Nevertheless, as long as the equation is used with care, it gives meaningful results and its qualitative implications are valid.

12.4 CONSEQUENCES OF BERNOULLI'S EQUATION

It follows from Bernoulli's equation that whenever a flowing fluid speeds up, there is a corresponding decrease in the pressure and/or the potential energy of the fluid. If the flow is horizontal, the whole of the velocity increase is accounted for by a decrease in pressure.

An aerofoil (e.g. an aircraft wing) is shaped so that air flows faster along the top of it than the bottom. There is, therefore, a greater pressure below the aerofoil than above it. It is this difference in pressure that provides the lift. A spinning ball experiences a similar effect. The spin drags air around with the ball (Fig. 12.5). The ball therefore has a resultant force acting on it towards the top of the page.

In accordance with the equation of continuity, fluids speed up at constrictions, and therefore there is a decrease in pressure at constrictions. This effect is made use of in such devices as filter pumps, Bunsen burners and carburettors.

The Venturi meter (Fig. 12.6) is a device which introduces a constriction into a pipe carrying a fluid, in order that the velocity of the fluid can be measured by measuring the resulting drop in pressure.

Fig. 12.5
To illustrate the effect of spin

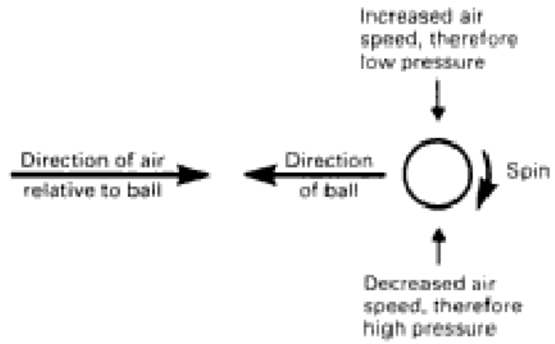
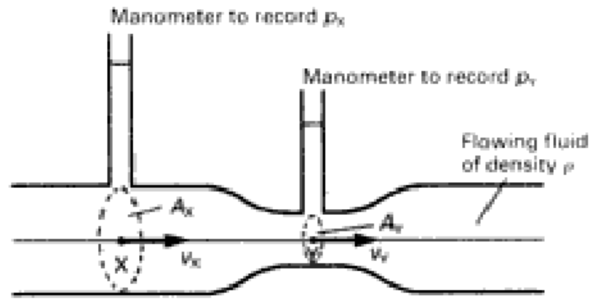


Fig. 12.6
The Venturi meter



Consider the fluid to be non-viscous, incompressible (of density ρ) and in horizontal steady flow. Let the pressure and velocity respectively be p_X and v_X at X, and be p_Y and v_Y at Y on the same streamline as X. Applying Bernoulli's equation at X and Y gives

$$p_X + \tfrac{1}{2}\rho v_X^2 = p_Y + \tfrac{1}{2}\rho v_Y^2$$

If the cross-sectional areas at X and Y are A_X and A_Y, then from the equation of continuity

$$A_X v_X = A_Y v_Y \quad \text{i.e.} \quad v_Y = \frac{A_X v_X}{A_Y}$$

$$\therefore \qquad p_X + \tfrac{1}{2}\rho v_X^2 = p_Y + \tfrac{1}{2}\rho\left(\frac{A_X v_X}{A_Y}\right)^2$$

$$\therefore \qquad p_X - p_Y = \tfrac{1}{2}\rho\left(\frac{A_X^2}{A_Y^2} - 1\right)v_X^2$$

Thus by measuring the pressures p_X and p_Y and knowing ρ, A_X and A_Y, it is possible to find the velocity v_X of the fluid in the unconstricted (main) section of the pipe.

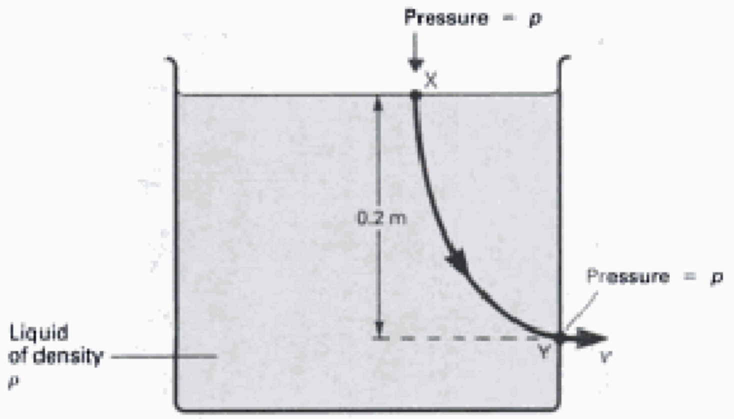
EXAMPLE 12.1

Calculate the velocity with which a liquid emerges from a small hole in the side of a tank of large cross-sectional area if the hole is 0.2 m below the surface (Assume $g = 10\,\mathrm{m\,s^{-2}}$.)

Solution

Refer to Fig. 12.7. We shall assume that we are dealing with a non-viscous, incompressible liquid in steady flow, in which case we may apply Bernoulli's equation to points X and Y on the streamline XY.

Fig. 12.7
Diagram for Example 12.1



Thus, taking the pressure, height and velocity at X to be p_X, h_X and v_X, and the pressure, height and velocity at Y to be p_Y, h_Y and v_Y, we may put

$$p_X + \rho g h_X + \tfrac{1}{2}\rho v_X^2 = p_Y + \rho g h_Y + \tfrac{1}{2}\rho v_Y^2 \qquad [12.3]$$

The pressure at both X and Y is atmospheric pressure p, and therefore

$$p_X = p_Y = p$$

Taking heights to be measured from the level of Y we have

$$h_X = 0.2\,\text{m} \qquad h_Y = 0$$

If we assume that the tank is wide enough for the rate at which the surface level falls to be negligible, then

$$v_X = 0 \qquad v_Y = v \text{ (the velocity of emergence)}$$

Substituting in equation [12.13] gives

$$p + \rho \times 10 \times 0.2 + 0 = p + 0 + \tfrac{1}{2}\rho v^2$$

i.e. $$v = \sqrt{2 \times 10 \times 0.2} = 2\,\text{m}\,\text{s}^{-1}$$

In general $v = \sqrt{2gh}$ and is equal to the velocity acquired by a body falling from rest through a height h – a result which is known as **Torricelli's theorem**. In practice v would be less than $2\,\text{m}\,\text{s}^{-1}$ because of viscous effects.

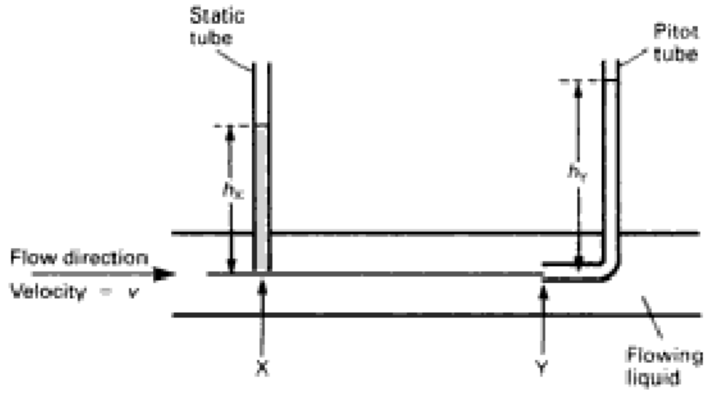
12.5 THE PITOT–STATIC TUBE

The Pitot–static tube is a device used to measure the velocity of a moving fluid. It consists of two manometer tubes – the Pitot tube and the static tube. The Pitot tube has its opening facing the fluid flow; the static tube has its opening at right angles to this.

When the Pitot–static tube is used to measure the velocity of a flowing liquid, the liquid itself can be used as the manometer liquid (Fig. 12.8). Providing the liquid has reached its equilibrium level in the Pitot tube, the liquid at Y will be stationary (i.e. Y is a **stagnation point**). Suppose that X is a point on the same streamline as Y, but sufficiently distant from it for the liquid there to have its full velocity, v. If the liquid is in steady flow and can be considered non-viscous and incompressible, we may apply Bernoulli's equation to X and Y. Bearing in mind that the flow is horizontal, this gives

$$p_X + \tfrac{1}{2}\rho v^2 = p_Y \qquad [12.4]$$

Fig. 12.8
Pitot-static tube to measure velocity of a liquid



where p_X and p_Y are the pressures in the liquid at X and Y respectively. Rearranging equation [12.4] gives

$$v = \sqrt{\frac{2}{\rho}(p_Y - p_X)} \qquad [12.5]$$

or

$$v = \sqrt{\frac{2}{\rho}\left(\begin{matrix}\text{Pressure at}\\ \text{stagnation point}\end{matrix} - \begin{matrix}\text{Pressure where}\\ \text{fluid velocity} = v\end{matrix}\right)} \qquad [12.6]$$

The pressure, p_Y, at Y is equal to the pressure exerted by the liquid in the Pitot tube plus atmospheric pressure, p_A. Therefore

$$p_Y = h_Y \rho g + p_A$$

The pressure, p_X, at X is equal to the pressure exerted by the liquid in the static tube plus atmospheric pressure, and therefore

$$p_X = h_X \rho g + p_A$$

Therefore

$$p_Y - p_X = \rho g(h_Y - h_X)$$

Therefore by equation [12.5]

$$v = \sqrt{2g(h_Y - h_X)} \qquad [12.7]$$

Gases cannot be used as manometer fluids, and therefore the type of Pitot–static tube used to measure gas velocities has the form shown in Fig. 12.9. The head of liquid in the manometer measures the difference ($h\rho_m g$) between the pressure at the stagnation point, Y, and the pressure at X, where the gas has velocity v. Therefore by equation [12.6]

$$v = \sqrt{\frac{2}{\rho}(h\rho_m g)}$$

The terms: 'static pressure', 'dynamic pressure' and 'total pressure' are often used in connection with flowing fluids.

Static Pressure

The static pressure at a point in a flowing fluid is the actual pressure at that point. As such, it is the pressure measured in such a way that the measurement does not affect, and is not affected by, the velocity of the fluid. One way of achieving this is with a manometer whose opening is parallel to the flow direction. (The static tube in Fig. 12.8 measures the static pressure at X.)

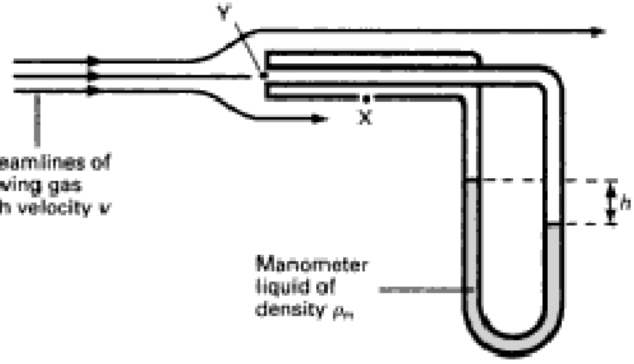


Fig. 12.9
Pitot–static tube to measure the velocity of a gas

Dynamic Pressure

For a fluid of density ρ, moving with velocity v, the dynamic pressure is $\frac{1}{2}\rho v^2$.

Dynamic pressure is not a true pressure – simply a quantity which has the same dimensions (see Appendix 2) as pressure.

Total Pressure

The total pressure is the sum of the static and dynamic pressures. The pressure and the total pressure are equal to each other at a stagnation point.

12.6 THE COEFFICIENT OF VISCOSITY (η)

The coefficient of viscosity of a fluid is a measure of the degree to which the fluid exhibits viscous effects. The higher the coefficient of viscosity, the more viscous the fluid – the coefficient of viscosity of golden syrup at room temperature is about 10^5 times that of water at the same temperature. The coefficients of viscosity of most fluids have a marked temperature dependence; those of liquids decrease with increasing temperature, whereas those of gases increase with increasing temperature.

Viscous effects are due to the frictional force which exists between two adjacent layers of fluid which are in relative motion. Consider a viscous fluid undergoing laminar flow, and consider in particular two parallel layers of area A separated by a small distance δy and whose velocities are v and $v+\delta v$ (Fig. 12.10). It was suggested by Newton that the frictional force F between the layers is proportional to A and to the **velocity gradient** $\delta v/\delta y$, i.e.

$$F \propto A\frac{\delta v}{\delta y}$$

(This is the opposite of the situation with solids – the frictional force between the surfaces of two solids is independent of the area of contact and of the relative velocity. See section 2.12.) Introducing a constant of proportionality, η, we have

$$F = \eta A\frac{\delta v}{\delta y} \qquad [12.8]$$

Equation [12.8] is sometimes called **Newton's law of viscosity**. It holds for all gases and for many liquids. Such liquids are called **Newtonian liquids**; water

Fig. 12.10
Friction between successive layers of a liquid

is an example. For any given values of A and $\delta v/\delta y$, F is large for those fluids which have high values of η, and therefore η is a measure of the viscosity of the fluid, and can meaningfully be called its **coefficient of viscosity**.

There are some liquids, called **non-Newtonian liquids**, for which F is not proportional to $\delta v/\delta y$ and which, therefore, do not have constant values of η – i.e. they do not have coefficients of viscosity in the normal sense. Oil-paint is an example of a non-Newtonian liquid.

Notes (i) The units of η are $\mathrm{N\,s\,m^{-2}} = \mathrm{kg\,m^{-1}\,s^{-1}}$.

(ii) By rearranging equation [12.8] as

$$\eta = \frac{F/A}{\delta v/\delta y}$$

and then comparing it with equation [11.7] of section 11.6 we can draw an analogy between η and the shear modulus G of a solid. In each case F/A is the shear stress. In the case of a solid, though, the stress produces a fixed strain ($\Delta x/y$) proportional to the stress, whereas with a Newtonian fluid the strain increases without limit as long as the stress is applied and it is the rate of change of the strain ($\delta v/\delta y$) which is proportional to the stress.

12.7 POISEUILLE'S FORMULA

Consider a viscous liquid undergoing steady flow through a pipe of circular cross-section. Because of viscous drag the velocity varies from a maximum at the centre of the pipe to zero at the walls. We shall use dimensional analysis (see Appendix 2) to derive an expression for the volume V of liquid passing any section of the pipe in time t.

It can reasonably be supposed that the **rate of volume flow** V/t depends on (i) the coefficient of viscosity η of the liquid, (ii) the radius r of the pipe, and (iii) the pressure gradient p/l, where p is the pressure difference between the ends of the pipe and l is its length. If we express the relationship as

$$\frac{V}{t} = k\eta^x r^y \left(\frac{p}{l}\right)^z$$

where k is a dimensionless constant and x, y and z are unknown indices, then since each side of the equation must have the same dimensions,

$$[V/t] = [\eta^x][r^y][(p/l)^z]$$

$$\therefore \quad L^3T^{-1} = (ML^{-1}T^{-1})^x(L)^y(ML^{-2}T^{-2})^z$$

i.e. $$L^3T^{-1} = M^{x+z}L^{y-x-2z}T^{-x-2z}$$

Equating the indices of M, L and T on both sides gives

$$0 = x + z \quad \text{(for M)}$$
$$3 = y - x - 2z \quad \text{(for L)}$$
$$-1 = -x - 2z \quad \text{(for T)}$$

Solving gives $z = 1$, $x = -1$, $y = 4$. The relationship is therefore

$$\frac{V}{t} = k\frac{r^4}{\eta}\left(\frac{p}{l}\right)$$

The value of k cannot be found by using dimensional analysis, however, mathematical analysis shows that its value is $\pi/8$, and therefore

$$\frac{V}{t} = \frac{\pi r^4 p}{8\eta l} \qquad [12.9]$$

This is called **Poiseuille's formula** in recognition of Poiseuille who in 1844 made the first thorough experimental investigation of the steady flow of a liquid (water) through a pipe. The formula applies only to Newtonian fluids (see section 12.6) which are undergoing steady flow.

The speed of bulk flow is defined as the rate of volume flow divided by the cross-sectional area of the pipe. Steady flow occurs only when the speed of bulk flow is less than a certain critical value v_c. Since Poiseuille's formula applies only to steady flow, it does not hold when the speed of bulk flow exceeds v_c. Experiment shows that for cylindrical pipes

$$v_c \approx \frac{1100\,\eta}{r\rho}$$

where ρ and η are the density and coefficient of viscosity of the fluid and r is the radius of the pipe.

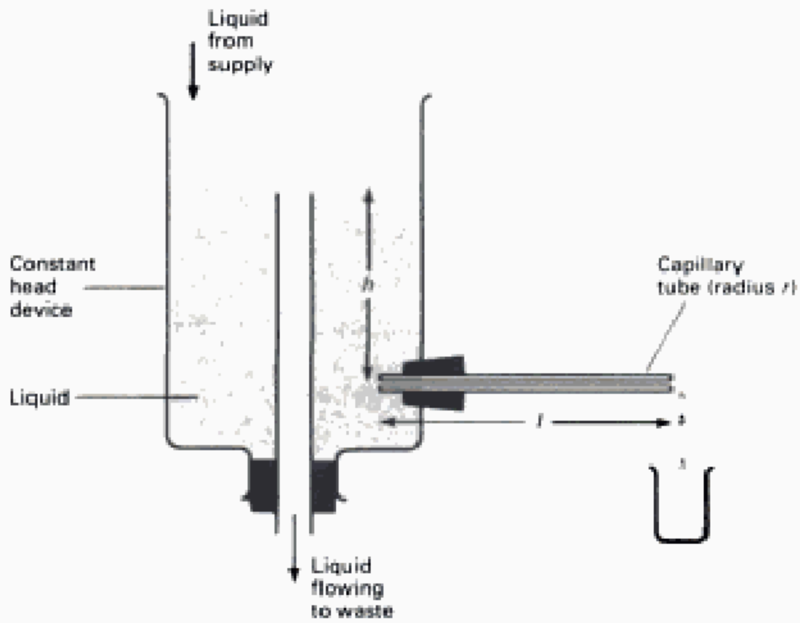
12.8 MEASUREMENT OF η BY USING POISEUILLE'S FORMULA

The method makes use of the apparatus shown in Fig. 12.11 and is suitable for liquids which flow easily (e.g. water). (For high-viscosity liquids see section 12.10.)

The liquid under test flows steadily through the capillary tube from a constant head device and the volume V of liquid which emerges in a known time t is measured. The pressure difference between the ends of the capillary tube is $h\rho g$ (where ρ is the density of the liquid and g is the acceleration due to gravity) and therefore from equation [12.9]

$$\frac{V}{t} = \frac{\pi r^4 h\rho g}{8\,\eta l}$$

Fig. 12.11
Apparatus for measuring η using Poiseuille's formula



Poiseuille's formula applies only if the flow is steady. In order to check that this is the case the measurements are repeated for different values of h and a graph of (V/t) against h is plotted. The graph is linear providing h has been kept below the value at which the rate of flow is so high that turbulence sets in. The gradient of the graph is $\pi r^4 \rho g/(8\eta l)$, enabling η to be calculated once r and l have been measured (ρ and g are found from tables). The mean radius r of the tube can be found by measuring the length and mass of a mercury thread introduced into the tube.

Notes

(i) Great care is needed when measuring r because it appears in the calculation of η as r^4. This makes the percentage error in η due to an error in r four times the percentage error in r.

(ii) A capillary tube is used because r needs to be small so that h is large enough to be measured accurately.

12.9 STOKES' LAW AND TERMINAL VELOCITY

Derivation of Stokes' Law

Consider a sphere of radius r moving with velocity v through a fluid whose coefficient of viscosity is η. The sphere experiences a viscous force F which acts in the opposite direction to that in which the sphere is moving. We shall use dimensional analysis (see Appendix 2) to obtain an expression for F.

It can reasonably be supposed that F depends only on r, η and v. (Though the mass of the sphere and the density of the fluid have a bearing on how the velocity varies under the effect of an applied force, they have no direct influence on the drag force.) If we express the relationship as

$$F = kr^x\eta^y v^z$$

where k is a dimensionless constant and x, y and z are unknown indices, then since each side of the equation must have the same dimensions

$$[F] = [r^x][\eta^y][v^z]$$

$$\therefore \quad \mathrm{MLT^{-2}} = (\mathrm{L})^x(\mathrm{ML^{-1}T^{-1}})^y(\mathrm{LT^{-1}})^z$$

$$\text{i.e.} \quad \mathrm{MLT^{-2}} = \mathrm{M}^y\mathrm{L}^{x+z-y}\mathrm{T}^{-y-z}$$

Equating the indices of M, L and T on both sides gives

$$1 = y \quad \text{(for M)}$$
$$1 = x + z - y \quad \text{(for L)}$$
$$-2 = -y - z \quad \text{(for T)}$$

Solving gives $y = 1$, $z = 1$, $x = 1$. The relationship is therefore

$$F = kr\eta v$$

A full mathematical analysis reveals that $k = 6\pi$, and therefore

$$F = 6\pi r\eta v \qquad [12.10]$$

Equation [12.10] was first derived by Stokes and is known as **Stokes' law**.

Notes (i) Strictly the law applies only to a fluid of infinite extent.

(ii) Stokes' law does not hold if the sphere is moving so fast that conditions are not streamline.

Terminal Velocity

Consider a sphere falling from rest through a viscous fluid. The forces acting on the sphere are its weight W, the upthrust U due to the displaced fluid, and the viscous drag F (see Fig. 12.12). Initially the downward force W is greater than the upward force, $U + F$, and the sphere accelerates downwards. As the velocity of the sphere increases so too does the viscous drag, and eventually $U + F$ is equal to W. The sphere continues to move downwards but, because there is now no net force acting on it, its velocity has a constant maximum value known as its **terminal velocity** v_t.

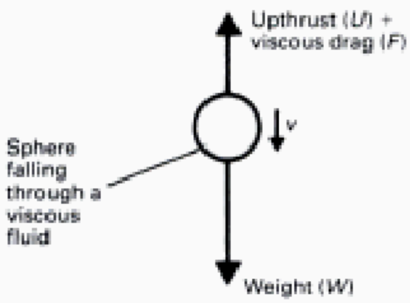


Fig. 12.12 Sphere falling through a viscous fluid

If ρ_f and ρ_s are the densities of the fluid and the sphere respectively, then

$$W = \tfrac{4}{3}\pi r^3 \rho_s g$$

and

$$U = \tfrac{4}{3}\pi r^3 \rho_f g$$

At the terminal velocity

$$U + F = W \qquad [12.11]$$

and

$$F = 6\pi r\eta v_t$$

where η is the coefficient of viscosity of the fluid. Substituting for W, U and F in equation [12.10] gives

$$\tfrac{4}{3}\pi r^3 \rho_f g + 6\pi r \eta v_t = \tfrac{4}{3}\pi r^3 \rho_s g$$

i.e. $$6\pi r \eta v_t = \tfrac{4}{3}\pi r^3 (\rho_s - \rho_f) g$$

i.e. $$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta} \qquad [12.12]$$

12.10 MEASUREMENT OF η BY USING STOKES' LAW

The method is suitable for liquids of high viscosity such as glycerine and treacle, and makes use of equation [12.12]. (For low-viscosity liquids see section 12.8.) The liquid whose coefficient of viscosity η is being determined is contained in a large measuring cylinder (Fig. 12.13). A small ball-bearing of radius r is dropped gently into the liquid. The time taken for the ball to fall from mark A to mark B is determined. Providing A is sufficiently far below the surface, the bearing will have reached its terminal velocity v_t before reaching A, in which case $v_t = \text{AB}/t$. If ρ_f and ρ_s are the densities of the liquid and the sphere respectively, then from equation [12.12]

$$\frac{\text{AB}}{t} = \frac{2r^2(\rho_s - \rho_f)g}{9\eta}$$

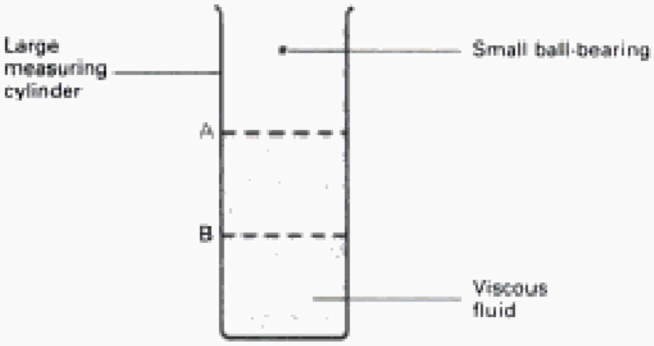


Fig. 12.13
Apparatus for measuring η using Stoke's law

A micrometer can be used to measure r; ρ_s and ρ_f are found from tables or are determined in additional experiments; hence η can be deduced.

Notes

(i) Stokes' law applies strictly only when the fluid is of infinite extent. The error due to the impossibility of fulfilling this condition is reduced by using a measuring cylinder which is wide compared with the diameter of the ball-bearing, and by having B well away from the bottom.

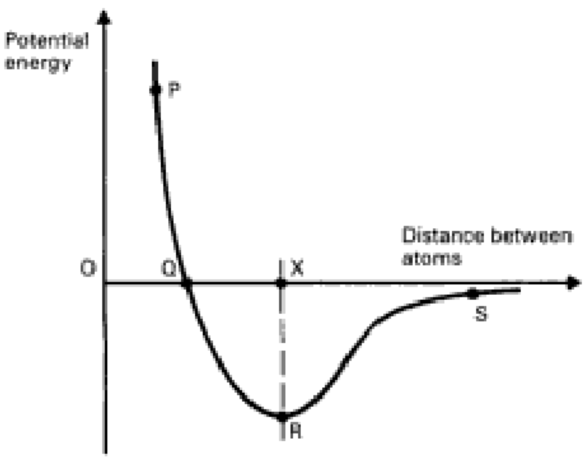
(ii) If the velocity of the bearing is so large that it produces turbulence, Stokes' law does not hold and equation [12.12] is not applicable. Using a highly viscous liquid and a small ball-bearing avoids this problem and also makes t large enough to be measured accurately.

QUESTIONS ON SECTION B

Assume $g = 10\,m\,s^{-2} = 10\,N\,kg^{-1}$ unless otherwise stated.

SOLIDS AND LIQUIDS (Chapter 9)

B1 The graph shows the potential energy of a pair of atoms at different distances apart.



(a) What distance represents the equilibrium separation of the atoms?

(b) What is the physical significance of the quantity represented by XR?

(c) Identify the region of the curve in which there is a net attractive force between the atoms.

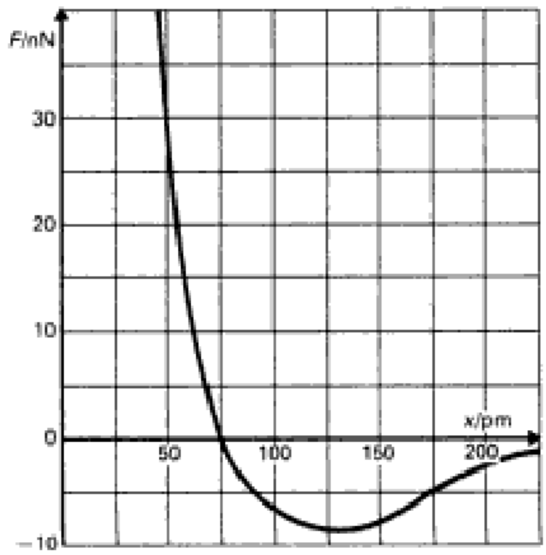
(d) What is the physical significance of the gradient of the tangent to this curve?

[J, '92]

B2 A certain molecule consists of two identical atoms, each of mass 1.7×10^{-27} kg. The equilibrium separation of the atoms in the molecule is x_0. The figure above shows the way in which the force F of repulsion between the atoms varies with their separation x.

(a) Account for the general shape of the graph and use it to find x_0.

(b) Sketch a graph of the potential energy V of the molecule as a function of x, marking the position of x_0 on the x-axis. How is V related to F?



For *very small* displacements from x_0, the force F is given by the approximate relation $F = -k(x - x_0)$.

(c) Find the value of k in this equation.

(d) Describe the motion of the atoms in the molecule when moving freely under the action of this force. By deriving the equation of motion of one of the atoms, or otherwise, find the frequency of the motion. [C]

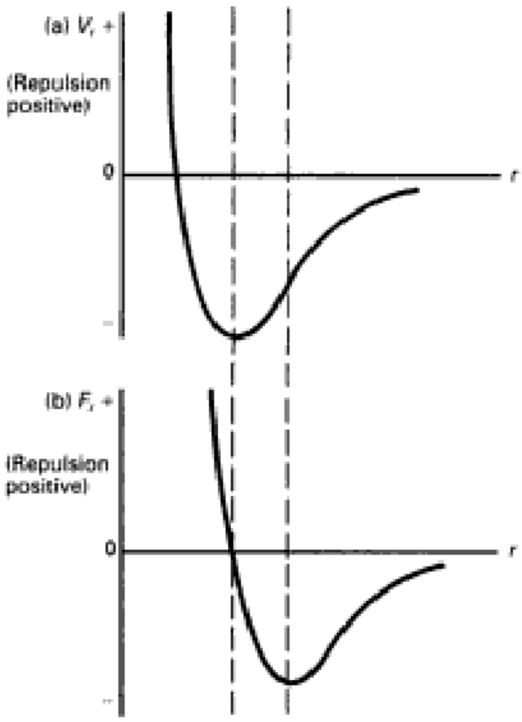
B3 The very simplified curves (p. 209) represent, for two adjacent atoms or molecules, the variation with the separation r between their centres of the potential energy V_r due to the interaction between them, and the force F_r between them.

(a) Explain the general relation between the F_r curve and the V_r curve, and the significance of the broken lines A and B.

(b) With reference to the V_r curve (a), explain how:

(i) the lower part is consistent with molecules in a solid oscillating about a mean position,

(ii) the effect of a rise in temperature could be represented,



(iii) the thermal expansion of a solid on heating is accounted for,

(iv) the latent heat of vaporization (or sublimation) per atom can be estimated.

(c) With reference to the F_r curve (b), explain how:

(i) the property of elasticity is represented,

(ii) Hooke's law is accounted for.

(d) How can it be forecast that a solid will rupture under a large enough stress, and will melt at a high enough temperature? [O]

B4 The specific latent heat of vaporization of a particular liquid is $2.0 \times 10^5\,\mathrm{J\,kg^{-1}}$, its relative molecular mass is 30 and its coordination number is 10. On the basis of this data, and given that the Avogadro constant $= 6 \times 10^{23}\,\mathrm{mol^{-1}}$, obtain a value for the binding energy of a pair of (adjacent) molecules of the liquid.

On freezing, the coordination number increases to 12. Estimate the specific latent heat of fusion.

B5 **(a)** Calculate the potential energy, in eV, per pair of atoms of a solid for which the latent heat of sublimation is $1.3 \times 10^4\,\mathrm{J\,mol^{-1}}$ and the number of neighbours per atom is 6. The Avogadro constant, $N_A = 6.0 \times 10^{23}\,\mathrm{mol^{-1}}$ and $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.

(b) For a pair of atoms, sketch a graph showing how the potential energy per atom pair varies with the distance between the atoms. Show on your graph **(i)** the equilibrium separation, r_0, **(ii)** the value of the energy calculated in (a).

(c) Mark on your sketch graph a point P corresponding to a separation *other than the equilibrium value* and explain how you would determine, from the graph, the force between the atoms at P. Indicate whether you consider the force at P to be attractive or repulsive. [J]

B6 The latent heat of vaporization of water is $4 \times 10^4\,\mathrm{J\,mol^{-1}}$ at the boiling point, and each water molecule has, on average, 10 near neighbours. Estimate the binding energy for a pair of water molecules.

Ignore the work done in expansion in your calculation, but explain whether this assumption leads to an overestimate or an underestimate of the binding energy.

(The Avogadro constant $N_A = 6 \times 10^{23}\,\mathrm{mol^{-1}}$.) [W]

B7 Calculate the average volume occupied by a single molecule of a solid whose density is $1.2 \times 10^3\,\mathrm{kg\,m^{-3}}$ and whose relative molecular mass is 90. (The Avogadro constant $= 6 \times 10^{23}\,\mathrm{mol^{-1}}$.)

Hence, stating any assumption that you make, estimate the distance between the centres of two adjacent molecules of the solid.

B8 **(a)** Estimate the diameter of a water molecule given that the relative molecular mass of water is 18 and its density is $1000\,\mathrm{kg\,m^{-3}}$.

(b) Using the value obtained in part (a), estimate the binding energy of water molecules, given that the surface tension of water is $0.072\,\mathrm{N\,m^{-1}}$ and that the number of near neighbours in the water is 10.

(Assume that $\gamma = N_s z\varepsilon/4$. Take $N_A = 6.0 \times 10^{23}\,\mathrm{mol^{-1}}$) [W, '91]

B9 An alloy contains two metals, X and Y, of densities $3.0 \times 10^3\,\mathrm{kg\,m^{-3}}$ and $5.0 \times 10^3\,\mathrm{kg\,m^{-3}}$ respectively. Calculate the density of the alloy
(a) if the volume of X is twice that of Y, and
(b) if the mass of X is twice that of Y.

B10 An alloy of two metals, X and Y, has a volume of $5.0 \times 10^{-4}\,\mathrm{m^3}$ and a density of $5.6 \times 10^3\,\mathrm{kg\,m^{-3}}$. The densities of X and Y are $8.0 \times 10^3\,\mathrm{kg\,m^{-3}}$ and $4.0 \times 10^3\,\mathrm{kg\,m^{-3}}$ respectively. Find the mass of X and the mass of Y.

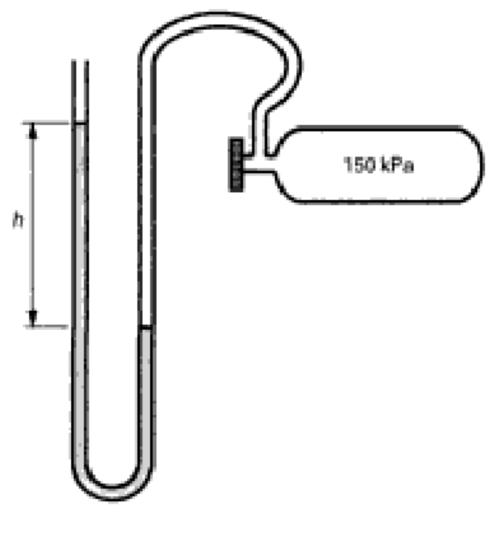
FLUIDS AT REST (Chapter 10)

B11 An open U-tube manometer containing mercury is used to measure the pressure of a gas. The mercury level in the open tube is 600 mm higher than that in the limb which is in contact with the gas. What is the pressure (in pascals) of the gas?

(Density of mercury $= 1.36 \times 10^4\,\mathrm{kg\,m^{-3}}$, atmospheric pressure $= 1.01 \times 10^5$ Pa, $g = 9.81\,\mathrm{m\,s^{-2}}$.)

B12 The diagram shows a mercury manometer recording a pressure of 150 kPa. The atmospheric pressure is 100 kPa.
(Take the density of mercury as 13 600 $\mathrm{kg\,m^{-3}}$.)

What is the height difference h of the mercury surfaces? [O, '91*]



B13 A body has a weight of 160 N when weighed in air and a weight of 120 N when totally immersed in a liquid of relative density 0.8. What is the relative density of the body?

B14 An object is suspended from a force meter ('spring balance') capable of reading forces to within ± 0.01 N. The object is found to have a weight of 4.92 N in air and 3.87 N when immersed in water.
(a) Calculate the density of the material from which the object is made.
(b) Discuss the reading which could be obtained if the object were suspended from the force meter within an evacuated enclosure.

(Density of air $= 1.3\,\mathrm{kg\,m^{-3}}$, density of water $= 1.0 \times 10^3\,\mathrm{kg\,m^{-3}}$.) [S]

B15 A tank contains a liquid of density $1.2 \times 10^3\,\mathrm{kg\,m^{-3}}$. A body of volume $5.0 \times 10^{-3}\,\mathrm{m^3}$ and density $9.0 \times 10^2\,\mathrm{kg\,m^{-3}}$ is totally immersed in the liquid and is attached by a thread to the bottom of the tank. What is the tension in the thread?

B16 A ball with a volume of $32\,\mathrm{cm^3}$ floats on water with exactly half of the ball below the surface. What is the mass of the ball? (Density of water $= 1.0 \times 10^3\,\mathrm{kg\,m^{-3}}$.)

B17 An object floats in a liquid of density $1.2 \times 10^3\,\mathrm{kg\,m^{-3}}$ with one quarter of its volume above the liquid surface. What is the density of the object?

B18 A hot-air balloon has a volume of $500\,\mathrm{m^3}$. The balloon moves upwards at a *constant* speed in air of density $1.2\,\mathrm{kg\,m^{-3}}$ when the density of the hot air inside it is $0.80\,\mathrm{kg\,m^{-3}}$.
(a) What is the combined mass of the balloon and the air inside it?
(b) What is the upward acceleration of the balloon when the temperature of the air inside it has been increased so that its density is $0.7\,\mathrm{kg\,m^{-3}}$?

B19 An object with a volume of $1.0 \times 10^{-5}\,\mathrm{m^3}$ and density $4.0 \times 10^2\,\mathrm{kg\,m^{-3}}$ floats on water in a tank of cross-sectional area $1.0 \times 10^{-3}\,\mathrm{m^2}$.
(a) By how much does the water level drop when the object is removed?
(b) Show that this decrease in water level reduces the force on the base of the tank by an amount equal to the weight of the object.

(Density of water $= 1.0 \times 10^3\,\mathrm{kg\,m^{-3}}$.)

B20 Derive, explaining the meaning of the terms on the right-hand side, the approximate relationship

$$\sigma \approx \tfrac{1}{4} n A \varepsilon_0$$

where σ is the work done in isothermally creating unit surface area of a liquid. Explain why the relationship is approximate.

B21 Show that σ, the work done in isothermally creating unit surface area of a liquid, is equal to γ, the force per unit length acting in the surface of the liquid at right-angles to one side of an imaginary line drawn in the surface.

B22 The specific latent heat of vaporization and the surface tension of a particular liquid are $7.5 \times 10^4\,\mathrm{J\,kg^{-1}}$ and $4.0 \times 10^{-2}\,\mathrm{N\,m^{-1}}$ respectively. The relative molecular mass of the liquid is 40. Estimate the number of molecules in $1\,\mathrm{cm^2}$ of the liquid surface. (The Avogadro constant $= 6 \times 10^{23}\,\mathrm{mol^{-1}}$.)

B23 Explain briefly, with the aid of a diagram, what you would expect to happen to a nearly spherical water droplet resting on a clean horizontal surface if a tiny amount of detergent were added to it.

How do you account for the change that might occur? [L]

B24 The velocity v of surface waves on a liquid may be related to their wavelength λ, the surface tension of the liquid σ and its density ρ by the following equation

$$v = k\lambda^{\alpha}\sigma^{\beta}\rho^{\gamma}$$

where k is a dimensionless constant.

Find values for α, β and γ by a dimensional argument. [W]

B25 A spherical drop of mercury of radius 2 mm falls to the ground and breaks into 10 smaller drops of equal size. Calculate the amount of work that has to be done. (Surface tension of mercury $= 4.72 \times 10^{-1}\,\mathrm{N\,m^{-1}}$.)

What is the minimum speed with which the original drop could have hit the ground? (Density of mercury $= 1.36 \times 10^4\,\mathrm{kg\,m^{-3}}$.)

B26 Two soap bubbles have radii of 3 cm and 4 cm. The bubbles are in a vacuum and they combine to form a single larger bubble. Calculate the radius of this bubble. (You may assume that the surface tension of soap solution is constant throughout.)

B27 A glass barometer tube has an internal radius of 3 mm. Calculate the actual atmospheric pressure on a day when the height of the mercury column is 760.2 mm. (Surface tension of mercury $= 4.72 \times 10^{-1}\,\mathrm{N\,m^{-1}}$, angle of contact of mercury with glass $= 137^\circ$, density of mercury $= 1.36 \times 10^4\,\mathrm{kg\,m^{-3}}$, acceleration due to gravity $= 9.81\,\mathrm{m\,s^{-2}}$.)

B28 A soap bubble whose radius is 12 mm becomes attached to one of radius 20 mm. Calculate the radius of curvature of the common interface.

B29 Define the terms *surface tension*, *angle of contact*.

The end of a clean glass capillary tube, having internal diameter 0.6 mm, is dipped into a beaker containing water, which rises up the tube to a vertical height of 5.0 cm above the water surface in the beaker. Calculate the surface tension of water. (Density of water $= 1000\,\mathrm{kg\,m^{-3}}$.)

What would be the difference if the tube were not perfectly clean, so that the water did not wet it, but had an angle of contact of 30° with the tube surface? [S]

B30 Define *surface tension*. Give a concise explanation of the origin of surface tension in terms of intermolecular forces.

The pressure difference on the two sides of a spherical liquid–gas interface is $2\gamma/R$; what do γ and R represent? (*You may use this expression in* (a) *if you so wish, but you are advised to use it in parts* (b) *and* (c).)

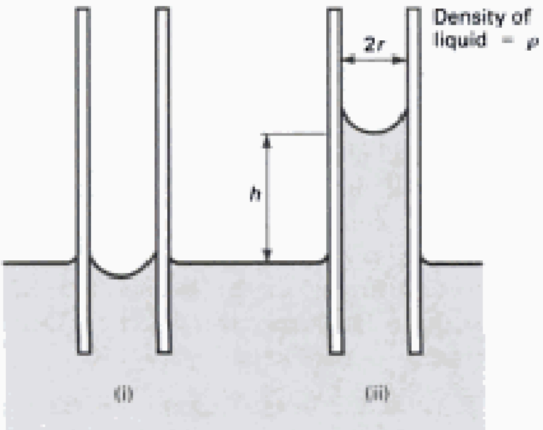
(a) Derive an expression for the height of the liquid column in a vertical, uniform capillary tube. (Neglect any correction for the mass of the meniscus and assume that the angle of contact is zero.) Describe the experimental determination of the surface tension of water by the capillary rise method giving with reasons, a suitable value for the radius of the tube.

(b) The two vertical arms of a manometer, containing water, have different internal radii of $10^{-3}\,\mathrm{m}$ and $2 \times 10^{-3}\,\mathrm{m}$ respectively. Determine the difference in height of the two liquid levels when the arms are open to the atmosphere.

(c) Explain why the pressure difference is not constant across the meniscus of the liquid column in a capillary tube, and discuss the general shape of the meniscus.

The surface tension and density of water are $7 \times 10^{-2}\,N\,m^{-1}$ and $10^3\,kg\,m^{-3}$ respectively. [W]

B31 By considering the work done per unit area in increasing the surface area of a bubble blown in a liquid, or otherwise, derive an expression for the excess pressure p inside a bubble of radius r.



The diagrams above represent glass capillary tubes dipping into a liquid. Explain why the situation represented by (i) is unstable while that in (ii) is stable.

Use the data given in diagram (ii) to derive an expression for the height h to which the liquid rises, given that the angle of contact between the liquid and glass is zero.

By considering intermolecular forces explain why the surface of a liquid is different from the bulk of the liquid.

Suggest why there might be a connection between the surface energies (surface tensions) of liquids and their normal boiling points. [L]

B32 The pressure difference p across a spherical surface of radius r between air and a liquid, where γ is the surface tension of the liquid, is given by

$$p = \frac{2\gamma}{r}$$

(a) Show that this expression is consistent with γ being measured in $N\,m^{-1}$. It can be shown that γ is also equal to the energy stored per unit area in the surface. Show that this is also consistent with γ being measured in $N\,m^{-1}$.

(b) Describe a method for measuring γ which is based on measuring the excess pressure in a bubble.

(c) Using the energy definition of γ given above calculate the energy stored in the surface of a soap bubble 2.0 cm in radius if its surface tension is $4.5 \times 10^{-2}\,N\,m^{-1}$. If the thickness of the surface is 6.0×10^{-7} m and the density of the soap solution is $1000\,kg\,m^{-3}$, calculate the speed with which the liquid fragments will fly apart when the bubble is burst. What assumptions have you made in your calculation? [L]

B33 **(a)** Draw and label a diagram of apparatus suitable for measuring the surface tension of water by Jaeger's method.

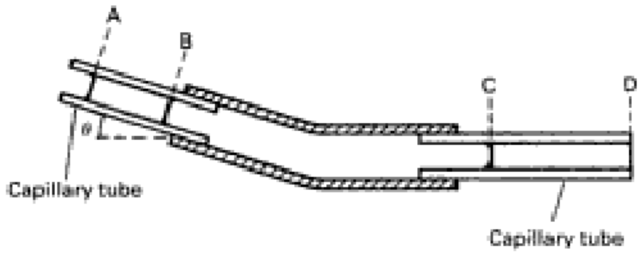
Assume that the pressure p within the apparatus *when it is assembled* equals the pressure p_0 of the atmosphere outside. Sketch a graph which shows how the pressure difference $p - p_0$ changes with time from the instant p begins to increase until the moment a bubble is about to break away from the bottom of the capillary for the third time.

How are the pressure differences shown in the graph related to **(i)** the position of the liquid meniscus in the capillary and **(ii)** the radius of the bubble formed at the bottom of the capillary?

State which quantities you would measure if you were using this apparatus to determine the surface tension of water and describe how you would measure them.

(b) The diagram, (p. 213) which is not to scale, shows two capillary tubes of uniform bore fitting tightly into a short length of rubber tubing. AB and CD are two threads of water. The capillary tube containing CD is kept horizontal while that containing AB is raised through an angle θ until the water surface at D is both flat and vertical.

(i) Calculate the surface tension of water given that θ is 10.5°, AB is 11.4 cm,



the radius of the capillary tube at C is 0.72 mm and the density of water is $1.00 \times 10^3\,kg\,m^{-3}$. The angle of contact between water and glass is zero. (You may assume the relation $\Delta p = 2\gamma/r$, and that $g = 9.8\,m\,s^{-2}$.)

(ii) Suggest an experimental procedure to determine when the water surface at D is flat. [L]

ELASTICITY (Chapter 11)

B34 Define *tensile stress, tensile strain, Young's modulus.*

A mass of 11 kg is suspended from the ceiling by an aluminium wire of length 2 m and diameter 2 mm. What is:

(a) the extension produced,

(b) the elastic energy stored in the wire?

The Young's modulus of aluminium is $7 \times 10^{10}\,Pa\,(N\,m^{-2})$. [S]

B35 The maximum upward acceleration of a lift of total mass 2500 kg is $0.5\,m\,s^{-2}$. The lift is supported by a steel cable, which has a maximum safe working stress of $1.0 \times 10^8\,Pa$. What minimum area of cross-section of cable should be used? [C(O)]

B36 An elastic string of cross-sectional area $4\,mm^2$ requires a force of 2.8 N to increase its length by one tenth. Find Young's modulus for the string. If the original length of the string was 1 m, find the energy stored in the string when it is so extended. [W]

B37 **(a)** For an elastic wire under tension there is, under certain conditions, a simple relation between the *applied stress* and the *strain produced*. Explain the meaning of the terms in italics, state the relation and indicate the conditions that must be fulfilled.

(b) A long thin vertical steel wire is fixed at the upper end. Describe, giving reasons for the design of the apparatus used, how you would measure the extensions caused by the addition of various loads at the lower end.

(c) A massive stone pillar 20 m high and of uniform cross-section rests on a rigid base and supports a vertical load of $5.0 \times 10^5\,N$ at its upper end. State, with reasons, where in the pillar the maximum compressive stress occurs. If the compressive stress in the pillar is not to exceed $1.6 \times 10^6\,N\,m^{-2}$, what is the minimum cross-sectional area of the pillar?

Density of the stone $= 2.5 \times 10^3\,kg\,m^{-3}$.
[J]

B38 A cylindrical copper wire and a cylindrical steel wire, each of length 1.000 m and having equal diameters are joined at one end to form a composite wire 2.000 m long. This composite wire is subjected to a tensile stress until its length becomes 2.002 m. Calculate the tensile stress applied to the wire.

(The Young modulus for copper $= 1.2 \times 10^{11}\,Pa$ and for steel $= 2.0 \times 10^{11}\,Pa$.)
[W, '91]

B39 **(a)** A heavy rigid bar is supported horizontally from a fixed support by two vertical wires, A and B, of the same initial length and which experience the same extension. If the ratio of the diameter of A to that of B is 2 and the ratio of Young's modulus of A to that of B is 2, calculate the ratio of the tension in A to that in B.

(b) If the distance between the wires is *D*, calculate the distance of wire A from the centre of gravity of the bar. [J]

B40 **(a)** Define *stress, strain* and the *Young modulus.*

(b) **(i)** Describe an experiment to determine the Young modulus for a material in the form of a wire.

(ii) Which measurement requires particular care, from the point of view of accuracy, and why?

(c) **(i)** Derive an expression for the potential energy stored in a stretched wire.

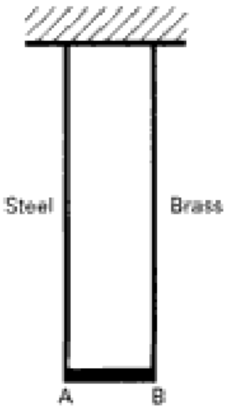
(ii) A steel wire of diameter 1 mm and length 1.5 m is stretched by a force of

50 N. Calculate the potential energy stored in the wire.
(Young modulus of steel = 2×10^{11} Pa.

(iii) The wire is further stretched to breaking. Where does the stored energy go? [W, 90]

B41 Define Young's modulus and describe a method to measure its value for a uniform elastic wire. State the precautions necessary to ensure an accurate result.
The ends of a uniform wire of cross-sectional area 10^{-6} m^2 and negligible mass are attached to fixed points A and B which are 1 m apart in the same horizontal plane. The wire is initially straight and unstretched. A mass of 0.5 kg is attached to the mid-point of the wire and hangs in equilibrium with the mid-point at a distance 10 mm below AB. Calculate the value of Young's modulus for the wire. [O & C]

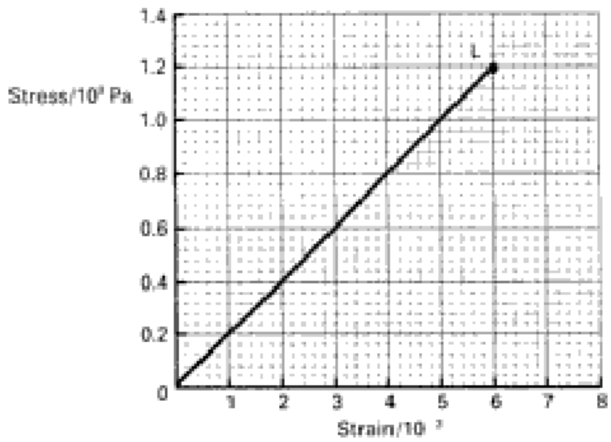
B42 **(a)** Describe an experiment using two long, parallel, identical wires to determine the Young modulus for steel. Explain why it is necessary to use two such wires. Indicate what quantities you would measure and what measuring instrument you would use in each case. State what graph you would plot, and show how it is used to calculate the Young modulus.



(b) A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass, as shown in the diagram. Each wire is 2.00 m long. The diameter of the steel wire is 0.60 mm and the length of the bar AB is 0.20 m. When a mass of 10.0 kg is suspended from the centre of AB the bar remains horizontal.

(i) What is the tension in each wire?
(ii) Calculate the extension of the steel wire and the energy stored in it.
(iii) Calculate the diameter of the brass wire.
(iv) If the brass wire were replaced by another brass wire of diameter 1.00 mm, where should the mass be suspended so that AB would remain horizontal?
(The Young modulus for steel = 2.0×10^{11} Pa, the Young modulus for brass = 1.0×10^{11} Pa.) [J, '91]

B43 **(a)** For moderate loads, most metals are elastic. What is meant by the term *elastic*?
(b) The graph shows a stress–strain diagram for a steel wire, of cross-section area 0.80×10^{-6} m^2, that is stretched to its elastic limit L.



Use this graph to estimate:
(i) the Young modulus of steel;
(ii) the tension in the wire at its elastic limit L;
(iii) the maximum elastic strain energy that can be stored in unit volume (1.0 m^3) of steel. [O, '91]

B44 Define *stress*, *strain*, and *Young's modulus* of an elastic material.

Describe an experiment for measuring the Young's modulus of a material in the form of a wire.

A rubber cord has a diameter of 5.0 mm, and an unstretched length of 1.0 m. One end of the cord is attached to a fixed support A. When a mass of 1.0 kg is attached to the other end of the cord, so as to hang vertically below A, the cord is observed to elongate by 100 mm. Calculate the Young's modulus of rubber.

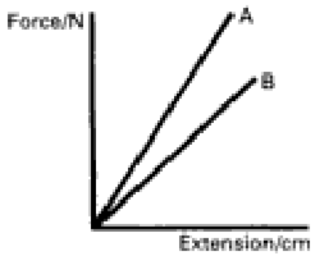
If the 1 kg mass is now pulled down a further short distance and then released, what is the period of the resulting oscillations? [S]

B45 Explain the term *Young's modulus.*

A nylon guitar string 62.8 cm long and 1 mm diameter is tuned by stretching it 2.0 cm. Calculate **(a)** the tension, **(b)** the elastic energy stored in the string.

Young's modulus of nylon $= 2 \times 10^9$ Pa. [S]

B46 Two copper wires A and B, of the same known areas of cross-section, are subjected to measured stretching forces and the corresponding extensions are measured. The results, on a force–extension graph, are shown in the diagram below.



Explain what deduction you could make about the difference between the two wires.
Define the quantities which you would plot to get the same graph for both wires. How would you use this second graph to evaluate an important physical constant of copper? [L]

B47 A submerged wreck is lifted from a dock basin by means of a crane to which is attached a steel cable 10 m long of cross-sectional area 5 cm^2 and Young's modulus 5×10^{10} N m^{-2}. The material being lifted has a mass 10^4 kg and mean density 8000 kg m^{-3}. Find the change in extension of the cable as the load is lifted clear of the water.

Assume that at all times the tension in the cable is the same throughout its length.

Density of water $= 1000$ kg m^{-3}. [J]

B48 A copper wire LM is fused at one end, M, to an iron wire MN. The copper wire has length 0.900 m and cross-section 0.90×10^{-6} m^2. The iron wire has length 1.400 m and cross-section 1.30×10^{-6} m^2. The compound wire is stretched; its total length increases by 0.0100 m.

Calculate:
(a) the ratio of the extensions of the two wires,
(b) the extension of each wire,
(c) the tension applied to the compound wire.
(Young's modulus for copper $= 1.30 \times 10^{11}$ N m^{-2}; Young's modulus for iron $= 2.10 \times 10^{11}$ N m^{-2}.) [L]

B49 Explain the terms tensile *stress* and tensile *strain* as applied to a specimen of material and explain the meaning of the word *tensile.*

A car breaks down and the driver asks a friend to tow it using a piece of nylon rope which is 10.00 m long and has a diameter of 10.0 mm. The rope obeys Hooke's law and has a Young modulus of 3.0×10^9 N m^{-2}. The mass of the car and driver is 750 kg.

(a) When towing on a level road at a constant speed, it is found that the rope extends by 0.025 m. Calculate the tension in the rope and hence the net resistive force acting on the towed car.

(b) The two cars now ascend a slope which rises 1.0 m vertically for every 15.0 m travelled on the road. They maintain the same speed as in part (a). What is the new length of the towrope?

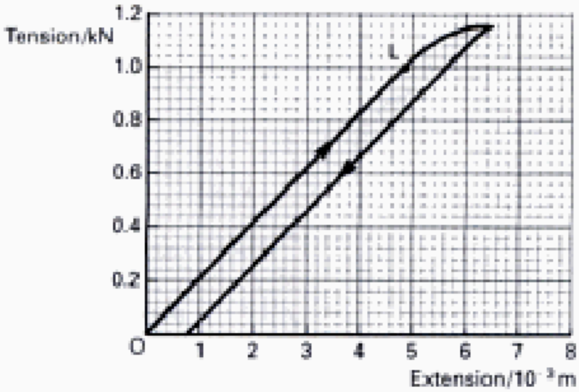
(c) How much elastic energy is stored in the rope while the cars are climbing?

(d) A stretched towrope must be regarded as dangerous because of the energy released should it break or become detached. This danger can be reduced by careful choice of towrope.

By comparison with the original rope in each case, state and explain how the energy stored in the rope could be reduced by using a rope with a different
(i) Young modulus,
(ii) area of cross-section. [O & C, '91]

B50 The graph (p. 216) represents the tension–extension graph for a copper wire of length 1.2 m and cross-sectional area 1.5×10^{-6} m^2.

The tension is gradually increased from zero to a maximum value, and then reduced back to zero.



(a) Use the region OL of the graph to find the Young modulus for the material of the wire.
(b) Why is the unloading curve displaced from the loading curve?
(c) Shade the area of the graph which represents the energy lost as heat during the loading–unloading cycle. [O, '92*]

B51 A 20 m length of continuous steel railway line of cross-sectional area $8.0 \times 10^{-3}\,m^2$ is welded into place after heating to a uniform temperature of 40 °C.

(Take Young's modulus for steel to be 2.0×10^{11} Pa, its linear expansivity to be $12 \times 10^{-6}\,K^{-1}$, its density to be $7800\,kg\,m^{-3}$, and its specific heat capacity to be $500\,J\,kg^{-1}\,K^{-1}$.)

Calculate, for normal operating conditions at 15 °C:
(a) the tensile strain,
(b) the tensile stress,
(c) the elastic strain energy in the rail.
How much heat would be required to return the rail to 40 °C? Explain briefly why your answer is not the same as that of (c). [O*]

B52 The diagrams show an apparatus designed to demonstrate the resistance to shear of a new material.

One end X of the steel bar is fixed. The other end has a hole of diameter 6 mm drilled in it. When the room temperature is 20 °C, the distance between the fixed end of the bar

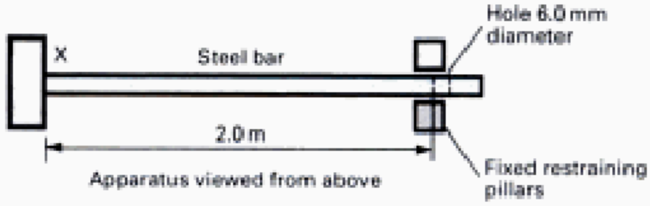


Fig. 1

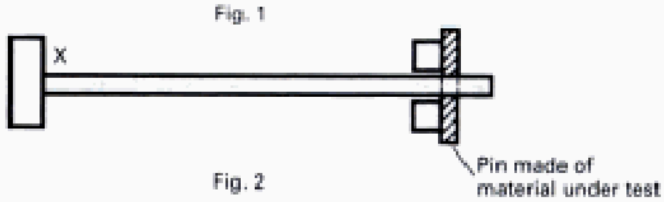


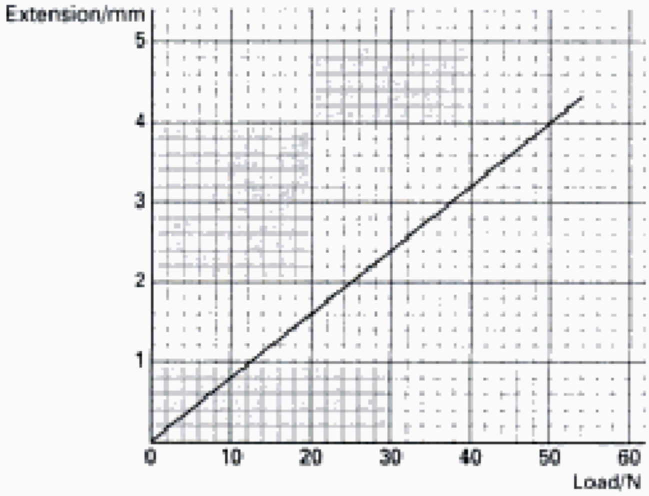
Fig. 2

and the nearer edge of the hole is 2.0 m as shown in Fig. 1. At this temperature half of the hole protrudes beyond the restraining pillars.

The bar is heated in a constant temperature enclosure until the hole just clears the restraining pillars. A pin, which just fits the hole and made of the material under test, is then inserted through the hole as shown in Fig. 2.
(a) Calculate the temperature of the enclosure.
(b) Given that the bar does not extend beyond its limit of proportionality, calculate the tensile stress in the steel bar when the temperature returns to 20 °C.
(Young modulus for steel = 1.2×10^{11} Pa, linear expansivity of steel = $1.5 \times 10^{-6}\,K^{-1}$.)
[AEB, '90]

B53 **(a)** Give the meanings of the terms *tensile stress* and *Young's modulus*. Define the quantity which relates these terms.
(b) When measuring Young's modulus of a material it is common to use a specimen which is **(i)** very long, and **(ii)** very thin. Give the reasons for this.

Describe how you would measure accurately the extension of such a wire under an applied load.
(c) The graph (p. 217) shows how the extension of a wire varies with the load applied to it. The wire used has a length 3.00 m and a diameter 5.0×10^{-4} m.
(i) Calculate the tensile stress produced by a load of 50 N.
(ii) Find the energy stored in the wire when this load is acting.

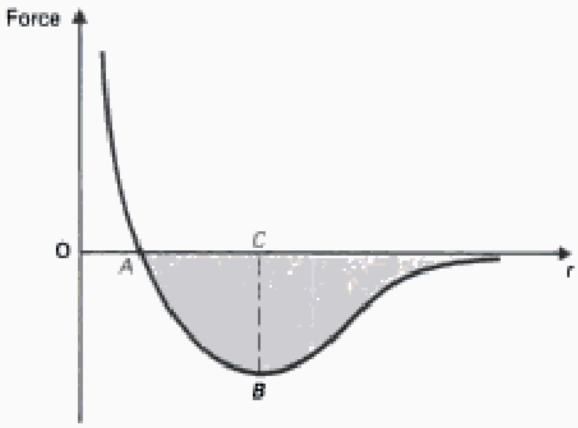


(iii) Calculate the reduction in gravitational potential energy of a 5.0 kg mass used to provide the load.

(iv) Suggest why the answers to (ii) and (iii) above are different.

(v) Calculate Young's modulus for the metal of the wire. [S]

B54 The sketch shows, approximately, how the resultant force between adjacent atoms in a solid depends on *r*, their distance apart.



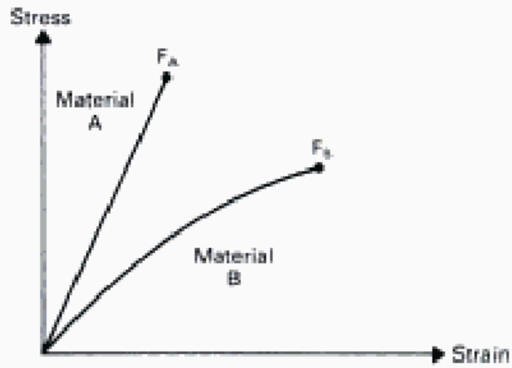
(a) Which distance on the graph represents the equilibrium separation of the atoms? Briefly justify your answer.

(b) What is the significance of the shaded area?

(c) Use the graph to explain why you would expect the solid to obey Hooke's Law for *small* extensions and compressions.

[W, '92]

B55 **(a)** The graphs represent stress–strain curves for two different materials, A and B. F_A and F_B are the respective points at which each material fractures.



State, giving your reasons, which material, A or B,

(i) obeys Hooke's law up to the point of fracture,

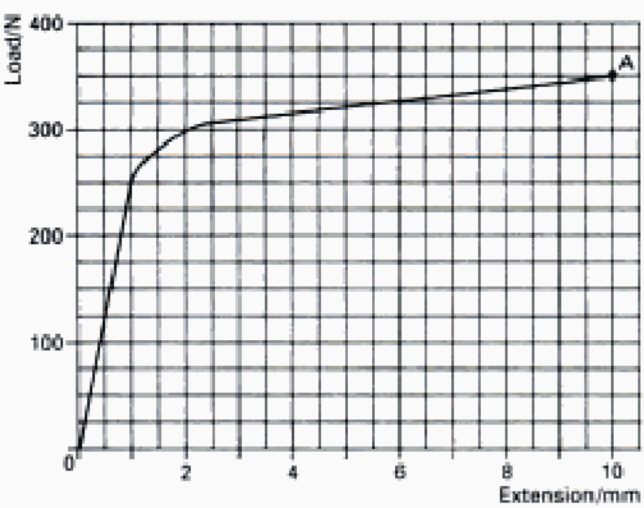
(ii) is the weaker,

(iii) has the greater value of Young's modulus.

(b) A thin steel wire initially 1.5 m long and of diameter 0.50 mm is suspended from a rigid support. Calculate **(i)** the final extension and **(ii)** the energy stored in the wire when a mass of 3.0 kg is attached to the lower end. Assume that the material obeys Hooke's law.

(Young's modulus for steel = $2.0 \times 10^{11}\,\mathrm{N\,m^{-2}}$.) [J]

B56 In the model of a crystalline solid the particles are assumed to exert both attractive and repulsive forces on each other. Sketch a graph of the potential energy between two particles as a function of the separation of the particles. Explain how the shape of the graph is related to the assumed properties of the particles.



The force F, in N, of attraction between two particles in a given solid varies with their separation d, in m, according to the relation

$$F = \frac{7.8 \times 10^{-20}}{d^2} - \frac{3.0 \times 10^{-96}}{d^{10}}$$

State, giving a reason, the resultant force between the two particles at their equilibrium separation. Calculate a value for this equilibrium separation.

The graph (p. 217) displays a load against extension plot for a metal wire of diameter 1.5 mm and original length 1.0 m. When the load reached the value at A the wire broke. From the graph deduce values of

(a) the stress in the wire when it broke,
(b) the work done in breaking the wire,
(c) the Young modulus for the metal of the wire.

Define *elastic* deformation. A wire of the same metal as the above is required to support a load of 1.0 kN without exceeding its elastic limit. Calculate the minimum diameter of such a wire. [O & C]

B57 **(a)** **(i)** Define *stress* and *strain* as related to the extension of a wire.

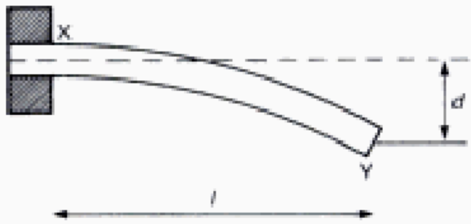
(ii) A rubber cord and a steel wire are each subjected to linear stress. Draw sketch graphs showing how the resultant strain of each sample depends on the applied stress, and point out any important differences between the graphs.

(iii) For some materials, the strain–stress curve obtained when the tension applied to the specimen is being increased may differ significantly from that when the tension is being decreased, even though no permanent extension has been caused. How may this phenomenon be interpreted?

(b) **(i)** Describe the important features of the structure of a polymeric solid, such as rubber.

(ii) Making reference to the curve you have drawn for rubber in (a)(ii) above, account for the behaviour of rubber under linear stress in terms of changes which may occur in the internal structure of the polymer. [I*]

B58 The end X of a uniform cylindrical rod XY is clamped in a fixed horizontal position. The free end Y is depressed under the action of the weight of the rod by a small amount d. The rod projects a distance l from the point of clamping X. The depression d is found to be directly proportional to the ratio g/A where g is the acceleration due to gravity and A the cross-sectional area of the rod. Also, d depends on l and the density ρ and the Young modulus E of the material of the rod. Use the method of dimensions to determine how d might depend on l, ρ and E.



How would you show experimentally the way in which d varies with the length and radius of the rod? [O & C*]

B59 **(a)** In order to determine *Young's modulus* for the material of a wire in a school laboratory, it is usual to apply a *tensile stress* to the wire and to measure the *tensile strain* produced. Explain the meanings of the terms in italics and state the relationship between them.

(b)

Additional load/kg	Scale reading/mm
0	2.8
2.0	3.8
4.0	4.5
6.0	5.1
8.0	5.7
10.0	6.3
12.0	6.9
14.0	7.5
16.0	8.1

The table shows readings obtained when stretching a wire supported at its upper end by suspending masses from its lower end. The unstretched length of the wire was 2.23 m and its diameter 0.71 mm. Using a graphical method, determine a value for Young's modulus for the material of the wire.

(c) Describe suitable apparatus for obtaining the readings shown and explain the important features of the design.

(d) A student noticed that when a mass of 10.0 kg suspended from a wire identical to that described above was pushed downwards and released, it executed vertical oscillations of small amplitude. Use the graph to explain briefly why you would expect the oscillations to be simple harmonic. [J]

B60 **(a)** When materials are stretched their behaviour may be either *elastic* or *plastic*. Distinguish carefully between these terms.

(b) Whilst stretching a length of thin copper wire it is noticed that

(i) at first a fairly strong pull is needed to stretch it by a small amount and that it stretches uniformly,

(ii) beyond a certain point the wire extends by a very much larger amount for no further increase in the pull,

(iii) finally the wire breaks.

Sketch a force–extension graph to illustrate the behaviour of this wire. Mark on it the region where the behaviour is elastic and the region where it is plastic. [L]

B61 The *force constant k* of a spring is the constant of proportionality in the Hooke's law relation $T = ke$ between tension T and extension e.

A spring A of force constant $6\,\text{N m}^{-1}$ is connected in series with a spring B of force constant $3\,\text{N m}^{-1}$, as shown in the diagram. One end of the combination is securely anchored and a force of 0.6 N is applied to the other end.

(a) By how much does each spring extend?

(b) What is the force constant of the combination? [C]

B62 **(a)** **(i)** Distinguish between *elastic* and *plastic* deformation of a material.

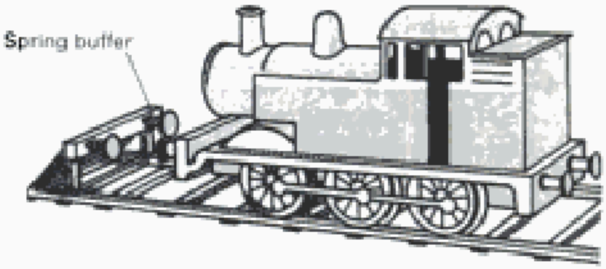
(ii) Sketch a graph to show how the extension x of a copper wire varies with F, the applied load. Mark on your sketch the region where the wire obeys Hooke's law.

(b) **(i)** A force is required to cause an extension of a spring. Explain why this causes energy to be stored in the spring.

(ii) A spring of spring constant k undergoes an elastic change resulting in an extension x. Deduce that W, its strain energy, is given by

$$W = \tfrac{1}{2}kx^2$$

(c) A toy train, mass m, travels along a track at speed v and is brought to rest by two spring buffers which are shown below.



Each buffer has spring constant k.

(i) By considering the energy transfer, derive an expression to show how the maximum compression of the buffers varies with the initial speed of the train.

(ii) Calculate the maximum compression of the buffers for a train of mass $m = 1.2\,\text{kg}$ travelling with an initial speed $v = 0.45\,\text{m s}^{-1}$ when the spring constant k of each buffer is $4.8 \times 10^3\,\text{N m}^{-1}$.

State and explain a reason why, in practice, spring buffers of this design are not used. [C, '92]

FLUID FLOW (Chapter 12)

B63 **(a)** Explain the terms *lines of flow* and *streamlines* when applied to fluid flow and deduce the relationship between them in laminar flow.

(b) State Bernoulli's equation, define the physical quantities which appear in it and the conditions required for its validity.

(c) The depth of water in a tank of large cross-sectional area is maintained at 20 cm and

water emerges in a continuous stream out of a hole 5 mm in diameter in the base. Calculate:
(i) the speed of efflux of water from the hole,
(ii) the rate of mass flow of water from the hole.
Density of water $= 1.00 \times 10^3\ \text{kg m}^{-3}$.
[J]

B64 **(a)** Distinguish between *static pressure*, *dynamic pressure* and *total pressure* when applied to streamline (laminar) fluid flow and write down expressions for these three pressures at a point in the fluid in terms of the flow velocity v, the fluid density ρ, pressure p, and the height h, of the point with respect to a datum.
(b) Describe, with the aid of a labelled diagram, the Pitot–static tube and explain how it may be used to determine the flow velocity of an incompressible, non-viscous fluid.
(c) The static pressure in a horizontal pipeline is 4.3×10^4 Pa, the total pressure is 4.7×10^4 Pa, and the area of cross-section is $20\ \text{cm}^2$. The fluid may be considered to be incompressible and non-viscous and has a density of $10^3\ \text{kg m}^{-3}$. Calculate:
(i) the flow velocity in the pipeline,
(ii) the volume flow rate in the pipeline.
[J]

B65 Air flows over the upper surfaces of the wings of an aeroplane at a speed of $120.0\ \text{m s}^{-1}$, and past the lower surfaces of the wings at $110.0\ \text{m s}^{-1}$. Calculate the 'lift' force on the aeroplane if it has a total wing area of $20.0\ \text{m}^2$.

(Density of air $= 1.29\ \text{kg m}^{-3}$.)

B66 A large tank contains water to a depth of 1.0 m. Water emerges from a small hole in the side of the tank 20 cm below the level of the surface. Calculate:
(a) the speed at which the water emerges from the hole,
(b) the distance from the base of the tank at which the water strikes the floor on which the tank is standing.
If a second hole were to be drilled in the wall of the tank vertically below the first hole, at what height above the base of the tank would this second hole have to be if the water issuing from it were to hit the floor at the same point as that from the first hole?

B67 **(a)** By considering the flow of an incompressible fluid along a horizontal pipe as shown in Fig. 1, derive Bernoulli's equation using the conservation of energy principle.

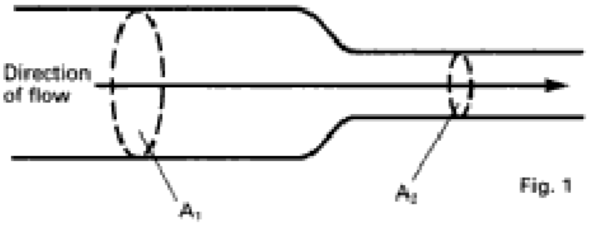


Fig. 1

(b) The water aspirator is a laboratory device used for the partial evacuation of air from a vessel. A jet of running water from a pipe constricted at X, as shown in Fig. 2, is directed into the expanded opening of a funnel at Y and passes out into the drain.

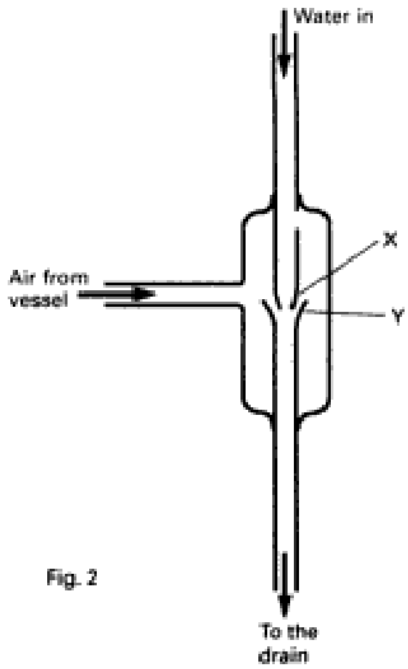


Fig. 2

(i) By considering the effect of the water jet on the air in the region of the jet, explain why this is a practical example of the Bernoulli effect.
(ii) Calculate the maximum reduction in pressure that could be achieved using this pump if the jet diameter is 2.0 mm and volume rate of flow of water is $1.3 \times 10^{-4}\ \text{m}^3\ \text{s}^{-1}$.
(Density of air $= 1.3\ \text{kg m}^{-3}$.) [J, '91]

B68 **(a)** State the equation of continuity for a *compressible* fluid flowing through a pipe.

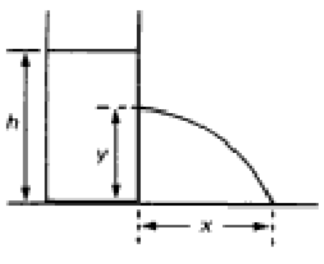
A horizontal pipe of diameter 36.0 cm tapers to a diameter of 18.0 cm at P. An ideal gas at a pressure of 2.00×10^5 Pa is moving along the wider part of the pipe at a speed of $30.0\,m\,s^{-1}$. The pressure of the gas at P is 1.80×10^5 Pa. Assuming that the temperature of the gas remains constant calculate the speed of the gas at P.

(b) State Bernoulli's equation for an incompressible fluid, giving the meanings of the symbols in the equation.

(c) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an *incompressible* fluid, and use Bernoulli's equation to calculate the corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still $30.0\,m\,s^{-1}$, the pressure is still 2.00×10^5 Pa and at this pressure the density of the gas is $2.60\,kg\,m^{-3}$.

(d) Draw a labelled diagram to show how you would use the change in pressure discussed in (c), treating the gas as an incompressible fluid, to obtain a value for the speed of the gas in the pipe. Show how the result is calculated. [J]

B69 **(a)** A cylinder of large cross-sectional area, containing water, stands on a horizontal bench. The water surface is at a height h above the bench. Water emerges horizontally from a hole in the side of the cylinder, at a height y above the bench.



(i) Use Bernoulli's equation to derive expressions for the speed at which the water emerges from the hole, and the speed at which it hits the bench.

(ii) Derive expressions for the time for the water to travel from the hole to the bench, and for x, the horizontal distance the water travels from the cylinder.

(b) Draw a diagram of a Pitot–static tube, and with reference to Bernoulli's equation explain how the tube may be used to measure the speed of a boat in sea water. [J]

B70 **(a)** Derive Bernoulli's equation for an incompressible fluid.

(b) State what you understand by *static pressure* and *dynamic pressure* and state how they are related to terms which appear in Bernoulli's equation.

(c) **(i)** Non viscous oil flows from the bottom of a tank in a horizontal pipe. State how you would measure the static pressure and the dynamic pressure of the oil in the pipe.

(ii) If the density of the oil is ρ and it is moving at a speed v in the pipe at a depth d below the surface of the oil in the tank derive from Bernoulli's equation an expression showing how the static pressure as measured in (i) is related to the atmospheric pressure, p_0, acting on the surface of the oil in the tank. (You may neglect any motion of the oil in the tank.) [J]

B71 **(a)** What do you understand by the *equation of continuity* as applied to a fluid in motion?

(b) Derive Bernoulli's equation for an incompressible fluid.

(c) A simple garden syringe used to produce a jet of water consists of a piston of area $4.00\,cm^2$ which moves in a horizontal cylinder which has a small hole of area $4.00\,mm^2$ at its end. If the force on the piston is 50.0 N calculate a value for the speed at which the water is forced out of the small hole, assuming the speed of the piston is negligible.
The density of water is $1.00 \times 10^3\,kg\,m^{-3}$.

(d) Explain why the speed of the piston may be ignored. [J]

B72 **(a)** Explain the meaning of the term *laminar flow*. Describe how, for a liquid flowing in a horizontal pipe, it can be shown whether or not laminar flow occurs.

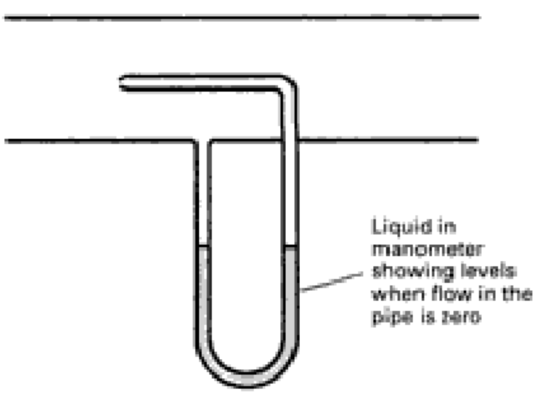
(b) (i) State both the equation of continuity and Bernoulli's equation for incompressible fluids.

(ii) Draw and label a diagram of a Venturi meter suitable for measuring the velocity of flow of a liquid in a horizontal pipe. Use the equations of (i) to obtain an expression from which the velocity of flow of the liquid in the Venturi meter can be calculated. What measurements must be made when the Venturi meter is used?

(iii) Explain how assumptions made in your derivation in (ii) could limit the usefulness of a Venturi meter.

[J]

B73 (a) (i) The Pitot tube shown in the figure below is used to measure the speed of flow of a gas in a pipe.



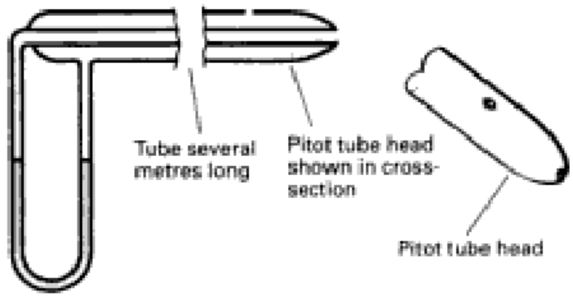
Redraw the diagram showing the direction of flow of the gas and the corresponding levels of liquid in the manometer.

(ii) By considering Bernoulli's equation show that the difference h in the levels of the liquid in the manometer is given by

$$h = \frac{\rho v^2}{2\rho_0 g}$$

where ρ is the density of the gas, ρ_0 is the density of the liquid in the manometer, v is the speed of the gas along the pipe, g is the acceleration due to gravity.

(b) The figure below shows a variation of the Pitot tube used to indicate the speed of aircraft.



(i) State *one* factor that would have to be taken into consideration when choosing where to position the Pitot tube head on the exterior of an aircraft if the true airspeed is to be indicated.

(ii) Would water be a suitable liquid for the manometer if airspeeds of up to 600 km h^{-1} are to be indicated in the cockpit of an aircraft? Justify your answer.

(c) The figure below shows a Venturi meter used to indicate the speed of flow of a liquid through a pipe.

(i) Redraw the diagram and show on it the levels of liquid in the three vertical tubes, assuming the pipe to be horizontal and the liquid to be incompressible and non-viscous.

(ii) How would your answer to (i) be different if the viscosity of the liquid was significant? Explain your answer.

[J, '92]

B74 (a) Give an account of an experiment which makes use of Poiseuille's formula to measure the viscosity of water.

(b) An empty vessel which is open at the top has a horizontal capillary tube of length 20 cm and internal radius 1.0 mm protruding from one of its side walls immediately above the base. Water flows into the vessel at a constant rate of 1.5 $cm^3\ s^{-1}$. At what depth does the water level stop rising?

(You may assume that the flow is steady. Coefficient of viscosity of water = 1.0 ×

10^{-3} N s m^{-2}, density of water = 1.0×10^3 kg m^{-3}, acceleration due to gravity = 10 m s^{-2}.)

B75 A liquid flows steadily through two pipes, A and B, which are joined end to end and whose internal radii are r and $2r$ respectively. If B is 8 times longer than A and the pressure difference between the ends of the composite pipe is 9000 N m^{-2}, what is the pressure difference across A?

B76 **(a)** Define coefficient of viscosity, η, and show that its dimensions in M, L and T are $ML^{-1}T^{-1}$. What is meant by laminar flow?

(b) Poiseuille's formula for the volume of liquid V flowing in time t through a uniform capillary of radius r under laminar conditions is

$$\frac{V}{t} = \frac{\pi r^4 p}{8\eta l}.$$

where p/l is the pressure gradient along the tube.

(i) Show that this equation is dimensionally consistent.

(ii) Describe how you would apply the equation to measure η for water at room temperature.

(iii) Laminar conditions should obtain provided that the value of

$$\frac{r^3 p\rho}{4\eta^2 l} < 1150,$$

where ρ is the density of the liquid. Taking η to be 1.2×10^{-3} Pa s (N s m^{-2}) and ρ to be 1000 kg m^{-3} for water, estimate the greatest head of water under which laminar flow should hold for a capillary of length 0.2 m and radius 0.7 mm. [O]

B77 **(a)** When a sphere of radius a moves slowly with a speed v through a fluid of viscosity η, Stokes' law tells us that the force F on the sphere due to viscous drag is given by the expression $F = 6\pi a\eta v$. Show that this expression is dimensionally correct.

(b) In an experiment to compare the viscosities of two oils, small spheres are allowed to fall through long columns of the liquids. What conditions are necessary in order that Stokes' law may be applied?

State what measurements are necessary to find the *terminal* speed of the spheres. [L*]

B78 Frictional forces and viscous drag both oppose relative motion. Suggest some similarities and differences between them.

Explain why a small sphere, falling through liquid in a deep tank, eventually moves with a constant speed (the terminal velocity). Sketch a graph showing how the acceleration of the sphere varies with time after its release at the surface of the liquid.

Describe how you would measure such a terminal velocity, explaining how you would use your measurements to ensure that your result was the true *terminal* velocity. [C*]

B79 The viscous force on a sphere, of radius r, moving through a fluid with velocity v can be expressed as $6\pi\eta r v$, where η is the coefficient of viscosity of the fluid. What is the limitation on the use of this expression? [L]

B80 **(a)** Stokes' law may be represented by the equation shown below.

$$F = 6\pi\eta r v$$

(i) State the physical quantities represented by the symbols F, η, r and v and the conditions under which the relationship is valid.

(ii) Tiny spherical particles of alumina, having a wide range of radii, are stirred up in a beaker of water 8.0 cm deep. Draw a diagram showing the forces acting on *one* such particle, including the force of upthrust (equal to the weight of water displaced by the particle), shortly after stirring has ceased and the water has achieved a still condition. Hence determine the radius of the largest particle to remain in suspension after 24 hours. You may assume that the particles fall through the water with terminal velocity.

(Density of water = 1.0×10^3 kg m^{-3}, density of alumina = 2.7×10^3 kg m^{-3}, (coefficient of viscosity of water = 1.0×10^{-3} N s m^{-2}.)

(b) **(i)** State Bernoulli's equation for an incompressible fluid.

(ii) Sketch a section through an aircraft wing and explain how the movement of such a wing through the air results in an upward force (lift) on the wing.

(iii) A particular aircraft design calls for a lift of about 1.2×10^4 N on each square metre of the wing when the speed of the aircraft through the air is $100\ \mathrm{m\,s^{-1}}$. Assuming that the air flows past the wing with streamline flow and the flow past the lower surface is equal to the speed of the aircraft, what is the required speed of the air over the upper surface of the wing? (Density of air = $1.3\ \mathrm{kg\,m^{-3}}$.) [J, '90]

B81 The stress σ between two planes of molecules in a moving liquid is given by

$$\sigma = \frac{\eta v}{x}$$

where v is the difference in the velocities of the planes, x their distance apart and η a constant for the liquid.

(a) Show that the dimensions of η are $\mathrm{ML^{-1}\,T^{-1}}$.

(b) The force F acting on a sphere moving through a liquid is known to depend upon
  (i) the radius r of the sphere,
  (ii) the speed u of the sphere,
  (iii) the constant η for the liquid.
Find how F depends on r, u and η. [W, '91]

B82 A body moving through air at a high speed v experiences a retarding force F given by

$$F = kA\rho v^x$$

where A is the surface area of the body, ρ is the density of the air and k is a numerical constant. Deduce the value of x.

A sphere of radius 50 mm and mass 1.0 kg falling vertically through air of density $1.2\ \mathrm{kg\,m^{-3}}$ attains a steady velocity of $11.0\ \mathrm{m\,s^{-1}}$. If the above equation then applies to its fall what is the value of k in this instance? [L]

B83 The drag force F exerted on a vehicle due to its motion through still air is given by

$$F = \frac{\rho D v^2}{2}$$

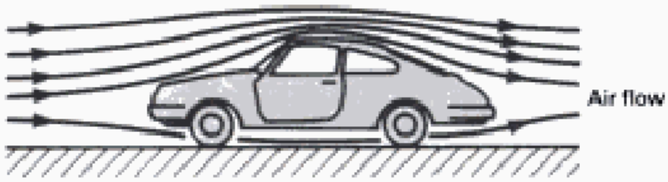
where ρ is the density of air, v is the speed of the car and D is the drag factor.

(a) Write down the units of F, ρ and v and hence determine the unit of D.

(b) The magnitude of the drag factor of a particular car is 0.33. Calculate the speed when the rate at which energy is dissipated in overcoming air resistance is 3.0 kW. (The density of air = $1.3\ \mathrm{kg\,m^{-3}}$.)

(c) State what happens to the energy dissipated in overcoming air resistance.

(d) Some cars are streamlined like the one in the diagram.

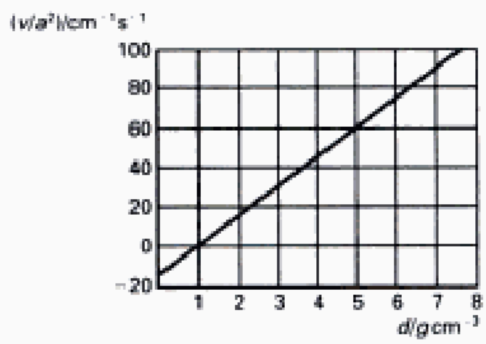


State and explain the effect of the shape of the car on the vertical forces acting on the vehicle when it starts from rest and accelerates. [AEB, '91]

B84 When a sphere of radius a and density d, falls through oil contained in a tank, it descends with uniform velocity v. The relation between v, a, and d is

$$v/a^2 = Ad - B$$

where A and B are constants.



The above graph shows the results of some experiments. Determine from the graph the numerical values of A and B. What is the radius of a steel sphere of density $7.5\ \mathrm{g\,cm^{-3}}$ which falls through the oil with velocity $3.9\ \mathrm{cm\,s^{-1}}$? [S]

B85 **(a)** Draw diagrams to show the forces acting on an object falling through a viscous liquid

(i) at the instant of release,
(ii) when it has reached its terminal velocity.

Write down an equation for the forces acting on the object in (ii). Describe and explain the motion of an object projected downwards through a viscous medium, assuming that the projection velocity of the object is greater than its terminal velocity.

(b) (i) Describe how the terminal velocity of a small sphere falling through motor oil could be measured.

(ii) In an experiment to determine the coefficient of viscosity of motor oil the following measurements were made.

Mass of glass sphere	1.2×10^{-4} kg
Diameter of sphere	4.0×10^{-3} m
Terminal velocity of sphere	5.4×10^{-2} m s^{-1}
Density of oil	860 kg m^{-3}

Calculate the coefficient of viscosity of the oil. [J]

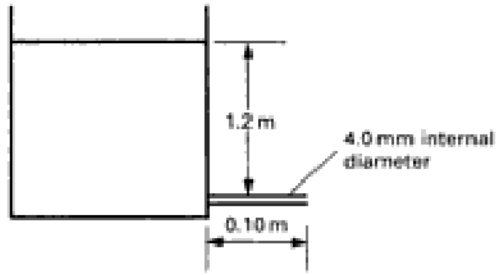
B86 **(a)** (i) State Newton's law of viscosity and hence deduce the dimensions of the coefficient of viscosity.

(ii) The rate of volume flow, $\frac{dV}{dt}$, of liquid of viscosity η, through a pipe of internal radius r and length l, is given by the equation

$$\frac{dV}{dt} = \frac{\pi p r^4}{8\eta l}$$

where p is the pressure difference between the ends of the pipe. Show that this equation is dimensionally correct.

(b) The figure shows a tank containing a light lubricating oil. The oil flows out of the tank through a horizontal pipe of length 0.10 m and internal diameter 4.0 mm.



(i) Calculate the volume of oil which flows through the pipe in one minute when the level of oil in the tank is 1.2 m above the pipe and does not significantly alter during this time.

Density of oil $= 9.2 \times 10^2$ kg m^{-3}
Coefficient of viscosity of oil $= 8.4 \times 10^{-2}$ N s m^{-2}

(ii) It is found that the volume flow is greater at higher temperatures. Assuming that density changes can be ignored, suggest an explanation for this effect in terms of the nature of the viscous force.

(c) Discuss how the lubricating properties of an oil are affected by:
(i) the coefficient of viscosity of the oil,
(ii) its variation with temperature. [J]

SECTION C

THERMAL PROPERTIES OF MATTER

13

THERMOMETRY AND CALORIMETRY

13.1 TEMPERATURE

The temperature of a body is its degree of hotness (or coldness). Thus, temperature is a measure of how hot (or cold) a body is, and should not be confused with the amount of heat it contains.

13.2 TEMPERATURE SCALES

There are many types of thermometer, but each makes use of a particular thermometric property (i.e. a property whose value changes with temperature) of a particular thermometric substance. For example: a mercury-in-glass thermometer makes use of the change in length of a column of mercury confined in a capillary tube of uniform bore; a platinum resistance thermometer makes use of the increase in the electrical resistance of platinum with increasing temperature.

In order to establish a temperature scale it is necessary to make use of **fixed points**: A fixed point is the single temperature at which it can confidently be expected that a particular physical event (e.g. the melting of ice under specific conditions) always takes place. Three such points are defined below.

The ice point is the temperature at which pure ice can exist in equilibrium with water at **standard atmospheric pressure** (i.e. at a pressure of 760 mm of mercury).

The steam point is the temperature at which pure water can exist in equilibrium with its vapour at standard atmospheric pressure.

The triple point of water is that unique temperature at which pure ice, pure water and pure water vapour can exist together in equilibium.

The triple point is particularly useful, since there is only one pressure at which all three phases (solid, liquid and gas) can be in equilibrium with each other.

The SI unit of temperature is the kelvin (K). **An interval of one kelvin is defined as being 1/273.16 of the temperature of the triple point of water as measured on the thermodynamic scale of temperature** (see later in this section and in section 16.6). The triple point of water is the fixed point of the scale and is assigned the value of 273.16 K. On this basis absolute zero is 0 K, the ice point is 273.15 K, and the steam point is 373.15 K.

Another unit, the **degree Celsius** (°C), is often used and is defined by

$$\boldsymbol{\theta = T - 273.15} \qquad [13.1]$$

where

θ = temperature in °C, and

T = temperature in K.

The Celsius scale was originally defined by using the ice and steam points as fixed points of the scale, and designating them as 0 °C and 100 °C respectively. Bearing in mind that these temperatures are respectively 273.15 K and 373.15 K, we can easily see that the more recent definition (equation [13.1]) is consistent with this. It also follows from equation [13.1] that **a temperature change of 1 K is exactly equal to a temperature change of 1 °C.**

A mercury-in-glass thermometer could be calibrated by marking the positions of the mercury when the thermometer is at the ice point and the steam point, and then dividing the interval between these two marks (designated 0 °C and 100 °C respectively) into a hundred equal divisions. If this procedure were to be adopted, the Celsius temperature θ corresponding to a length l_θ of the mercury column would be given by

$$\theta = \frac{l_\theta - l_0}{l_{100} - l_0} \times 100 \qquad [13.2]$$

where l_0 and l_{100} are the lengths of the mercury column at 0 °C and 100 °C respectively. Such a calibration regards equal increases in the length of the mercury column as being due to equal increases in temperature. There is of course no valid reason for making this assumption, and so if such a thermometer is used, it is important to stress that the measured temperatures are according to the mercury-in-glass scale of temperature. If a platinum resistance thermometer were to be calibrated by making an equivalent assumption, i.e. that equal increases in temperature produce equal increases in the resistance of platinum, then temperatures measured by this thermometer would be according to the platinum resistance scale. These two scales coincide only at the fixed points (0 °C and 100 °C), because, as might be expected, the volume of mercury and the resistance of platinum do not vary in the same way.

The thermodynamic scale of temperature is totally independent of the properties of any particular substance and is therefore an absolute scale of temperature. Although this scale is theoretical, it can be shown (see section 16.6) that it is identical with the scale based on the pressure variation of an ideal gas (see Chapter 14) at constant volume. The fixed point of both scales is the triple point of water (273.16 K) and **the kelvin temperature *T* on both the ideal gas scale and the thermodynamic scale can be found from**

$$T = \frac{p_T}{p_{Tr}} \times 273.16 \qquad [13.3]$$

where p_T is the pressure of an ideal gas at temperature T, and p_{Tr} is the pressure of the same volume of the gas at the triple point of water. Ideal gases do not exist, but real gases at low pressures are a good approximation to them. This means that results obtained using constant-volume gas thermometers incorporating real gases can be adjusted to coincide exactly with the theoretically correct temperatures of the thermodynamic scale. (The unknown temperature is estimated on the basis of equation [13.3] at a number of different (low) pressures. The results are then extrapolated to what would be obtained at zero pressure if such a measurement were possible, because at zero pressure a real gas would behave like an ideal gas.) In practice, therefore, the various types of thermometer are calibrated in terms of the constant-volume gas thermometer. As a result, the measured value of any particular temperature is the same (within the limits of accuracy of the instrument being used) no matter what type of thermometer is used to measure it.

13.3 LIQUID-IN-GLASS THERMOMETERS

These are simple to use and cheap to buy, but cannot be used for accurate work because:

(i) parallax errors prevent the scale being read to better than about 0.1 °C;

(ii) non-uniform bore limits the accuracy to about 0.1 °C;

(iii) the glass expands and contracts and can take many hours to reach its correct size, and therefore spoils the calibration;

(iv) the accuracy of the calibration depends on whether or not the thermometer is upright, and on how much of the stem is exposed.

This type of thermometer is easily adjusted to the constant-volume gas thermometer scale by suitably spacing the degree markings on the glass. Liquid-in-glass thermometers have relatively large heat (thermal) capacities, and this limits their use in two distinct ways:

(i) they cannot be used to follow rapidly changing temperatures; and

(ii) they can considerably affect the temperature of the body whose temperature they are being used to measure.

The majority of liquid-in-glass thermometers use mercury as the thermometer liquid. This is because:

(i) mercury is opaque and therefore easily seen;

(ii) mercury is a good conductor of heat and therefore can rapidly take up the temperature of its surroundings;

(iii) mercury does not wet (i.e. stick to) the glass.

The range of such a thermometer is from −39 °C (the freezing point of mercury) to something below its normal boiling point of 357 °C. This upper limit can, however, be extended by filling the thermometer with an inert gas such as nitrogen; this increases the pressure on the mercury so that its boiling point can be increased to about 800 °C. Ordinary soda-lime glass or Pyrex would soften at such a temperature, and therefore the thermometer would probably be made from fused quartz. If the mercury is replaced by ethyl alcohol, temperatures as low as −114.9 °C (the freezing point of alcohol) can be measured. Alcohol is also more sensitive to temperature change than mercury but its expansion is very non-linear. The use of liquid pentane can reduce the lower limit even more, to about −200 °C.

EXAMPLE 13.1

A particular resistance thermometer has a resistance of 30.00 Ω at the ice point, 41.58 Ω at the steam point and 34.59 Ω when immersed in a boiling liquid. A constant-volume gas thermometer gives readings of 1.333×10^5 Pa, 1.821×10^5 Pa and 1.528×10^5 Pa at the same three temperatures. Calculate the temperature at which the liquid is boiling: (a) on the scale of the gas thermometer, (b) on the scale of the resistance thermometer.

Solution

The Celsius temperature θ_g, according to the gas thermometer scale, is given by

$$\theta_g = \frac{p_\theta - p_0}{p_{100} - p_0} \times 100$$

where p_θ is the gas pressure at the temperature of the boiling liquid and p_0 and p_{100} are the gas pressures at 0 °C and 100 °C respectively. Thus

$$\theta_g = \frac{1.528 \times 10^5 - 1.333 \times 10^5}{1.821 \times 10^5 - 1.333 \times 10^5} \times 100$$

$$= \frac{0.195}{0.488} \times 100$$

$$= 39.96\ ^\circ\text{C}$$

The Celsius temperature θ_r according to the resistance scale is given by

$$\theta_r = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100$$

where R_θ is the resistance at the temperature of the boiling liquid and R_0 and R_{100} are the resistance values at 0 °C and 100 °C respectively. Thus

$$\theta_r = \frac{34.59 - 30.00}{41.58 - 30.00} \times 100$$

$$= \frac{4.59}{11.58} \times 100$$

$$= 39.64\ ^\circ\text{C}$$

EXAMPLE 13.2

The resistance R_θ of a particular resistance thermometer at a Celsius temperature θ as measured by a constant-volume gas thermometer is given by

$$R_\theta = 50.00 + 0.1700\theta + 3.00 \times 10^{-4}\theta^2$$

Calculate the temperature as measured on the scale of the resistance thermometer which corresponds to a temperature of 60 °C on the gas thermometer.

Solution

A resistance R_θ corresponds to a temperature θ_r on the scale of the resistance thermometer which is given by

$$\theta_r = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100$$

where R_0 and R_{100} are the resistances at 0 °C and 100 °C respectively. It follows that the resistance temperature which corresponds to a temperature of 60 °C on the gas thermometer scale is given by

$$\theta_r = \frac{R_{60} - R_0}{R_{100} - R_0} \times 100$$

where R_{60} is the resistance at 60 °C on the gas thermometer scale.

$$R_\theta = 50.00 + 0.1700\theta + 3.00 \times 10^{-4}\theta^2$$

$$\therefore \quad R_0 = 50.00\,\Omega$$

$$\text{and} \quad R_{60} = 50.00 + 10.20 + 1.08 = 61.28\,\Omega$$

$$\text{and} \quad R_{100} = 50.00 + 17.00 + 3.00 = 70.00\,\Omega$$

Therefore

$$\theta_r = \frac{61.28 - 50.00}{70.00 - 50.00} \times 100$$

$$= \frac{11.28}{20.00} \times 100$$

$$= 56.40\,°\text{C}$$

EXAMPLE 13.3

Derive equation [13.2].

Solution

If equal increases in the length of a mercury column are regarded as being due to equal increases in temperature, then a graph of length of column against temperature is a straight line (Fig. 13.1).

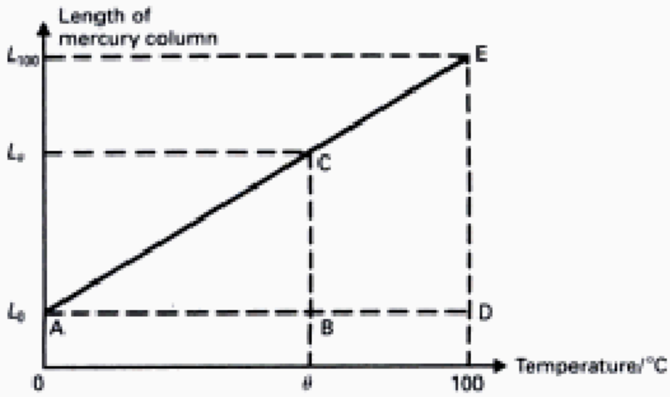


Fig. 13.1 Length of mercury column against temperature measured on the mercury-in-glass scale

Since △ABC and △ADE are similar,

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\therefore \quad \frac{\theta - 0}{100 - 0} = \frac{l_\theta - l_0}{l_{100} - l_0}$$

$$\text{i.e.} \quad \theta = \frac{l_\theta - l_0}{l_{100} - l_0} \times 100$$

QUESTIONS 13A

1. A resistance thermometer has a resistance of 21.42 Ω at the ice point, 29.10 Ω at the steam point and 28.11 Ω at some unknown temperature θ. Calculate θ on the scale of this thermometer.

2. A particular constant-volume gas thermometer registers a pressure of 1.937×10^4 Pa at the triple point of water and 2.618×10^4 Pa at the boiling point of a liquid. What is the boiling point of the liquid according to this thermometer?

3. The temperature measurement described in question 2 was repeated using the same thermometer but with a different quantity of (the same) gas. The readings on this occasion were 4.068×10^4 Pa at the triple point of water and 5.503×10^4 Pa at the boiling point of the liquid. **(a)** What is the boiling point of the liquid according to this measurement? **(b)** Which of the two values is the better approximation to the ideal gas temperature, and why? **(c)** Estimate the ideal gas temperature.

13.4 THERMOCOUPLES

Whenever two dissimilar metals are in contact an EMF is set up at the point of contact. The magnitude of this EMF depends on the temperature at the junction of the two metals, and therefore the effect (known as the **thermoelectric or Seebeck effect**) can be used in thermometry. The devices which are used in this way are called thermocouples, and at their simplest consist of two wires of different metals joined to each other and to a high-resistance millivoltmeter as shown in Fig. 13.2. The reading on the millivoltmeter increases as the temperature of the junction increases, due to the increased EMF at the junction.

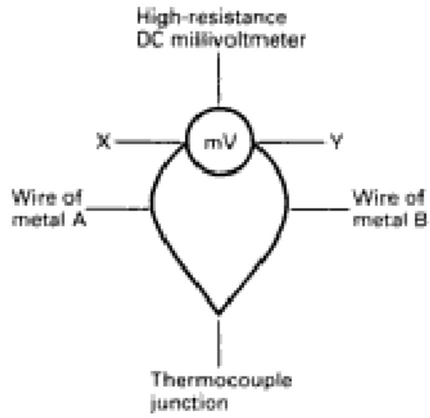
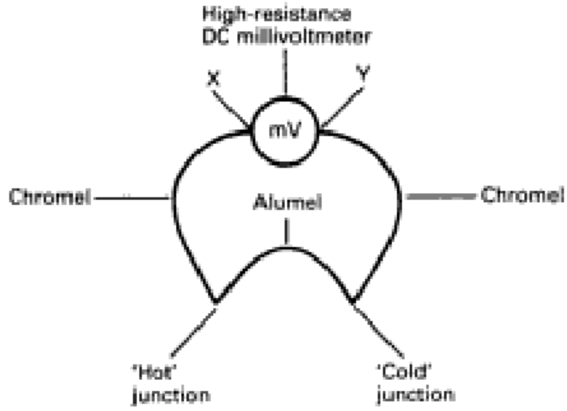


Fig. 13.2
Simple thermocouple

This simple arrangement has a serious disadvantage. Suppose metal A is chromel and metal B is alumel (these two alloys are commonly used in the manufacture of thermocouples), and that the terminal posts of the meter are brass. At X then, there is an EMF due to a chromel/brass thermocouple, and at Y there is a different EMF due to a brass/alumel thermocouple. The meter reading will be the algebraic sum of the three EMFs, and not the EMF of the actual chromel/alumel thermocouple which is required.

This difficulty can be overcome by using a second junction as shown in Fig. 13.3. With this arrangement, the EMFs produced at the meter terminals are equal and opposite, and therefore cancel each other. The extra junction that has been introduced, the so-called 'cold' junction, acts as a reference junction. The hot junction acts as the temperature measuring junction. The cold junction is normally placed in crushed ice and water so that it is always at 0 °C. The EMF at

Fig. 13.3
Thermocouple with reference junction



the cold junction is therefore always the same, and so it is a simple matter to adjust the meter reading to allow for this EMF. (**Note**. The use of the terms 'hot junction' and 'cold junction' arises because thermocouples are normally used to measure temperatures above 0 °C, in which case the reference junction is the colder of the two.)

Thermocouples have very small heat capacities, and so have very little effect on the temperature of the body whose temperature they are measuring, and can measure rapidly fluctuating temperatures. In both these respects thermocouples are superior to other types of thermometer. In addition, they are cheap and easy to use, and are ideal for use with a pen-recorder.

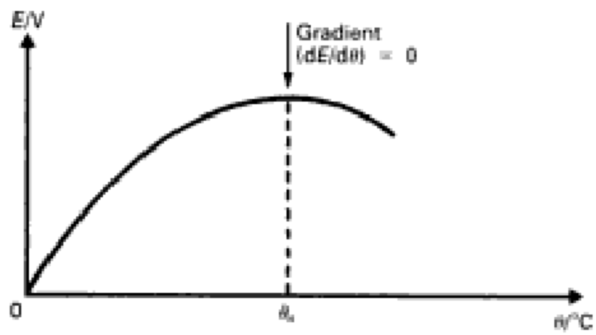
The thermoelectric EMFs of many pairs of metals have been measured as a function of the hot junction temperature θ as measured by a constant-volume gas thermometer and expressed in degrees Celsius. In every case if the cold junction is maintained at 0 °C, it is found that to a good approximation the EMF E is given by

$$E = \alpha\theta + \beta\theta^2 \qquad [13.4]$$

where the values of α and β depend on the particular pair of metals concerned. This relationship is, of course, parabolic and therefore there exists a value of θ, known as the **neutral temperature**, θ_n, for which $dE/d\theta = 0$ (Fig. 13.4). It is clearly not desirable to use a thermocouple to measure temperatures close to its neutral temperature, because the variation of EMF with temperature is small and the thermometer is therefore insensitive in this region.

The particular pair of metals used depends on the temperature range for which the thermocouple is intended. Chromel/alumel thermocouples are normally used up to about 1100 °C, and produce a thermoelectric EMF of about 4 mV for every 100 °C difference in temperature between the hot and cold junctions. Above 1100 °C and up to about 1700 °C platinum/platinum–rhodium is used on account of the high melting points of platinum and platinum–rhodium. All these metals, particularly platinum and platinum–rhodium alloy, are readily available in states of high purity and so can be used to make thermocouples which give highly

Fig. 13.4
Thermocouple EMF as a function of temperature



reproducible results. The disadvantage of platinum/platinum-rhodium is its relatively low thermoelectric EMF, about 1 mV per 100 °C.

For the most accurate work the millivoltmeter is replaced by a potentiometer (see Chapter 38). The use of a potentiometer, however, prevents the thermocouple being used to measure rapidly changing temperatures.

The values of α and β of equation [13.4] which are relevant to the commonly used thermocouple materials can be obtained from tables, and can be used in equation [13.4] to determine θ once E has been measured. Alternatively, calibration charts (plots of E against θ) are available.

13.5 RESISTANCE THERMOMETERS

Resistance thermometers rely on the fact that the resistances of metals are temperature-dependent, and therefore a measurement of resistance can be used as a measurement of temperature. They are usually made of platinum because of its high temperature coefficient of resistance and high melting point (1773 °C); features which make platinum resistance thermometers both sensitive and useful over large ranges of temperature. Also, platinum is readily available in a state of high purity, so that the measurements made with one particular platinum resistance thermometer are likely to match those made with another. The platinum is in the form of wire coiled on a suitable insulator such as mica or alumina. In use, the thermometer forms one arm of a Wheatstone bridge (see Chapter 37). This arrangement allows very slight changes in resistance, and therefore in temperature, to be measured. Platinum resistance thermometers are extremely accurate from −200 °C up to 1200 °C. The main disadvantage of thermometers of this type is that they have relatively large heat capacities. This means that they take a considerable time to come into thermal equilibrium with their surroundings, and therefore prevents them following rapidly changing temperatures. This is precluded anyway because a Wheatstone bridge has to be used.

When calibrated against constant-volume gas thermometers the resistance R of platinum is found to vary with Celsius temperature θ according to

$$R = R_0(1 + \alpha\theta + \beta\theta^2) \qquad [13.5]$$

where R_0 is the resistance of the platinum at 0 °C and α and β are constants. The values of R_0, α and β pertaining to any particular thermometer are found by measuring its resistance at the ice point, the steam point and at the melting point of sulphur (444.6 °C), and inserting the three pairs of values of R and θ in equation [13.5]. Once R_0, α and β have been found equation [13.5] can be used to determine θ for any measured value of R.

13.6 THERMISTORS

These devices, like resistance thermometers, rely on their change of electrical resistance with temperature as a means of measuring temperature. Unlike resistance thermometers, however, they have negative temperature coefficients of resistance; their resistance decreasing approximately exponentially with increasing temperature. Thermistors are semiconducting devices cheaply manufactured out of several different mixtures of semiconducting oxide powders

($Fe_3O_4 + MgCr_2O_4$ is a common mixture). They are very robust. When a Wheatstone bridge circuit is used to measure their resistance they are about twenty times as sensitive as resistance thermometers. The resistance of the connecting wires is of no significance, since the devices themselves typically have a resistance of 1 kΩ. Thermistors have very small thermal capacities, and therefore respond quickly and have little effect on the temperature they are measuring. The range is typically −70 °C to 300 °C. They are less stable than resistance thermometers, and therefore less accurate.

13.7 THE CONSTANT-VOLUME GAS THERMOMETER

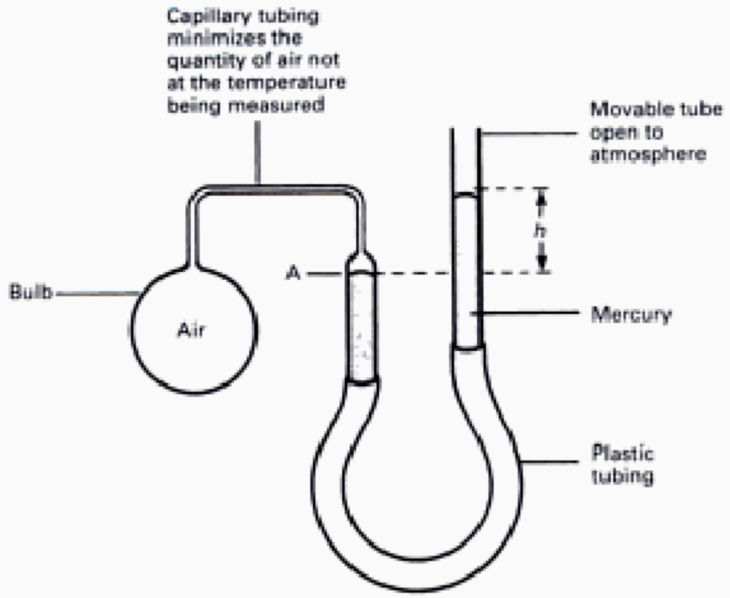
A simple constant-volume gas thermometer is shown in Fig. 13.5. When the thermometer is in use the bulb is placed inside the enclosure whose temperature is required. The gas in the bulb (air in the simplest versions) expands and forces mercury up the movable tube. The height of this tube is then adjusted to bring the mercury in the left-hand tube back to its original position at a fixed mark A. The gas now has its original volume. At this stage the head of mercury h is measured and the pressure p_θ of the gas is calculated from $p_\theta = p_A + h$ where p_A is the prevailing atmospheric pressure expressed in mm of mercury.

If p_0 and p_{100} are the pressures at 0 °C and 100 °C respectively, the temperature of the enclosure can be found from

$$\theta = \frac{p_\theta - p_0}{p_{100} - p_0} \times 100$$

where θ is the desired temperature in °C according to the constant-volume gas scale.

Fig. 13.5
Constant-volume gas thermometer



There are a number of sources of error:

(i) the bulb expands;

(ii) air is not an ideal gas;

(iii) the air in the capillary tube is not at the temperature being measured.

13.8 HEAT CAPACITY

The temperature rise produced by the addition of any given amount of heat to a body is determined by the mass of the body and the substance(s) of which it is composed.

The heat capacity* (C) of a body is defined as being the heat required to produce unit temperature rise.

It follows that if the temperature of a body whose heat capacity is C rises by $\Delta\theta$ when an amount of heat ΔQ is added to it, then

$$\Delta Q = C\Delta\theta \qquad [13.6]$$

Unit of heat capacity $= \text{J K}^{-1} = \text{J °C}^{-1}$ (see note (ii)).

The term **specific heat capacity** refers to the heat capacity of unit mass of a substance.

The specific heat capacity (c) of a substance is the heat required to produce unit temperature rise in unit mass of the substance.

It follows that if the temperature of a body of mass m and specific heat capacity c rises by $\Delta\theta$ when an amount of heat ΔQ is added to it, then

$$\Delta Q = mc\Delta\theta \qquad [13.7]$$

Unit of specific heat capacity $= \text{J kg}^{-1}\,\text{K}^{-1} = \text{J kg}^{-1}\,\text{°C}^{-1}$ (see note (ii)).

Notes

(i) The value of c depends on the temperature at which it is measured. However, over moderate changes in temperature, the variation is slight (except at low temperatures) and is normally ignored at this level.

(ii) Equations [13.6] and [13.7] involve only changes in temperature and so the numerical values of C and c when expressed in J °C^{-1} and $\text{J kg}^{-1}\,\text{°C}^{-1}$ are the same as those expressed in J K^{-1} and $\text{J kg}^{-1}\,\text{K}^{-1}$ respectively.

QUESTIONS 13B

1. Calculate the quantity of heat required to raise the temperature of a metal block with a heat capacity of 23.1 J °C^{-1} by 30.0 °C.

2. An electrical heater supplies 500 J of heat energy to a copper cylinder of mass 32.4 g. Find the increase in temperature of the cylinder.
(Specific heat capacity of copper = 385 J kg^{-1} °C^{-1}.)

3. How much heat must be removed from an object with a heat capacity of 150 J °C^{-1} in order to reduce its temperature from 80.0 °C to 20.0 °C?

4. A metal block of heat capacity 36.0 J °C^{-1} at 70 °C is plunged into an insulated beaker containing 200 g of water at 18 °C. The block and the water eventually reach a common temperature of θ °C. **(a)** Write down expressions in terms of θ for **(i)** the decrease in temperature of the block, **(ii)** the increase in temperature of the water. **(b)** Find in terms of θ, **(i)** the heat lost by the block, **(ii)** the heat gained by the water. **(c)** Assuming that no heat is used to heat the beaker and that no heat is lost to the surroundings, find the value of θ.
(Specific heat capacity of water = 4.2×10^3 J kg^{-1} °C^{-1}.)

*Sometimes called **thermal capacity**.

13.9 THE COOLING CORRECTION

Experimental determinations of specific heat capacities usually involve some loss of heat to the surroundings. Losses due to conduction and convection can be reduced by lagging, or by surrounding the apparatus with a layer of still air, or by evacuating the region around the substance under test. Losses due to radiation are significant at high temperatures and can be reduced by using polished surfaces. One way of reducing the effect of the heat losses which remain is to apply a cooling correction.

Suppose that in some experiment the temperature is recorded both during heating and after heating has been discontinued, and that the temperature is found to vary with time as shown by the solid curve in Fig. 13.6. If there had been no heat loss, the maximum temperature would have been $\theta_m + \Delta\theta$. It can be shown that

$$\Delta\theta = \frac{A}{A'}\Delta\theta' \qquad [13.8]$$

Measuring $\Delta\theta'$ and the areas A and A' enables the correction ($\Delta\theta$) to be found.

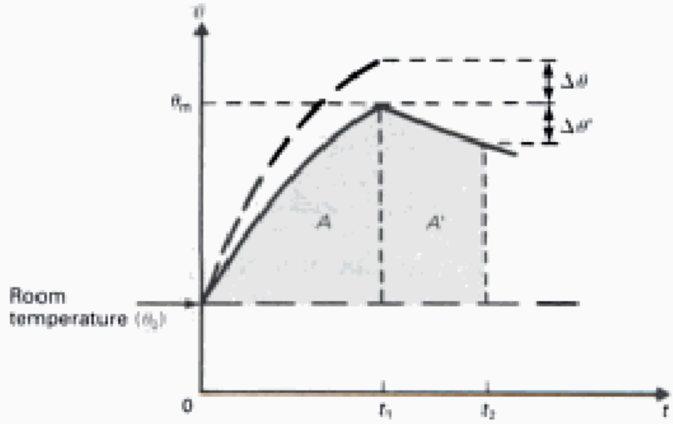


Fig. 13.6
Temperature against time with and without cooling

Theory of the Cooling Correction

The cooling correction is based on Newton's law of cooling (section 13.15). The reader should be familiar with this before proceeding.

If Newton's law of cooling applies, the rate of loss of heat to the surroundings, $\mathrm{d}Q/\mathrm{d}t$, both during heating and during cooling is given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = k(\theta - \theta_0) \qquad [13.9]$$

where k is a constant of proportionality. The total loss of heat, Q, in the interval between $t = 0$ and $t = t_1$ is given by

$$Q = \int_0^{t_1} \frac{\mathrm{d}Q}{\mathrm{d}t}\,\mathrm{d}t$$

Therefore by equation [13.9]

$$Q = k\int_0^{t_1} (\theta - \theta_0)\,\mathrm{d}t$$

i.e. $\quad Q = k \times \text{Area } A$

Similarly, the loss of heat, Q', between $t = t_1$ and $t = t_2$ is given by

$$Q' = k \times \text{Area } A'$$

Therefore

$$\frac{Q}{Q'} = \frac{\text{Area } A}{\text{Area } A'}$$

But $Q = C\Delta\theta$ and $Q' = C\Delta\theta'$ where C is the heat capacity of the body, and therefore

$$\frac{\Delta\theta}{\Delta\theta'} = \frac{A}{A'}$$

i.e. $$\Delta\theta = \frac{A}{A'}\Delta\theta'$$

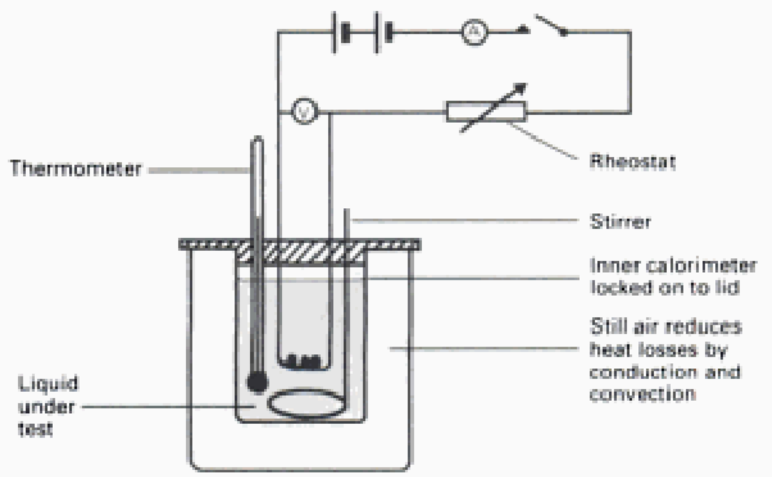
Note Calorimetry experiments are usually carried out in still air and it may seem surprising therefore that Newton's law of cooling (which applies to conditions of forced convection) is the basis of the cooling correction. It is used because it simplifies the theory, and is justified because the error it introduces is an error only in a correction term and is therefore of little significance overall.

13.10 ELECTRICAL METHODS OF MEASURING SPECIFIC HEAT CAPACITIES

Specific Heat Capacity of a Liquid

The apparatus is shown in Fig. 13.7. The rheostat should be adjusted to give a suitable current through the heating coil. The inner calorimeter contains a known mass of the liquid under test. The temperature θ_0 of the liquid is recorded. The switch is closed and the heater current and PD are recorded. The liquid is stirred continuously and its temperature is measued at one-minute intervals. Heating is continued until the temperature has risen by about 50 °C. The current and PD change slightly due to the increased resistance of the heating coil at higher temperatures, and their values should be recorded immediately before switching off the heater. The heater is switched off and the temperature is recorded until it has fallen to about 10 °C below its maximum value θ_m.

Fig. 13.7 Apparatus for determining the specific heat capacity of a liquid



If the specific heat capacity of the liquid and the heat capacity of the inner calorimeter are c and C respectively, and $\Delta\theta$ is the cooling correction found from equation [13.8], then

$$VIt = (mc + C)(\theta_m + \Delta\theta - \theta_0)$$

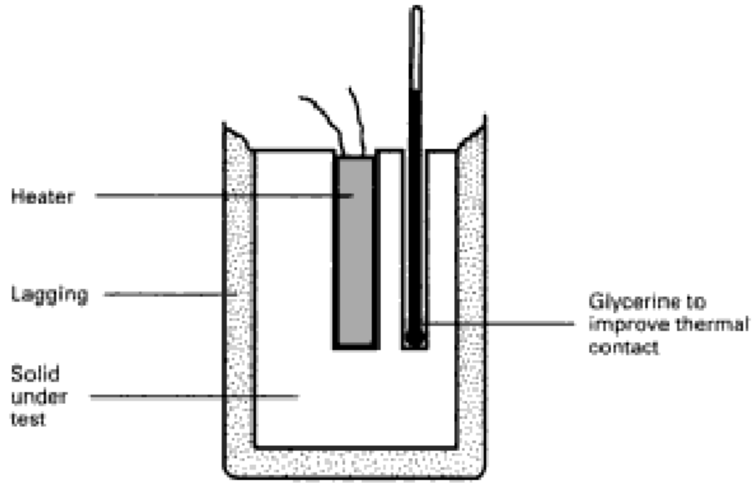
where V and I are the average heater PD and current and t is the time for which heating is carried out; hence c.

Specific Heat Capacity of a Solid

The apparatus is shown in Fig. 13.8. The material under test is in the form of a solid cylinder of mass m, into which two holes have been drilled to accommodate a heater and a thermometer. The procedure is basically the same as that for a liquid. The specific heat capacity c is calculated from

$$VIt = mc(\theta_m + \Delta\theta - \theta_0)$$

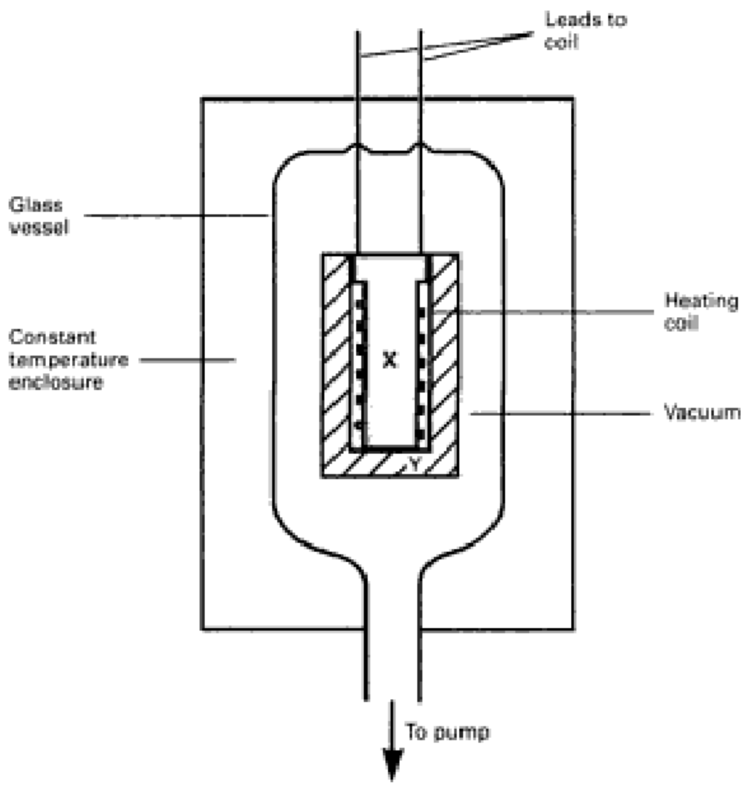
Fig. 13.8
Apparatus for determining the specific heat capacity of a solid



Nernst's Method for a Solid

The apparatus is shown in Fig. 13.9. A platinum heating coil is wound on paraffin-waxed paper around a cylindrical plug X of the metal under test. The paper insulates the coil from X so that its turns are not shorted out. The plug and coil are inserted into a cylindrical block Y of the same metal as X. A layer of paraffin wax around the coil insulates it from Y. The leads to the heating coil are used to

Fig. 13.9
Nernst's apparatus



suspend the metal inside a glass vessel which can be evacuated. The apparatus is surrounded by a constant-temperature enclosure, the temperature of which is the temperature at which the specific heat measurement is required. (The specific heat capacity of a substance depends on the temperature at which it is measured.) The apparatus is left until the metal acquires this temperature and then the glass vessel is evacuated. Since the metal is in a vacuum and is at the temperature of its surroundings, heat losses are almost entirely eliminated.

The electrical energy used in a measured time t to raise the temperature of the specimen by a small amount $\Delta\theta$ is determined by measuring the current I through the coil and the PD V across it. The temperature rise is found by using the coil as a platinum resistance thermometer. In order to do this the resistance of the coil is measured immediately before the heater current is switched on, and immediately after it has been switched off. Temperature rises of as little as 10^{-3} K can be used.

13.11 THE CONTINUOUS FLOW METHOD FOR THE SPECIFIC HEAT CAPACITY OF A LIQUID

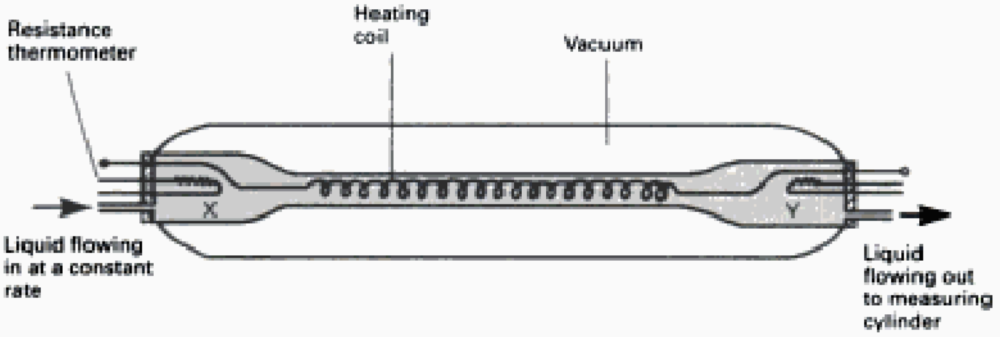
The method is due to Callendar and Barnes (1899).

Liquid is passed through the continuous flow calorimeter (Fig. 13.10) at a constant rate until all conditions are steady. At this stage the temperatures θ_X and θ_Y at X and Y, and the mass m_1 of liquid flowing through the calorimeter in time t are measured, together with the current I_1 through the heating coil and the PD V_1 across it. Under steady conditions none of the electrical energy which is being supplied is being used to heat the calorimeter, and therefore

$$V_1 I_1 t = m_1 c(\theta_Y - \theta_X) + Q \qquad [13.10]$$

where Q is the heat lost to the surroundings in time t.

Fig. 13.10
Callender and Barnes' continuous flow calorimeter



The rate of flow is altered so that the mass of liquid flowing in time t is m_2. The current and PD are adjusted (to I_2 and V_2) to bring the temperature at Y back to its original value θ_Y. The temperature at X is that of the tank supplying the liquid and is constant at θ_X. Since all temperatures are the same as they were with the initial flow rate, the heat lost in time t is again Q. Therefore

$$V_2 I_2 t = m_2 c(\theta_Y - \theta_X) + Q \qquad [13.11]$$

Subtracting equation [13.10] from equation [13.11] gives

$$(V_2 I_2 - V_1 I_1)t = (m_2 - m_1)c(\theta_Y - \theta_X)$$

Hence c.

Advantages

(i) The presence of the vacuum prevents heat losses by convection, and the effect of losses due to conduction and radiation is eliminated.

(ii) The temperatures which are measured are steady and therefore can be determined accurately by using platinum resistance thermometers. This allows small temperature rises to be used (typically 2 °C) and the method is therefore suitable for determining the manner in which the specific heat capacity changes with temperature.

(iii) The calculation does not involve the heat capacities of the various parts of the apparatus and so there is no need to know their values.

Disadvantage

A large quantity of liquid is required.

Further Points

(i) The percentage error is least when the difference between the two flow-rates is large.

(ii) Continuous flow methods can also be used for gases.

13.12 LATENT HEAT

It is necessary to supply energy (heat) to a solid in order to melt it, even if the solid is already at its melting point. This energy is called **latent heat**. It is distinct from any heat that might have been used to bring the solid up to its melting point in the first place, and from that which might be used to raise the temperature of the liquid once the solid has melted.

The energy is used to provide the increased molecular potential energy of the liquid phase and, when the phase change results in expansion, to do external work in pushing back the atmosphere. The energy used to do external work is usually much less than that used to increase the potential energy of the molecules, and in the case of ice, which contracts on melting, is negative.

The conversion of a liquid to a vapour (vaporization) and the direct conversion of a solid to a vapour (sublimation) also require latent heat to be supplied. These two processes usually involve large changes in volume, and the proportion of the latent heat which is used to do external work is greater than in melting.

In terms of the first law of thermodynamics (section 14.15) melting (i.e. fusion), vaporization and sublimation are represented by

$$L = \Delta U + \Delta W$$

where

L = the latent heat supplied in order to cause the phase change

ΔU = the increase in internal potential energy which accompanies the phase change. (There is no change in temperature and therefore no change in kinetic energy.)

ΔW = the external work done as a result of the phase change. This term is positive for expansion and negative for contraction.

The specific latent heat (l) of fusion (or vaporization or sublimation) of a substance is defined as the energy required to cause unit mass of the substance to change from solid to liquid (or liquid to vapour, or solid to vapour) without temperature change. (Unit = $J\,kg^{-1}$.)

Note The value of l depends on the temperature (and therefore the pressure) at which it is measured.

It follows that the heat, ΔQ, which must be added to change the phase of a mass, m, of substance is given by

$$\Delta Q = ml$$

where l is the specific latent heat of fusion, vaporization or sublimation according to the particular phase change which is taking place. For the reverse processes (liquid to solid, vapour to liquid, and vapour to solid) ΔQ represents the amount of heat that must be removed from the substance.

EXAMPLE 13.4

A calorimeter with a heat capacity of $80\,J\,^\circ C^{-1}$ contains 50 g of water at 40 °C. What mass of ice at 0 °C needs to be added in order to reduce the temperature to 10 °C? Assume no heat is lost to the surroundings. (Specific heat capacity of water $= 4.2 \times 10^3\,J\,kg^{-1}\,^\circ C^{-1}$, specific latent heat of ice $= 3.4 \times 10^5\,J\,kg^{-1}$.)

Solution

Heat lost by calorimeter cooling to 10 °C

$$= 80(40 - 10) = 2400\,J$$

Heat lost by water cooling to 10 °C

$$= 50 \times 10^{-3} \times 4.2 \times 10^3(40 - 10) = 6300\,J$$

$\therefore$ Total heat lost $= 2400 + 6300 = 8700\,J$

Let mass of ice $= m$

Heat used to melt ice at 0 °C

$$= m \times 3.4 \times 10^5 = 3.4 \times 10^5\,m$$

Heat used to increase temperature of melted ice to 10 °C

$$= m \times 4.2 \times 10^3(10 - 0) = 4.2 \times 10^4\,m$$

$\therefore$ Total heat used $= 3.4 \times 10^5\,m + 4.2 \times 10^4\,m = 3.82 \times 10^5\,m$

Since no heat is lost to the surroundings,

$$3.82 \times 10^5\,m = 8700$$

$\therefore$ $m = 0.0228\,kg$

i.e. Mass of ice required $= 23\,g$

QUESTIONS 13C

1. Calculate the heat required to melt 200 g of ice at 0 °C.
(Specific latent heat of ice = $3.4 \times 10^5\,\mathrm{J\,kg^{-1}}$.)

2. Calculate the heat required to turn 500 g of ice at 0 °C into water at 100 °C.
(Specific latent heat of ice = $3.4 \times 10^5\,\mathrm{J\,kg^{-1}}$, specific heat capacity of water = $4.2 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}}$.)

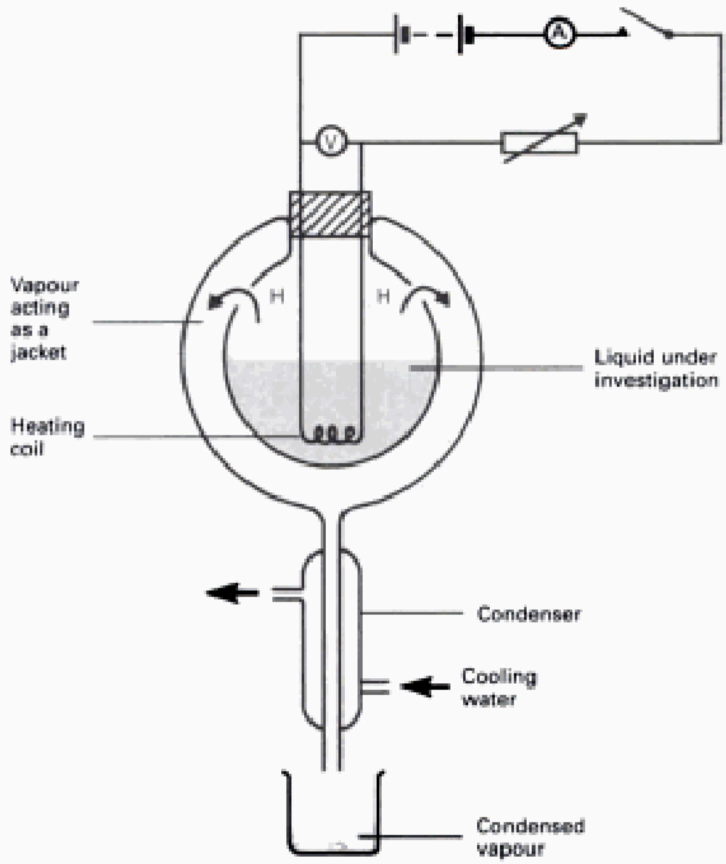
3. Calculate the heat given out when 600 g of steam at 100 °C condenses to water at 20 °C.
(Specific latent heat of steam = $2.26 \times 10^6\,\mathrm{J\,kg^{-1}}$, specific heat capacity of water = $4.2 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}}$.)

13.13 EXPERIMENTAL DETERMINATION OF THE SPECIFIC LATENT HEAT OF VAPORIZATION OF A LIQUID

The method about to be described is a continuous flow method and makes use of a self-jacketing vaporizer.

The apparatus is shown in Fig. 13.11. The liquid under investigation is heated to boiling point and the vapour which is produced passes to the condenser by way of holes (H) in the inner wall of the vessel. Boiling is continued, and eventually the temperatures of all parts of the apparatus become steady. At this stage the condensed vapour is collected, over a measured time t, and its mass m_1

Fig. 13.11
Apparatus for determining the specific latent heat of vaporization of a liquid



determined. If V_1 and I_1 are the heater PD and current, then the electrical energy supplied in time t is V_1I_1t. Since the temperatures are steady, this energy is used only to vaporize the liquid and to offset heat losses, and therefore

$$V_1I_1t = m_1l + Q \qquad [13.12]$$

where l is the specific latent heat of vaporization of the liquid and Q is the heat lost to the surroundings in time t.

The heater PD and current are now changed to V_2 and I_2 and the new mass m_2 of vapour which condenses in the same time t is measured.

Each part of the apparatus is at the same temperature as it was with the initial rate of heating and the energy lost in time t is again Q. Therefore.

$$V_2I_2t = m_2l + Q \qquad [13.13]$$

Subtracting equation [13.12] from equation [13.13] gives

$$(V_2I_2 - V_1I_1)t = (m_2 - m_1)l$$

from which l can be determined.

Note The liquid which is being vaporized is surrounded by its vapour (hence self-jacketing vaporizer). Any heat lost by the vapour causes it to condense, not to cool, and therefore the liquid is surrounded by a constant temperature enclosure which is at its own temperature; this considerably reduces heat losses from the liquid.

13.14 EXPERIMENTAL DETERMINATION OF THE SPECIFIC LATENT HEAT OF FUSION OF ICE BY THE METHOD OF MIXTURES

A calorimeter of mass m_c is about two-thirds filled with water of mass m_w which is about 5 °C above room temperature. The water and the calorimeter are left for a short time until they reach the same temperature as each other. This temperature (θ_1) is measured using a sensitive ($\frac{1}{10}$ °C) thermometer.

A lump of melting ice (i.e. ice at 0 °C) is then dried with blotting paper and immediately added to the water. The mixture is then stirred gently until the lump has melted. This procedure is repeated with further lumps until the temperature of the mixture is approximately as far below room temperature as θ_1 was above. The lowest temperature attained (θ_2) is recorded.

The calorimeter and its contents are weighed to determine the mass (m_i) of the ice.

$$\begin{bmatrix}\text{Heat lost by}\\ \text{water cooling}\\ \text{from } \theta_1 \text{ to } \theta_2\end{bmatrix} + \begin{bmatrix}\text{Heat lost by}\\ \text{calorimeter}\\ \text{cooling from}\\ \theta_1 \text{ to } \theta_2\end{bmatrix} = \begin{bmatrix}\text{Heat used}\\ \text{to melt ice}\\ \text{at } 0^\circ\text{C}\end{bmatrix} + \begin{bmatrix}\text{Heat used to}\\ \text{increase}\\ \text{temperature}\\ \text{of melted ice}\\ \text{from } 0^\circ\text{C to } \theta_2\end{bmatrix}$$

Therefore

$$m_wc_w(\theta_1 - \theta_2) + m_cc_c(\theta_1 - \theta_2) = m_il + m_ic_w(\theta_2 - 0)$$

from which the specific latent heat of fusion of ice (l) can be found, providing the specific heat capacities, c_w and c_c, of water and of the calorimeter material respectively, are known.

Notes (i) It is very important that the ice is dry when it is added to the water. If it is not, the mass of ice that is melted is less than m_i.

(ii) No cooling correction is applied because θ_1 is as far above room temperature as θ_2 is below it, and therefore, to a reasonable approximation, the mixture gains as much heat from the surroundings whilst it is below room temperature as it loses to them whilst it is above room temperature.

13.15 COOLING LAWS

Newton's Law of Cooling

This applies when a body is cooling under conditions of forced convection (i.e. when it is in a steady draught). It states that the rate of loss of heat of a body is proportional to the difference in temperature between the body and its surroundings, i.e.

$$\begin{pmatrix}\text{Rate of loss of}\\ \text{heat to surroundings}\end{pmatrix} \propto \begin{pmatrix}\text{Excess}\\ \text{temperature}\end{pmatrix}$$

or

$$\begin{pmatrix}\text{Rate of loss of}\\ \text{heat to surroundings}\end{pmatrix} = k(\theta - \theta_0)$$

where

θ = temperature of body

θ_0 = temperature of surroundings

k = a constant of proportionality whose value depends on both the nature and the area of the body's surface.

The law can be taken to be a good approximation for cooling under conditions of natural convection (a body cooling in still air for example) provided the excess temperature is not greater than about 30 °C. For higher excess temperatures than this the five-fourths power law should be used.

The Five-Fourths Power Law

This applies when a body is cooling under conditions of natural convection. It can be stated as

$$\begin{pmatrix}\text{Rate of loss of}\\ \text{heat to surroundings}\end{pmatrix} = k(\theta - \theta_0)^{5/4}$$

Experimental Investigation of Newton's Law of Cooling

The rate at which a body loses heat is proportional to the rate at which its temperature falls provided that its heat capacity does not vary with temperature (since $\Delta Q = C\Delta\theta$, where C = heat capacity). Therefore for a body cooling under conditions where Newton's law of cooling applies

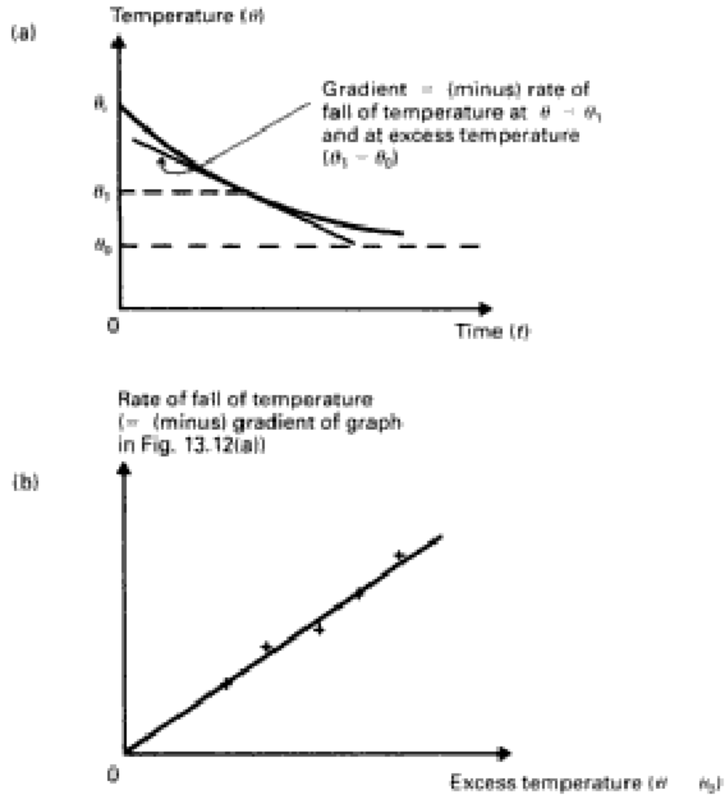
$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

where $d\theta/dt$ is (minus) the rate of fall of temperature of the body and K is a constant of proportionality whose value depends on the nature and the area of the body's surface, and on the heat capacity of the body. (Since the temperature is falling, the presence of the minus sign makes K a positive constant.) It follows that Newton's law of cooling can be verified by verifying this equation.

Method

A calorimeter containing hot water and standing on an insulating surface (e.g. a wooden block) is placed in the stream of air from an electric fan or an open window. The temperature of the water is measured at one-minute intervals using a $\frac{1}{10}$°C thermometer. The water should be stirred gently prior to each temperature measurement. A graph of temperature (θ) versus time (t) is plotted (Fig. 13.12(a)). The gradient of this graph at any temperature θ is the rate of fall of temperature at that value of θ. The gradients are measured (by constructing tangents to the curve) at various values of θ and are then plotted against the corresponding excess temperature ($\theta - \theta_0$) as in Fig. 13.12(b). If this plot is a straight line through the origin, Newton's law of cooling has been verified.

Fig. 13.12
Plot of (a) temperature against time during cooling, (b) rate of fall of temperature against excess temperature



If Newton's law of cooling applies, the graph of temperature against time is exponential. This is easily proved:

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$$\therefore \quad \int_{\theta_i}^{\theta} \frac{d\theta}{(\theta - \theta_0)} = -K \int_0^t dt$$

where θ_i is the initial temperature.

$$\therefore \qquad \left[\log_e(\theta - \theta_0)\right]_{\theta_i}^{\theta} = -K\left[t\right]_0^t$$

$$\therefore \qquad \log_e\left(\frac{\theta - \theta_0}{\theta_i - \theta_0}\right) = -Kt$$

$$\therefore \qquad (\theta - \theta_0) = (\theta_i - \theta_0)e^{-Kt}$$

$$\text{i.e.} \qquad \begin{pmatrix}\text{Excess}\\ \text{temperature}\end{pmatrix} = \begin{pmatrix}\text{Initial excess}\\ \text{temperature}\end{pmatrix} e^{-Kt}$$

CONSOLIDATION

The temperature of a body is a measure of how hot it is, not how much heat it contains.

Celsius temperature, θ, based on some thermometric property, X, (e.g. the length of a column of mercury) is given by

$$\theta = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100$$

Thermometers calibrated on the basis of this equation necessarily agree with each other only at the fixed points. They may agree at other temperatures too.

Kelvin temperature, T, on the thermodynamic scale and on the ideal gas scale is given by

$$T = \frac{p_T}{p_{Tr}} \times 273.16$$

where p_T and p_{Tr} are the pressures of a fixed volume of an ideal gas at temperature T and the triple point of water respectively.

The thermodynamic scale and the ideal gas scale are absolute scales, i.e. they do not depend on the properties of any particular substance.

An interval of one kelvin is defined as 1/273.16 of the temperature of the triple point of water.

Celsius temperature, θ, is defined by

$$\theta = T - 273.15$$

where T is the corresponding temperature in kelvins.

Heat capacity (C) is a property of a body. It is the heat required to produce unit temperature rise in the body. (Unit $=$ J $^\circ$C^{-1} or J K^{-1}.)

Specific heat capacity (c) is a property of a substance. It is the heat required to produce unit temperature rise in unit mass of the substance. (Unit $=$ J kg^{-1} $^\circ$C^{-1} or J kg^{-1} K^{-1}.)

For a change of temperature $\Delta Q = C\Delta\theta \qquad \Delta Q = mc\Delta\theta$

For a change of phase $\Delta Q = ml$

14

GASES

14.1 THE GAS LAWS

The experimental relationships between the pressures, volumes and temperatures of gases were investigated by various workers in the seventeenth and eighteenth centuries. These early experiments resulted in three laws – the so-called **gas laws**.

Boyle's Law

For a fixed mass of gas at constant temperature, the product of pressure and volume is constant.

On the basis of the other two laws the Kelvin scale of temperature was introduced, and these laws are stated below in terms of that scale.

Charles' Law

For a fixed mass of gas at constant pressure, the volume is directly proportional to the temperature measured in kelvins.

The Pressure Law

For a fixed mass of gas at constant volume, the pressure is directly proportional to the temperature measured in kelvins.

Representing pressure, volume, and temperature in kelvins by p, V and T respectively, we can formulate the three laws as:

At constant T	pV =	a constant	or	$p \propto 1/V$
At constant p	V/T =	a constant	or	$V \propto T$
At constant v	p/T =	a constant	or	$p \propto T$

It should be noted that the three laws are not independent; any one of them can be derived from the other two. The experimental investigation of the gas laws is dealt with in sections 14.9 to 14.11.

14.2 CONCEPT OF AN IDEAL GAS AND THE IDEAL GAS EQUATION

No gas obeys the gas laws exactly. Nevertheless they provide a fairly accurate description of the way gases behave when they are at low pressures and are at temperatures which are well above those at which they liquefy. A useful concept is that of an **ideal (or perfect) gas** – a gas which obeys the gas laws exactly. The behaviour of such a gas can be accounted for by

$$pV = nRT \tag{14.1}$$

where

p = the pressure of the gas ($\mathrm{N\,m^{-2}}$ = pascals, Pa)

V = the volume of the gas ($\mathrm{m^3}$)

n = the number of moles (see section 14.3) of gas (mol)

R = the universal molar gas constant (= $8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$)

T = the temperature of the gas **in kelvins**.

Equation [14.1] is known as the equation of state of an ideal gas (or simply as the **ideal gas equation**); it embodies the three gas laws and Avogadro's law (section 14.6). It can be shown that a gas which obeys this equation exactly must be subject to the assumptions inherent in the kinetic theory of gases (section 14.4). In particular, there would be no forces between the molecules of such a gas and therefore the **internal energy** (i.e. the energy of the molecules) of such a gas would be entirely kinetic and would depend only on its temperature.

Summary

(i) An ideal gas obeys the gas laws and $pV = nRT$ exactly. No such gas exists.

(ii) The internal energy of an ideal gas is entirely kinetic and depends only on its temperature.

(iii) The behaviour of real gases and unsaturated vapours (see Chapter 15) can be described by $pV = nRT$ if they are at low pressures and are at temperatures which are well above those at which they liquefy.

Note For a gas at pressure p_1, volume V_1 and temperature T_1 equation [14.1] gives

$$p_1V_1 = nRT_1 \qquad \text{i.e.} \qquad p_1V_1/T_1 = nR$$

If the same sample of gas is at pressure p_2, volume V_2 and temperature T_2

$$p_2V_2 = nRT_2 \qquad \text{i.e.} \qquad p_2V_2/T_2 = nR$$

(The number of moles is the same in each case (n) because we are dealing with the same sample of gas, i.e. with a fixed mass of gas and therefore with a fixed number of moles.) Combining these equations gives

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \qquad \text{for a fixed mass of gas}$$

Any unit of pressure can be used for p_1 providing the same unit is used for p_2. Similarly, any unit of volume can be used for V_1 as long as the same unit is used for V_2, but **both T_1 and T_2 must be expressed in kelvins**.

EXAMPLE 14.1

A gas (which can be considered ideal) has a volume of 100 cm^3 at 2.00×10^5 Pa and 27 °C. What is its volume at 5.00×10^5 Pa and 60 °C?

Solution

$$p_1 = 2.00 \times 10^5\ \text{Pa} \qquad p_2 = 5.00 \times 10^5\ \text{Pa}$$

$$V_1 = 100\ \text{cm}^3 \qquad V_2 = V_2$$

$$T_1 = 27\ ^\circ\text{C} = 300\ \text{K} \qquad T_2 = 60\ ^\circ\text{C} = 333\ \text{K}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore \quad \frac{2.00 \times 10^5 \times 100}{300} = \frac{5.00 \times 10^5 \times V_2}{333}$$

$$\therefore \quad V_2 = \frac{2.00 \times 10^5 \times 100 \times 333}{300 \times 5.00 \times 10^5} = 44.4\ \text{cm}^3$$

Note that V_1 was expressed in cm^3 and therefore V_2 is in cm^3.

EXAMPLE 14.2

Refer to Fig 14.1. Initially A contains 3.00 m^3 of an ideal gas at a temperature of 250 K and a pressure of 5.00×10^4 Pa, whilst B contains 7.20 m^3 of the same gas at 400 K and 2.00×10^4 Pa. Find the pressure after the connecting tap has been opened and the system has reached equilibrium, assuming that A is kept at 250 K and B is kept at 400 K.

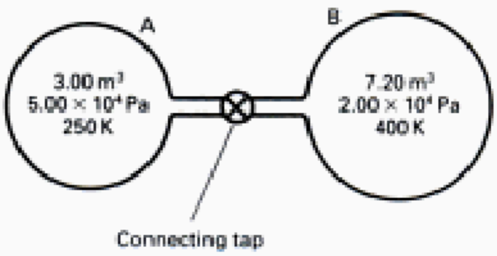


Fig. 14.1 Diagram for Example 14.2

Solution

On opening the tap some gas moves from A to B, reducing the pressure in A and increasing it in B. This continues until, at equilibrium, the pressure in A is equal to that in B. Let this final pressure be p. The 'trick' is to recognize that the total mass of gas, and therefore the total number of moles, is the same after the tap is opened as it was before.

Therefore, for 1 mole of gas at STP

$$V = \frac{(1)(8.31)(273)}{1.013 \times 10^5} = 22.4 \times 10^{-3}\,\text{m}^3$$

QUESTIONS 14A

1. What is the temperature of $19.0\,\text{m}^3$ of an ideal gas at a pressure of 600 mmHg if the same gas occupies $12.0\,\text{m}^3$ at 760 mmHg and 27 °C?

2. A gas has a volume of $60.0\,\text{cm}^3$ at 20 °C and 900 mmHg. What would its volume be at STP, i.e. at 273 K and 760 mmHg?

3. A cylinder contains $2.40 \times 10^{-3}\,\text{m}^3$ of hydrogen at 17 °C and 2.32×10^6 Pa. The relative molecular mass of hydrogen = 2, $R = 8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$ and the Avogadro constant, $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$. Calculate:
(a) the number of moles of hydrogen in the cylinder,
(b) the number of molecules of hydrogen in the cylinder,
(c) the mass of the hydrogen,
(d) the density of hydrogen under these conditions.

14.4 THE KINETIC THEORY OF GASES (DERIVATION OF $p = \frac{1}{3}\rho\overline{c^2}$)

This is an attempt to explain the experimentally observed properties of gases by considering the motion of the molecules (or atoms) of which they are composed. A number of assumptions are made.

(i) The molecules of a particular gas are identical.

(ii) Collisions between the molecules and with the container are (perfectly) elastic (see section 2.8).

(iii) The molecules exert no forces on each other except during impacts (which are assumed to have negligible duration anyway) and the effect of gravity is ignored so that:

(a) between collisions the molecules move in straight lines at constant speed, and

(b) the motion is random.

(iv) There is a sufficiently large number of molecules for statistics to be meaningfully applied.

(v) The size of the molecules is negligible compared to their separation.

(vi) The laws of Newtonian mechanics apply.

Some of these assumptions run through the entire analysis; others are used more specifically.

Consider a gas enclosed in a cubical container of side L (Fig. 14.2). Let each molecule of the gas have mass m (assumption (i)). Consider, initially, a single molecule which is moving towards wall X, and suppose that its x-component of velocity is u_1. This molecule will have an x-component of momentum mu_1 towards the wall. The molecule will eventually reverse the direction of its momentum by

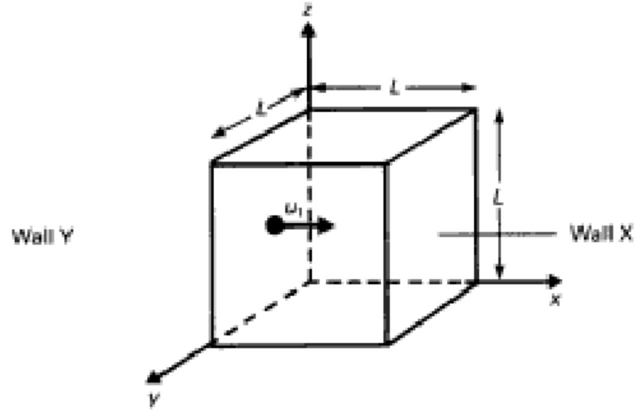


Fig. 14.2
Derivation of $p = \frac{1}{3}\rho\overline{c^2}$

colliding with the wall. Since the collision will be elastic (assumption (ii)), it will rebound with the same speed so that its momentum will now be $-mu_1$. The change in the x-component of momentum is therefore $2mu_1$.

The molecule has to travel a distance $2L$ (from X to Y and back to X) before it next collides with wall X. The time for such a trip is $2L/u_1$, and therefore this molecule's rate of change of momentum due to collision with X will be

$$\frac{2mu_1}{2L/u_1} = \frac{mu_1{}^2}{L}$$

By Newton's second law, rate of change of momentum is equal to force, and therefore $mu_1{}^2/L$ is the force exerted on the molecule by the wall. By Newton's third law, the molecule exerts an equal but oppositely directed force on the wall, and therefore

$$\text{Force on X} = mu_1{}^2/L$$

Therefore

$$\text{Force per unit area on X} = \frac{mu_1{}^2/L}{L^2} \qquad (\text{since area of X} = L^2)$$

Therefore

$$\text{Pressure on X} = \frac{mu_1{}^2}{L^3}$$

If there are N molecules in the container and their x-components of velocity are u_1, $u_2, \ldots, u_N$, the total pressure, p, on wall X will be given by

$$p = \frac{m}{L^3}(u_1{}^2 + u_2{}^2 + \cdots + u_N{}^2)$$

Therefore

$$p = \frac{m}{L^3}N\overline{u^2} \qquad [14.2]$$

where

$\overline{u^2}$ is the mean square velocity in the x-direction.

Since mN is the total mass of gas in the container, mN/L^3 is the density, ρ, of the gas and therefore, by equation [14.2],

$$p = \rho\overline{u^2} \qquad [14.3]$$

If

c = the resultant velocity of a molecule whose x-, y- and z-components of velocity are u, v and w respectively, then

$$c^2 = u^2 + v^2 + w^2$$

Therefore

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2} \qquad [14.4]$$

where

$\overline{c^2}$ is the mean square velocity of the molecules

$\overline{v^2}$ is the mean square velocity in the y-direction

$\overline{w^2}$ is the mean square velocity in the z-direction.

Since there is a large number of molecules and they are moving randomly (assumptions (iv) and (iii)b)

$$\overline{u^2} = \overline{v^2} = \overline{w^2}$$

Therefore, from equation [14.4],

$$\overline{u^2} = \tfrac{1}{3}\overline{c^2}$$

Therefore, from equation [14.3]

$$p = \tfrac{1}{3}\rho\overline{c^2} \qquad [14.5]$$

14.5 RELATIONSHIP BETWEEN MOLECULAR KINETIC ENERGY AND TEMPERATURE

On the basis of the kinetic theory of gases,

$$p = \tfrac{1}{3}\rho\overline{c^2}$$

Therefore, for any volume V of gas,

$$pV = \tfrac{1}{3}\rho V\overline{c^2}$$

Therefore,

$$pV = \tfrac{1}{3}M\overline{c^2} \qquad [14.6]$$

where

M = the mass of volume V of the gas.

Equation [14.6] may be rewritten as

$$pV = \tfrac{2}{3}N(\tfrac{1}{2}m\overline{c^2}) \qquad [14.7]$$

where

N = the total number of molecules in volume V, and

m = the mass of one molecule.

The ideal gas equation for n moles of a gas of volume V and pressure p is

$$pV = nRT \qquad [14.8]$$

where

R = the universal molar gas constant, and

T = the temperature in kelvins.

Thus the predictions of the kinetic theory of gases (represented by equation [14.7]) are in agreement with idealized experimental observation (represented by equation [14.8]) if

$$\tfrac{2}{3}N(\tfrac{1}{2}m\overline{c^2}) = nRT$$

i.e. $$\tfrac{1}{2}m\overline{c^2} = \frac{3}{2}\frac{nR}{N}T$$

i.e. $$\tfrac{1}{2}m\overline{c^2} = \frac{3}{2}\frac{R}{N_A}T$$

(since N/n is the number of molecules per mole, i.e. N_A, the Avogadro constant). Both R and N_A are universal constants, and therefore so also is R/N_A; it is called **Boltzmann's constant,** k ($= 1.38 \times 10^{-23}\,\text{J K}^{-1}$) and is the gas constant per molecule. The left-hand term is the average translational* kinetic energy of a single molecule, and therefore

$$\text{Average translational KE of a molecule} = \tfrac{3}{2}kT = \frac{3}{2}\frac{R}{N_A}T \qquad [14.9]$$

Thus, in order to make the kinetic theory consistent with the ideal gas equation we need to accept the validity of equation [14.9], i.e. in addition to assumptions (i) to (vi) of section 14.4, we need to make the further assumption that **the average translational kinetic energy of a molecule is equal to (3/2) kT**. Such an assumption is reasonable, since putting heat energy into a gas increases its temperature and must also increase the kinetic energy of its molecules becuase there is no other way that the energy can be absorbed. (An ideal gas can have no potential energy because it has no intermolecular forces, and there is nothing other than molecules present.)

Note The three gas laws (section 14.1) can be combined as $pV \propto T$. In order to make the kinetic theory consistent with this expression, rather than with the more demanding $pV = nRT$, we need only make the assumption that $\frac{1}{2}m\overline{c^2}$ is proportional to T, for it then follows from equation [14.7] that $pV \propto T$.

QUESTIONS 14B

1. By what factor does **(a)** the mean square speed, **(b)** the root mean square speed of the molecules of a gas increase when its temperature is doubled?

2. The temperature of a gas is increased in such a way that its volume doubles and its pressure quadruples. If the root mean square speed of the molecules was originally $250\,\text{m s}^{-1}$, what is it at the higher temperature?

3. Find the value of the ratio

$$\frac{\text{Root mean square speed of hydrogen molecules}}{\text{Root mean square speed of oxygen molecules}}$$

(a) when the two gases are at the same temperature, **(b)** when the oxygen is at 100 °C and the hydrogen is at 30 °C.
(Relative molecular mass of hydrogen = 2, relative molecular mass of oxygen = 32.)

*In section 14.16 diatomic and polyatomic molecules are considered. These have both translational and rotational kinetic energies.

14.6 AVOGADRO'S LAW

Equal volumes of all gases at the same temperature and pressure contain the same number of molecules.

The law was announced by Avogadro in 1811 and was well-established before the kinetic theory was developed. It is embodied in the ideal gas equation, $pV = nRT$. In order to illustrate this we shall consider two gases distinguished by the subscripts $_1$ and $_2$. Applying the ideal gas equation gives

$$p_1V_1 = n_1RT_1 \quad \text{and} \quad p_2V_2 = n_2RT_2$$

For equal volumes at the same pressure $p_1V_1 = p_2V_2$, and therefore

$$n_1RT_1 = n_2RT_2 \qquad [14.10]$$

If the gases are at the same temperature, $T_1 = T_2$, and therefore from equation [14.10]

$$n_1 = n_2$$

Thus equal volumes of two gases which are at the same temperature and pressure contain the same number of moles. It follows from the definition of the mole (section 14.3) that the gases also contain the same number of molecules. Thus, under the conditions to which Avogadro's law relates, the number of molecules in each gas is the same, i.e. Avogadro's law is embodied in $pV = nRT$.

Note Avogadro's law can be applied to real gases which are at low pressures and are at temperatures well above those at which they liquefy.

14.7 DALTON'S LAW OF PARTIAL PRESSURES

The total pressure of a mixture of gases, which do not interact chemically, is equal to the sum of the partial pressures, i.e. to the sum of the pressures that each gas would exert if it alone occupied the volume containing the mixture.

Suppose that a volume V contains n_1 moles of a gas whose partial pressure is p_1 and n_2 moles of a gas whose partial pressure is p_2. If the temperature of the gases is T, then by equation [14.1]

$$p_1V = n_1RT \quad \text{and} \quad p_2V = n_2RT$$

Dividing gives

$$\frac{p_1}{p_2} = \frac{n_1}{n_2} \qquad [14.11]$$

From Dalton's law, the total pressure p is given by

$$p = p_1 + p_2$$

Substituting for p_2 from equation [14.11] gives

$$p_1 = \left(\frac{n_1}{n_1 + n_2}\right)p$$

and substituting for p_1 gives

$$p_2 = \left(\frac{n_2}{n_1 + n_2}\right)p$$

Note Dalton's law still applies if one or more of the components of the mixture is a vapour (saturated or unsaturated) – see Example 15.1.

14.8 THE MAXWELLIAN DISTRIBUTION OF MOLECULAR SPEEDS

It can be shown, on the basis of statistical mechanics, that the speeds of gas molecules are distributed as illustrated in Fig. 14.3. The curve, which is known as the Maxwellian distribution of molecular speeds, agrees well with that obtained by experiment. The quantity $N(c)$ is such that $N(c)\delta c$ is the number of molecules whose speeds are in the narrow range c to $c + \delta c$. Theoretically

$$c_0 : \bar{c} : \sqrt{\overline{c^2}} = 1 : 1.13 : 1.23$$

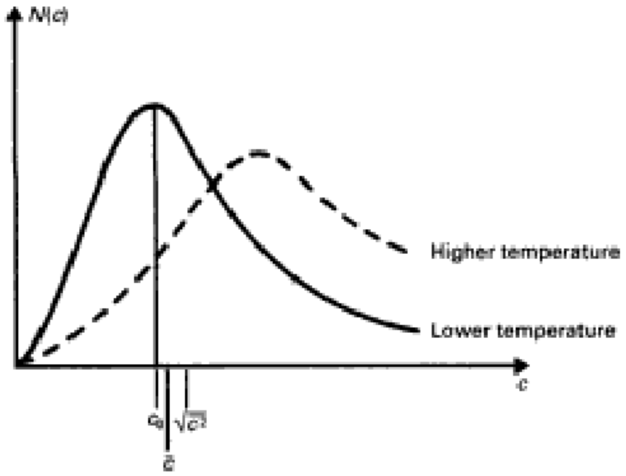
where

c_0 = most probable speed

$\bar{c}$ = mean speed

$\sqrt{\overline{c^2}}$ = root mean square speed.

Fig. 14.3 Maxwellian distribution of molecular speeds

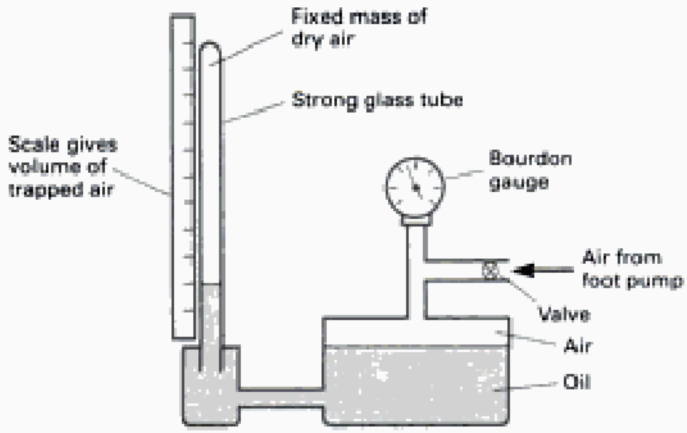


14.9 EXPERIMENTAL INVESTIGATION OF BOYLE'S LAW

The apparatus is shown in Fig. 14.4. The gas under investigation is the air trapped above the oil in the glass tube. The volume, V, of the air is read directly from the scale. It is compressed by using a foot pump to increase the pressure above the oil in the reservoir. The pressure, p, of the trapped air is the same as that of the air in the oil reservoir. (Whether or not this can be read directly from the Bourdon gauge depends on the particular type of gauge being used – see section 10.7.) The pressure is increased in stages, allowing a number of pairs of values of p and V to be taken. Compressing the air warms it slightly – it should be allowed to cool to room temperature (indicated by a steady volume reading) before each measurement is made.

A graph of V against $1/p$ is plotted. If the graph is a straight line through the origin, Boyle's law has been verified for the particular temperature and range of pressures investigated.

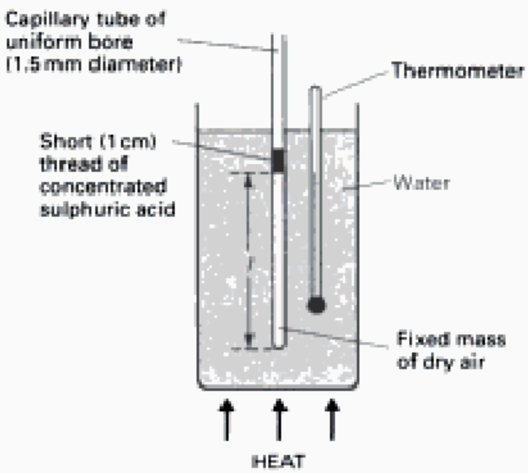
Fig. 14.4
Boyle's law apparatus



14.10 EXPERIMENTAL INVESTIGATION OF CHARLES' LAW

The apparatus is shown in Fig. 14.5. A column of air is trapped inside a capillary tube by a short thread of concentrated sulphuric acid. The reason for using the acid (rather than mercury, say) is that it absorbs any water that might be in the air and so allows meaningful results to be obtained. The tube has a uniform bore and therefore the volume of the trapped air is proportional to the length of the air column. The water is heated and the length, l, of the air column is measured for a number of different temperatures, θ. The water should be heated slowly, and stirred before each reading, to allow the air to reach the temperature of the water. The pressure of the air throughout the experiment is constant (equal to atmospheric pressure plus the pressure exerted by the acid thread).

Fig. 14.5
Charles' law apparatus



A graph of l against Celsius temperature, θ, is plotted. The graph will be a straight line (Fig. 14.6(a)) showing that (for the particular pressure and range of temperatures investigated) the volume of a fixed mass of dry air at a constant pressure increases uniformly with temperature. This is one form of Charles' law.

Alternatively, a graph of l against Kelvin temperature, T, where $T = \theta + 273$, could be plotted (Fig. 14.6(b)). This graph passes through the origin and therefore verifies that the volume of a fixed mass of gas (dry air) at constant pressure is directly proportional to the temperature measured in Kelvins. This is the form of Charles' law given in section 14.1.

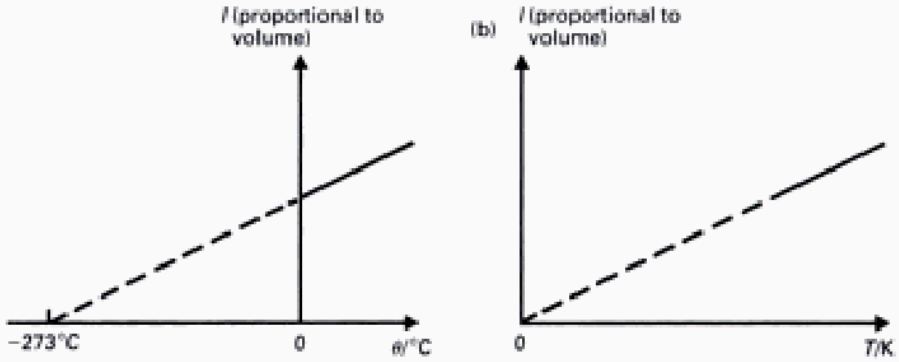


Fig. 14.6
Graphs for Charles' law investigation

Note Small values of l should be avoided; otherwise the rounded end of the capillary tube introduces a significant error into the assumption that the volume of the trapped air is proportional to l.

14.11 EXPERIMENTAL INVESTIGATION OF THE PRESSURE LAW

The apparatus is shown in Fig. 14.7.* It enables the pressure variation with temperature of a fixed mass of dry air at constant volume to be investigated.

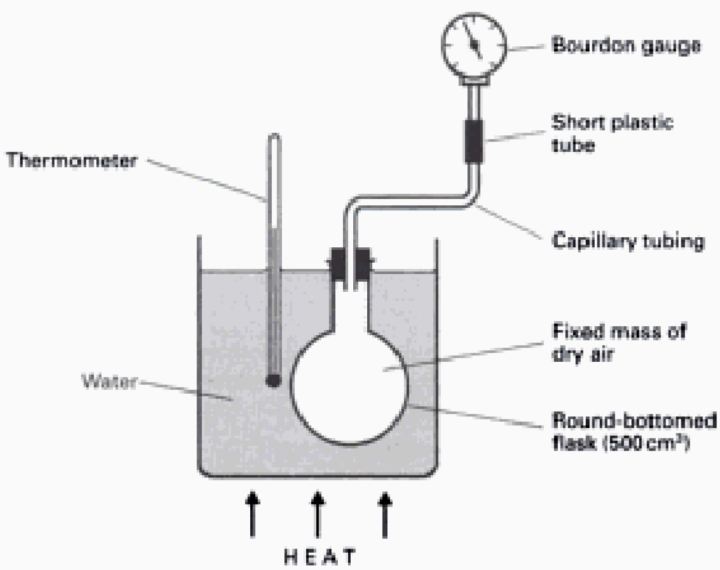


Fig. 14.7
Pressure law apparatus

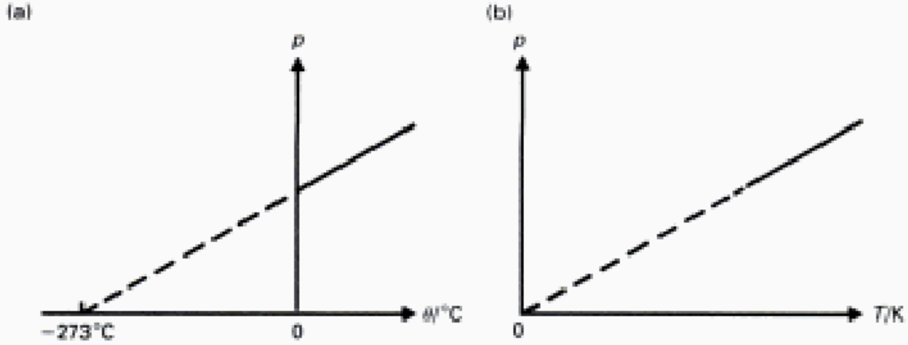
The water is heated, and the pressure, p, of the air in the flask is recorded for a number of different temperatures, θ. (Whether or not the pressure can be read directly from the Bourdon gauge depends on the particular type of gauge being used—see section 10.7.) The water should be heated slowly, and stirred before each reading, to allow the air in the flask to reach the temperature of the water.

The air in the Bourdon gauge and connecting tube is not at the same temperature as that in the flask. Using a large flask and capillary tubing reduces the significance of the error that this causes.

*An alternative form of apparatus is shown in Fig. 13.5.

A graph of p against Celsius temperature, θ, is plotted. The graph will be a straight line (Fig. 14.8(a)) showing that (for the particular volume and range of temperatures investigated) the pressure of a fixed mass of dry air at constant volume increases uniformly with temperature. This is one form of the pressure law.

Fig. 14.8
Graphs for pressure law investigation



Alternatively, a graph of p against Kelvin temperature, T, where $T = \theta + 273$, could be plotted (Fig. 14.8(b)). This graph passes through the origin and therefore verifies that the pressure of a fixed mass of gas (dry air) at constant volume is directly proportional to the temperature measured in Kelvins. This is the form of the pressure law given in section 14.1.

14.12 VOLUME AND PRESSURE COEFFICIENTS OF GASES

The expansivity of a gas at constant pressure (or **volume coefficient**) α_p is defined by

$$\alpha_p = \frac{V - V_0}{V_0\theta} \quad \text{i.e.} \quad V = V_0(1 + \alpha_p\theta)$$

where

V_0 = volume of gas at 0 °C

V = volume of gas at Celsius temperature θ.

It follows from equation [14.1] that for an ideal gas at a constant pressure p

$$V = nR(273 + \theta)/p \quad \text{and} \quad V_0 = nR(273)/p$$

$$\therefore \quad \alpha_p = \frac{[nR(273 + \theta)/p] - [nR(273)/p]}{[nR(273)/p]\theta}$$

i.e. $\alpha_p = 1/273\,\text{K}^{-1}$ (or °C^{-1})

The coefficient of pressure increase at constant volume (or pressure coefficient) α_v is defined by

$$\alpha_v = \frac{p - p_0}{p_0\theta} \quad \text{i.e.} \quad p = p_0(1 + \alpha_v\theta)$$

where

p_0 = pressure of gas at 0 °C

p = pressure of gas at Celsius temperature θ.

It can be shown (by the method used for α_p) that

$$\alpha_v = 1/273\,\text{K}^{-1} \quad (\text{or}\,{}^\circ\text{C}^{-1})$$

14.13 REVERSIBLE PROCESSES

If, at every stage, a process can be made to go in the reverse direction by an infinitesimal change in the conditions which are causing it to take place, it is said to be a reversible process.

It follows that when the state of a system is changed reversibly:

(i) the system is in **thermodynamic equilibrium** (i.e. all parts of the system are at the same temperature and pressure) at every instant, and

(ii) at the completion of the process the system could be returned to its initial state by passing through the intermediate states in reverse order, and without there being any net change in the rest of the Universe.

In practice, it is impossible to produce a perfect reversible change. However, processes which take place very slowly and which do not involve friction are often good approximations to reversible changes. The slow compression of a gas by the movement of a light, frictionless piston in a non-conducting cylinder is an example of an approximately reversible process, because a slight decrease in the force on the piston would allow the gas to expand and no energy will have been dissipated as heat. Other examples include the changes of pressure, volume and temperature which are associated with the passage of a sound wave through air, and the movement of a pendulum about a frictionless support in a vacuum.

14.14 EXTERNAL WORK DONE BY AN EXPANDING GAS

Consider a gas enclosed in a cylinder by a frictionless piston of cross-sectional area A (Fig. 14.9). Suppose that the piston is in equilibrium under the action of the force pA exerted by the gas and an external force F. Suppose now, that the gas

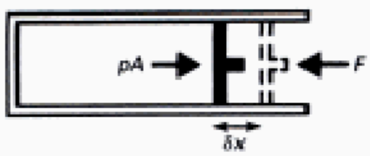


Fig. 14.9 Gas expanding in a cylinder

expands and moves the piston outwards through a distance δx, where δx is so small that p can be considered to be constant. The external work done δW by the expansion is given, by equation [5.1], as

$$\delta W = pA\,\delta x$$

i.e.

$$\delta W = p\delta V \qquad [14.12]$$

where δV is the small increase in volume of the gas. The total work done W by the gas if its volume changes by a finite amount from V_1 to V_2 is therefore given by

$$W = \int_{V_1}^{V_2} p\,dV \qquad [14.13]$$

Equation [14.13] holds no matter what the relationship between p and V. For the particular case of an **isobaric process** (i.e. one in which p is constant)

$$W = \int_{V_1}^{V_2} p\,dV = p\int_{V_1}^{V_2} dV$$

i.e. $$W = p(V_2 - V_1) \qquad [14.14]$$

In the general case, if a plot of p against V is available (known as an **indicator diagram**), the work done can be obtained graphically. Suppose that the pressure of a gas varies with volume as shown in Fig. 14.10. The work done W by the gas as its volume changes from V_1 to V_2 is given by

$$W = \int_{V_1}^{V_2} p\,dV = \text{Area of shaded region}$$

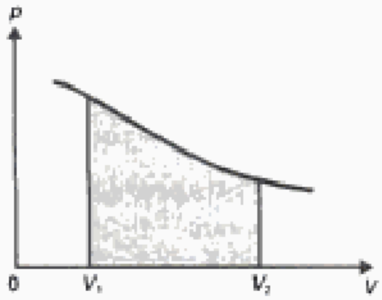


Fig. 14.10 Indicator diagram for a gas

Notes

(i) Equation [14.13] also applies when a gas is compressed, in which case work is being done on the gas.

(ii) Strictly, equation [14.13] can be applied only if the change takes place reversibly – if it does not, the values of pressure and temperature at any instant will be different in different regions of the gas.

(iii) Equation [14.13] also applies to solids and liquids. In these cases, though, the increases in volume are small and therefore the amounts of external work done are small compared with increases in internal energy.

14.15 THE FIRST LAW OF THERMODYNAMICS

Thermodynamics is the study of the relationship between heat and other forms of energy. When the principle of conservation of energy is stated with reference to heat and work it is known as the **first law of thermodynamics**.

The heat energy (ΔQ) supplied to a system is equal to the increase in the internal energy (ΔU) of the system plus the work done (ΔW) by the system on its surroundings.

i.e. $$\Delta Q = \Delta U + \Delta W \qquad [14.15]$$

The internal energy of a system is the sum of the kinetic and potential energies of the molecules of the system. It follows from equation [14.15] that it may be increased by:

(i) putting heat energy into the system, and/or

(ii) doing work on the system.

When the internal energy of a system changes the change depends only on the initial and final states of the system, and not on how the change was brought about. This is equivalent to saying that the internal energy of a system depends only on the state that it is in, and not on how it reached that state. (Note, a system is said to have changed 'state' if some observable property of the system, e.g. its temperature, pressure, or phase, has changed.)

It is not possible to determine the absolute value of the internal energy of a real system*. This is no real problem, though, because we are always concerned with changes in internal energy, and these can be determined. To do so we make use of equation [14.15] – either directly or indirectly. Either we find the values of ΔQ and ΔW and use them (directly) in equation [14.15] to calculate ΔU, or we use the measured values of the quantities in an equation which is based on equation [14.15] – equation [14.16] for example.

An **isolated system** is one which is cut off from any form of external influence. In particular, no work can be done on it or by it (i.e. $\Delta W = 0$), and no heat can enter it or leave it (i.e. $\Delta Q = 0$). It follows from equation [14.15] that $\Delta U = 0$, and therefore that **the internal energy of an isolated system is constant**.

When a system undergoes an **adiabatic process** (see section 14.21) $\Delta Q = 0$, and equation [14.15] reduces to $\Delta U = -\Delta W$. Bearing in mind that ΔW represents work done by the system, $(-\Delta W)$ represents work done on the system. Thus, **when a system undergoes an adiabatic process the increase in internal energy of the system is equal to the work done on it**.

Notes

(i) We stated immediately after equation [14.15] that the internal energy of a system is the sum of the kinetic and potential energies of the molecules of the system. This should not be taken to imply that we are defining internal energy in this way. Absolute values of internal energy are not defined at all for real systems; changes in internal energy are defined by equation [14.15].

(ii) The internal energy of an ideal gas is due entirely to the kinetic energy of the molecules. It therefore follows from equation [14.9] that the internal energy, U, of one mole of an ideal monatomic gas at kelvin temperature T is given by

$$U = \tfrac{3}{2}RT$$

The increase in internal energy, ΔU, due to an increase in temperature, ΔT, is given by

$$\Delta U = \tfrac{3}{2}R\Delta T$$

*It is possible for an ideal gas – see Note (ii).

14.16 THE PRINCIPAL MOLAR HEAT CAPACITIES OF A GAS

The molar heat capacity of a substance is the heat required to produce unit temperature rise in one mole of the substance.

A change in temperature involves a change in pressure and/or volume. With solids and liquids such changes are small and are normally ignored. Large changes occur with gases, and in order to define the heat capacity of a gas it is necessary to specify the particular conditions of pressure and volume. Two cases are of special interest: (i) when the pressure is constant, (ii) when the volume is constant. The heat capacities measured under these conditions are called the principal heat capacities.

The molar heat capacity of a gas at constant pressure (C_p) is the heat required to produce unit temperature rise in one mole of the gas when the pressure remains constant.

The molar heat capacity of a gas at constant volume (C_v) is the heat required to produce unit temperature rise in one mole of the gas when the volume remains constant.

When a gas is heated at constant pressure it expands, and therefore some of the heat which is supplied to the gas is used:

(i) to do external work, and (in the case of a real gas)

(ii) to increase the potential energy of its molecules.

When a gas is heated at constant volume, on the other hand, all of the heat which is supplied to it is used to increase the temperature. It follows that the amount of heat required to raise the temperature of a gas at constant pressure is greater than that required to raise its temperature by the same amount at constant volume. In particular, C_p is greater than C_v.

Note The principal heat capacities for unit mass of gas are called the principal specific heat capacities at constant pressure and constant volume and are denoted by c_p and c_v respectively.

14.17 TO SHOW THAT $C_p - C_v = R$ FOR AN IDEAL GAS

Suppose that one mole of an ideal gas is heated so that its temperature increases by ΔT at constant volume. It follows from the definition of C_v that the heat supplied ΔQ is given by

$$\Delta Q = C_v \Delta T$$

Since there is no change in volume, the external work done ΔW is zero. From the first law of thermodynamics (equation [14.15])

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad C_v \Delta T = \Delta U \qquad [14.16]$$

where ΔU is the increase in internal energy of the gas. It is important to note that the internal energy of an ideal gas depends only on its temperature, and therefore equation [14.16] holds whenever the temperature of one mole of an ideal gas increases by ΔT, it is not restricted to situations in which the temperature increase occurs at constant volume.

Suppose now that one mole of the same gas is heated so that its temperature increases by the same amount ΔT at constant pressure. It follows from the definition of C_p that the heat supplied ΔQ is given by

$$\Delta Q = C_p \Delta T$$

The external work done ΔW by the gas is given (by equation [14.14]) as

$$\Delta W = p\Delta V$$

where ΔV is the (non-zero) change in volume and p is the constant pressure. From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad C_p \Delta T = \Delta U + p\Delta V$$

Substituting for ΔU from equation [14.16] gives

$$C_p \Delta T = C_v \Delta T + p\Delta V \qquad [14.17]$$

If the initial volume and temperature of the gas are V and T respectively, then since we are concerned with one mole of an ideal gas

$$pV = RT$$

and

$$p(V + \Delta V) = R(T + \Delta T)$$

Subtracting gives

$$p\Delta V = R\Delta T$$

Substituting for $p\Delta V$ in equation [14.17] gives

$$C_p \Delta T = C_v \Delta T + R\Delta T$$

i.e.

$$C_p - C_v = R \qquad [14.18]$$

14.18 CALCULATION OF C_p/C_v FOR AN IDEAL MONATOMIC GAS

The internal energy of an ideal gas is entirely kinetic. The moment of inertia of a monatomic molecule can be considered to be zero and therefore the kinetic energy of such a molecule is associated with its translational motion only*. It follows that the average total kinetic energy of a monatomic molecule is given by equation [14.9], and that the internal energy U of one mole of such a gas is given by

$$U = \tfrac{3}{2} N_A kT$$

i.e.

$$U = \tfrac{3}{2} RT$$

If the temperature changes by ΔT, the corresponding change in internal energy ΔU is given by

$$\Delta U = \tfrac{3}{2} R\Delta T$$

*If the moment of inertia were not zero, the molecule would have additional kinetic energy due to its rotational motion.

By equation [14.16]

$$\Delta U = C_v \Delta T$$

$$\therefore \quad C_v \Delta T = \tfrac{3}{2} R \Delta T$$

i.e. $$C_v = \tfrac{3}{2} R$$

By equation [14.18]

$$C_p - C_v = R$$

$$\therefore \quad \frac{C_p - C_v}{C_v} = \frac{R}{\tfrac{3}{2} R}$$

i.e. $$\frac{C_p}{C_v} - 1 = \tfrac{2}{3}$$

i.e. $$\frac{C_p}{C_v} = \tfrac{5}{3} = 1.67$$

14.19 C_p/C_v FOR DIATOMIC AND POLYATOMIC GASES

Molecules which contain more than one atom have non-negligible moments of inertia and therefore possess rotational kinetic energy in addition to translational kinetic energy. When this is taken into account it can be shown that

$$\frac{C_p}{C_v} = \tfrac{7}{5} = 1.40 \quad \text{for a diatomic gas, and}$$

$$\frac{C_p}{C_v} = \tfrac{4}{3} = 1.33 \quad \text{for a polyatomic gas.}$$

The ratio C_p/C_v is denoted by γ, and therefore:

$\gamma = 1.67$ for a monatomic gas
$\gamma = 1.40$ for a diatomic gas
$\gamma = 1.33$ for a polyatomic gas

Note $C_p - C_v = R$ holds no matter what the atomicity of the gas.

14.20 ISOTHERMAL PROCESSES

An isothermal process is a process which takes place at constant temperature.

It follows from the ideal gas equation that when a gas expands or contracts isothermally

$$pV = \text{a constant} \qquad [14.19]$$

The internal energy of an ideal gas depends only on its temperature and therefore, for an ideal gas which is involved in an isothermal process, $\Delta U = 0$ and the first law of thermodynamics reduces to $\Delta Q = \Delta W$. Thus if the gas expands and does external work ΔW, an amount of heat ΔQ has to be supplied to the gas in order to maitain its temperature. Conversely, if the gas contracts, work is being done on it and an amount of heat ΔQ has to be allowed to leave the gas.

Any attempt to produce an isothermal change requires that the gas is contained in a vessel which has thin, good-conducting walls and which is surrounded by a constant temperature reservoir. In addition, the expansion or contraction must take place slowly. If these conditions are not fulfilled when, say, a gas expands, then the energy used by the gas in doing external work has to be provided at the expense of the kinetic energy of its molecules, and the temperature of the gas falls.

Note Equation [14.19] can be expressed as

$$p_1V_1 = p_2V_2$$

where p_1 and V_1 are the initial pressure and volume of the gas, and p_2 and V_2 are the pressure and volume after the isothermal change has taken place.

14.21 ADIABATIC PROCESSES

An adiabatic process is one which takes place in such a way that no heat enters or leaves the system during the process.

It can be shown that when an ideal gas undergoes a reversible adiabatic expansion or contraction

$$pV^{\gamma} = \text{a constant} \qquad [14.20]$$

where γ is the ratio of the principal heat capacities of the gas.

Since $\Delta Q = 0$, the first law of thermodynamics reduces to $\Delta U = -\Delta W$. Thus if the gas expands and does external work, its temperature falls. Conversely, an adiabatic compression causes the temperature of the gas to rise.

A truly adiabatic process is an ideal which cannot be realized. However, when a gas expands rapidly, the expansion is nearly adiabatic, particularly if the gas is contained in a vessel which has thick, badly conducting walls. Two examples of approximately adiabatic processes are:

(i) the rapid escape of air from a burst tyre,

(ii) the rapid expansions and contractions of air through which a sound wave is passing.

Notes (i) Equation [14.20] can also be expressed in the form

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \qquad [14.21]$$

where p_1 and V_1 are the initial pressure and volume of the gas, and p_2 and V_2 are the pressure and volume after the adiabatic change has taken place.

(ii) $pV = nRT$ applies to any change of the state of an ideal gas and can be expressed as

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \qquad [14.22]$$

Dividing equation [14.21] by equation [14.22] gives

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

from which the final temperature T_2 can be calculated.

14.22 ISOTHERMAL AND ADIABATIC PROCESSES COMPARED

Fig. 14.11 illustrates isothermal and adiabatic expansions of an ideal gas which is initially at a pressure p_1 and volume V_1. The temperature fall which accompanies the adiabatic expansion results in a lower final pressure than that produced by the isothermal expansion. Note that the area under the isothermal is greater than that under the adiabatic, i.e. more work is done by the isothermal expansion than by the adiabatic expansion. Note also that the adiabatic through any point is steeper than the isothermal through that point.

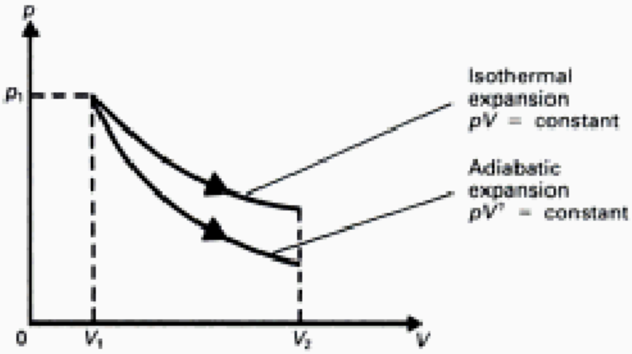


Fig. 14.11
p–V **curves for isothermal and adiabatic expansions**

Fig. 14.12 illustrates the p–V curves of a gas which is expanded adiabatically from a volume V_1 to a volume V_2, and is then compressed isothermally to its original volume.

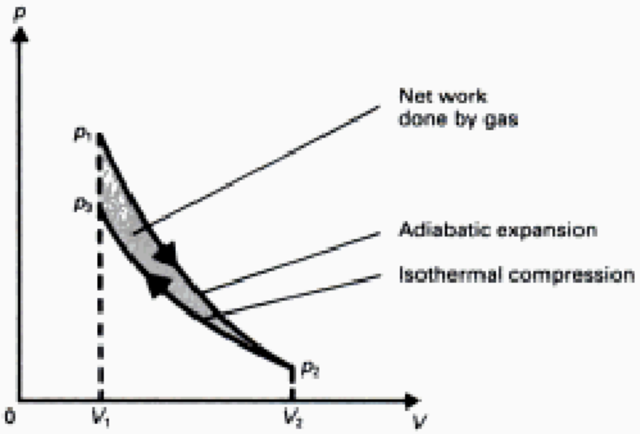


Fig. 14.12
p–V **curves for adiabatic expansion followed by isothermal compression**

Work done by gas in expanding $= \text{Area } V_1 p_1 p_2 V_2$

Work done on gas in contracting $= \text{Area } V_1 p_3 p_2 V_2$

Net work done by gas $= \text{Area } p_1 p_2 p_3$

14.23 MEASUREMENT OF γ FOR AIR BY CLÉMENT AND DÉSORMES' METHOD

The apparatus is shown in Fig. 14.13. The principal component is a vessel of large volume (~10 litres) containing air and a little phosphorus pentoxide to dry it.

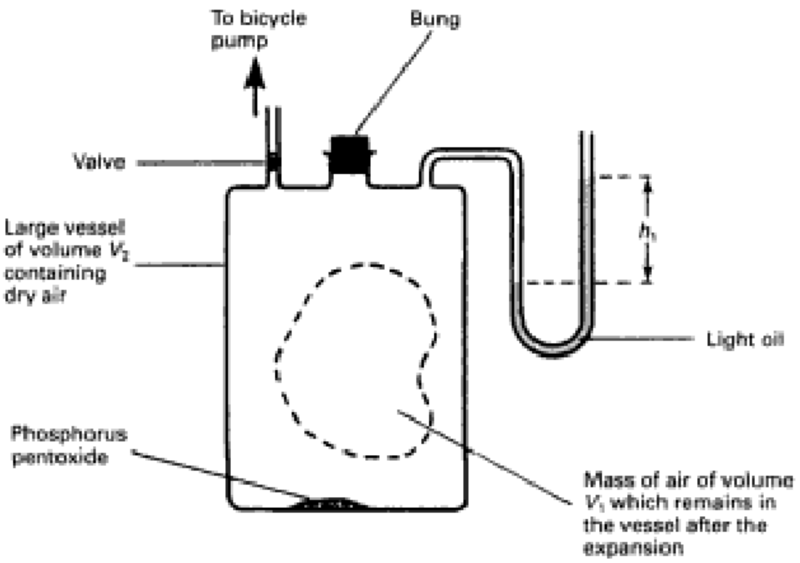


Fig. 14.13 Clément and Désormes' apparatus

Procedure

(i) Air is pumped into the vessel until the pressure inside it is a little above atmospheric. The air is then allowed to cool to room temperature and the (now steady) manometer reading h_1 is recorded.

(ii) The bung is removed for about one second allowing the air to undergo an (approximately) reversible adiabatic expansion. The pressure falls to atmospheric and the air cools.

(iii) The air is now left to regain room temperature and the new (steady) manometer reading h_2 is recorded.

Theory

Suppose that the air which remains in the vessel initially occupied a volume V_1 at a pressure p_1. Immediately after the expansion this same mass of air is at atmospheric pressure p and now occupies the whole of the vessel so that its volume is V_2. Since the expansion is adiabatic

$$p_1 V_1^{\gamma} = p V_2^{\gamma} \quad [14.23]$$

Suppose that p_2 is the pressure of the air when it has regained room temperature. Thus, a mass of air which initially had volume V_1 and pressure p_1 at room temperature, now has volume V_2 and pressure p_2, also at room temperature, and therefore

$$p_1 V_1 = p_2 V_2 \quad [14.24]$$

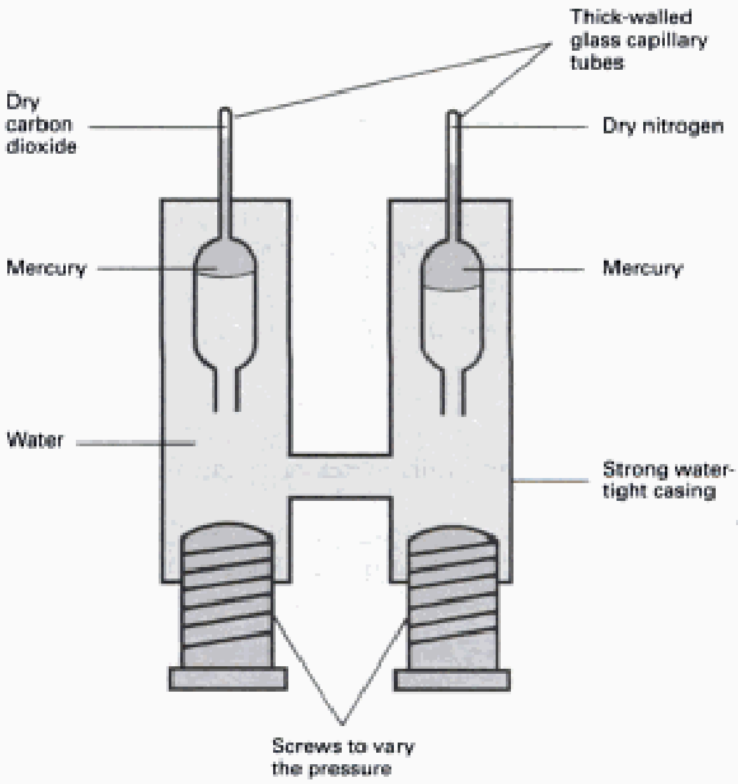
An increase in pressure or a decrease in temperature clearly reduces the validity of assumption (ii). Assumption (i) also becomes less valid because there is ample evidence (see Chapter 9) that the closely packed molecules of solids and liquids do exert forces on each other.

The extent of the departure from ideal gas behaviour varies from gas to gas, but of the common gases carbon dioxide shows considerable non-ideal characteristics.

14.25 ANDREWS' EXPERIMENTS ON CARBON DIOXIDE

The apparatus which Andrews used to investigate the behaviour of carbon dioxide is shown, schematically, in Fig. 14.14. By tightening the screws, Andrews was able to force water into the glass tubes and so increase the pressures and decrease the

Fig. 14.14
Andrews' apparatus



volumes of the gases trapped in the upper portions of the tubes. These tubes had been calibrated beforehand, so that it was a simple matter for Andrews to read off the volumes of the trapped gases by noting the positions of the tops of the mercury columns. By assuming that the nitrogen obeyed Boyle's law (a reasonable assumption at the pressures and temperatures involved as Andrews knew), he was able to calculate the pressure of the nitrogen once he had measured its volume. Since both gases were at the same pressure, this gave him the pressure of the carbon dioxide as desired. The capillary tubes were surrounded by a water bath, the purpose of which was to maintain the gases at a constant temperature. In this way then, Andrews measured the volume of the carbon dioxide as a function of its pressure at a fixed temperature. Altering the water bath temperature allowed him to obtain this information for a number of different temperatures. He presented his results as a series of **isothermals** (i.e. a series of plots of pressure against volume, each at a fixed temperature). Some of these curves are shown in Fig. 14.15.

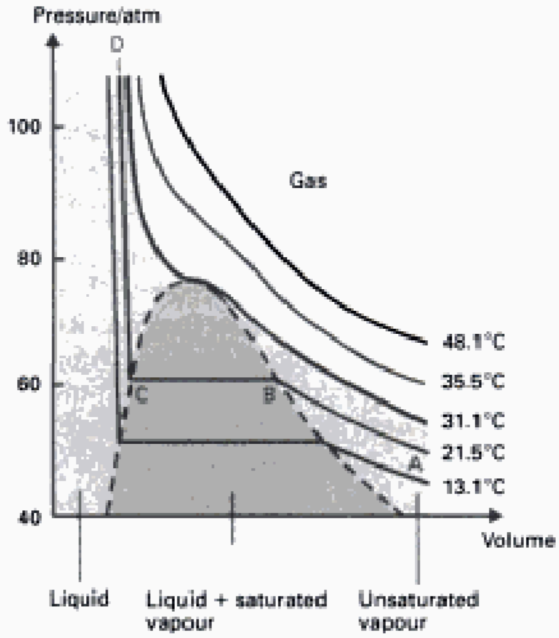


Fig. 14.15
Andrews' isothermals (p–V curves) for a fixed mass of carbon dioxide

The diagram shows the critical nature of the 31.1 °C isothermal. Above 31.1 °C the carbon dioxide exists as a gas no matter how high the pressure, and the curves are approximately hyperbolic (the shape they would be if the carbon dioxide were an ideal gas). Below 31.1 °C the carbon dioxide can exist in both the gaseous state (as a vapour) and the liquid state. Consider the carbon dioxide to be in the state of pressure, volume and temperature that is represented by the point A on the 21.5 °C isothermal. In this state the carbon dioxide is an unsaturated vapour (see section 15.2), and if it is compressed, the p–V curve is very nearly hyperbolic until the pressure reaches that represented by B. At B the carbon dioxide begins to liquefy. Between B and C the volume decreases as the screws are turned in, but there is no increase in pressure. The decrease in volume is due to the fact that in moving from B to C more and more liquid forms, so that at C the carbon dioxide is entirely liquid. From C to D and beyond, large increases in pressure produce very little decrease in volume – as might be expected, since liquds are virtually incompressible.

14.26 TERMINOLOGY

It is now possible to define some useful terms.

Critical temperature (T_c) is the temperature above which a gas cannot be liquefied, no matter how great the pressure. ($T_c = 31.1$ °C for carbon dioxide.)

Critical pressure (p_c) is the minimum pressure that will cause liquefaction of a gas at its critical temperature. ($p_c = 73$ atm for carbon dioxide.)

Specific critical volume (V_c) is the volume occupied by 1 kg of a gas at its critical temperature and critical pressure.

Gas is the term applied to a substance which is in the gaseous phase and is above its critical temperature.

Vapour is the term applied to a substance which is in the gaseous phase and is below its critical temperature.

Thus, a vapour can be liquefied simply by increasing the pressure on it; a gas cannot.

Notes (i) Oxygen, nitrogen and hydrogen are traditionally called **permanent gases**, since it was originally thought that they could not be liquefied. This misconception arose because the early workers had no knowledge of the necessity for a gas to be below its critical temperature, and each of these gases has a critical temperature which is well below room temperature (−118 °C, −146 °C and −240 °C respectively).

(ii) It can be seen from the p–V curves of carbon dioxide (Fig. 14.15), for example, that when a liquid at its critical temperature (and critical pressure) becomes gaseous, then it does so without any change of volume. Under these conditions then, the liquid and its saturated vapour have the same density. Therefore, if a liquid and its saturated vapour are in equilibrium at their critical temperature, there is no meniscus, i.e. no distinction between liquid and vapour.

14.27 CURVES OF pV AGAINST p

A convenient way to show the departure from ideal gas behaviour at some temperature, is to plot pV against p at that temperature. For an ideal gas such a plot is, of course, a straight line parallel to the p axis, but for a fixed mass of real gas the curves typically have the form shown in Fig. 14.16.

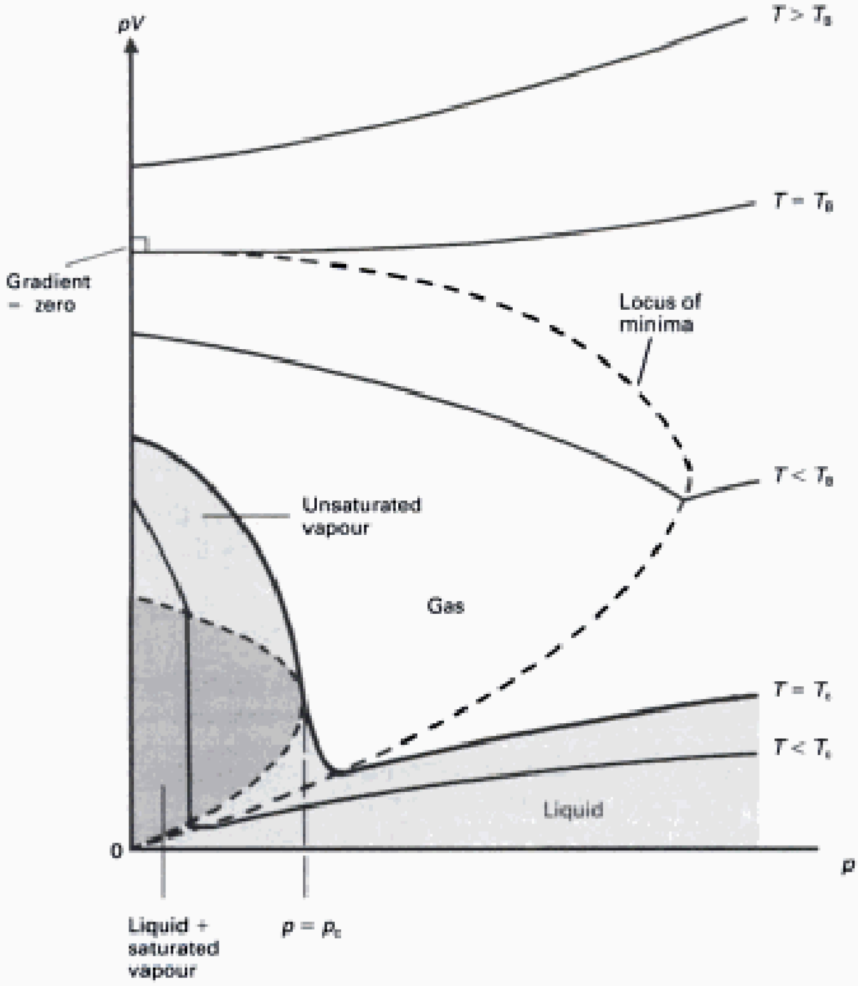


Fig. 14.16
Plots of ***pV*** against ***p*** for a typical real gas

15

VAPOURS

The distinction between a gas and a vapour is given in section 14.26.

15.1 EVAPORATION

Evaporation is the process by which a liquid* becomes a vapour. It can take place at all temperatures, but occurs at the greatest rate when the liquid is at its boiling point.

The kinetic theory supposes that the molecules of liquids are in continual motion and make frequent collisions with each other. Although the average kinetic energy of a molecule is constant at any particular temperature, it may gain kinetic energy as a result of collisions with other molecules. If a molecule which is near the surface and is moving towards the surface gains enough energy to overcome the attractive forces of the molecules behind it, it escapes from the surface. It follows that the rate of evaporation can be increassed by:

(i) increasing the area of the liquid surface;

(ii) increasing the temperature of the liquid (since this increases the average kinetic energy of all the molecules without increasing the strength of the intermolecular forces of attraction);

(iii) causing a draught to remove the vapour molecules before they have a chance to return to the liquid;

(iv) reducing the air pressure above the liquid (since this decreases the possibility of a vapour molecule rebounding off an air molecule).

When a liquid evaporates it loses those of its molecules which have the greatest kinetic energies, and therefore **when a liquid evaporates it cools.**

15.2 SATURATED AND UNSATURATED VAPOURS

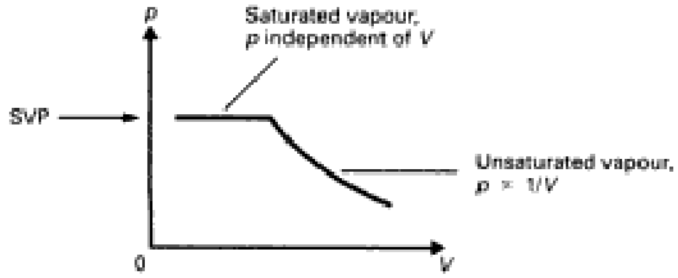
Suppose that a container is partly filled with a liquid and then sealed. Some molecules escape from the liquid by the process of evaporation and exist as a vapour in the region above. The vapour molecules move about at random, and some of them return to the liquid. The rate of condensation (i.e. the rate at which molecules return to the liquid) is determined by the number of molecules in the vapour phase. Initially this is low, and the rate of evaporation exceeds the rate of condensation. There is, therefore, a net gain of molecules by the vapour, and eventually a **dynamic equilibrium** is established in which the rate at which

*Solids evaporate but the rate of evaporation of a solid is negligible unless it is close to its melting point.

molecules enter the vapour is equal to the rate at which they return to the iquid. The region above the iquid is said to be **saturated** with vapour, for it now contains the maximum possible number of molecules which the conditions will allow. (If the number of vapour molecules were to increase by some means, the rate of condensation would become greater than the rate of evaporation and the equilibrium would be re-established.) The pressure exerted by a saturated vapour is called the **saturated vapour pressure** (SVP) and its value depends only on temperature.

If the volume of the space above the liquid is increased, there is a momentary decrease in the density of the vapour, in particular, immediately above the liquid surface. This decreases the rate of condensation and restores the pressure to its previous value, i.e. SVP is independent of volume. If the increase in volume is continued, more and more liquid evaporates and eventually there is none left. Any further increase in volume causes the vapour to become unsaturated. Once this happens the pressure varies with volume in a manner which is approximately consistent with Boyle's law. A plot of pressure against volume at a fixed temperature is shown in Fig. 15.1. (Note that Andrews' isothermals in Fig. 14.15 for carbon dioxide at temperatures below its critical temperature are also of this form.)

Fig. 15.1
p–V **curve for vapour in a sealed container**



It follows from what has been said so far that **a saturated vapour can be defined as being a vapour which is in equilibrium with its own liquid.** If the temperature of such a system is increased there are two distinct consequences.

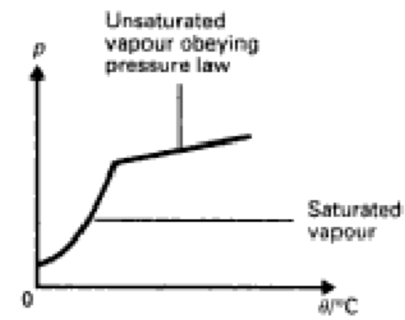
(i) The kinetic energy of the vapour molecules increases.

(ii) The rate of evaporation increases and therefore there is an increase in the number of molecules in the vapour phase.

If the volume of the system is constant, each of these effects produces an increase in pressure. The effect of (i) alone would be to give a pressure increase of the form predicted by the pressure law (approximately); the additional effect of (ii) means that the increase in pressure with increasing temperature is much more rapid than this (see Fig. 15.2).

If the temperature is increased at constant pressure, the volume increases, but because of (ii) it increases much more rapidly than required by Charles' law.

Fig. 15.2
p–θ **curve for vapour in a sealed container**



Summary

(i) A saturated vapour is a vapour which is in equilibrium with its own liquid.

(ii) The gas laws refer to fixed masses of gases. Changing the state of a saturated vapour involves condensation or evaporation and therefore changes its mass. It follows that saturated vapours do not obey the gas laws. In particular SVP depends only on temperature.

(iii) Unsaturated vapours, like real gases, obey the gas laws approximately. In carrying out calculations at this level unsaturated vapours can be taken to obey the gas laws exactly.

15.3 MIXTURES OF GASES AND SATURATED VAPOURS

Dalton's law of partial pressure applies. The total pressure is that of the gas plus that of the vapour. It must be borne in mind that the gas obeys the gas laws, the saturated vapour does not (see Example 15.1).

15.4 SUPERSATURATED VAPOURS

If the temperature of a saturated vapour is reduced suddenly, there is a brief period* during which the vapour contains more molecules than it should at the new temperature. Such a vapour is called a supersaturated vapour and it is not in equilibrium with its liquid.

15.5 BOILING

A liquid boils when its temperature is such that bubbles of vapour form throughout its volume. The pressure inside these bubbles is the SVP of the liquid at the temperature concerned, and must be at least as big as the pressure outside the bubbles otherwise they would collapse. Thus:

The boiling point of a liquid is that temperature at which its SVP is equal to the external pressure.

The external pressure is equal to

(i) the pressure of the atmosphere above the liquid, plus

(ii) the hydrostatic pressure due to the liquid itself, plus

(iii) the pressure due to surface tension effects.

The last two of these are normally ignored but, in particular, (ii) accounts for the lower part of a boiling liquid being hotter than the upper part.

*If there are no nucleating sites present (e.g. dust), the vapour may remain supersaturated for a long time.

If the pressure above a boiling liquid is increased, it stops boiling because the external pressure is now greater than the SVP. If the temperature of the liquid is increased, its SVP rises and eventually becomes equal to the new external pressure. Thus the boiling point of a liquid increases with pressure and a plot of external pressure against boiling point is identical to a plot of SVP against temperature. The SVP of water is shown as a function of temperature in Fig. 15.3.

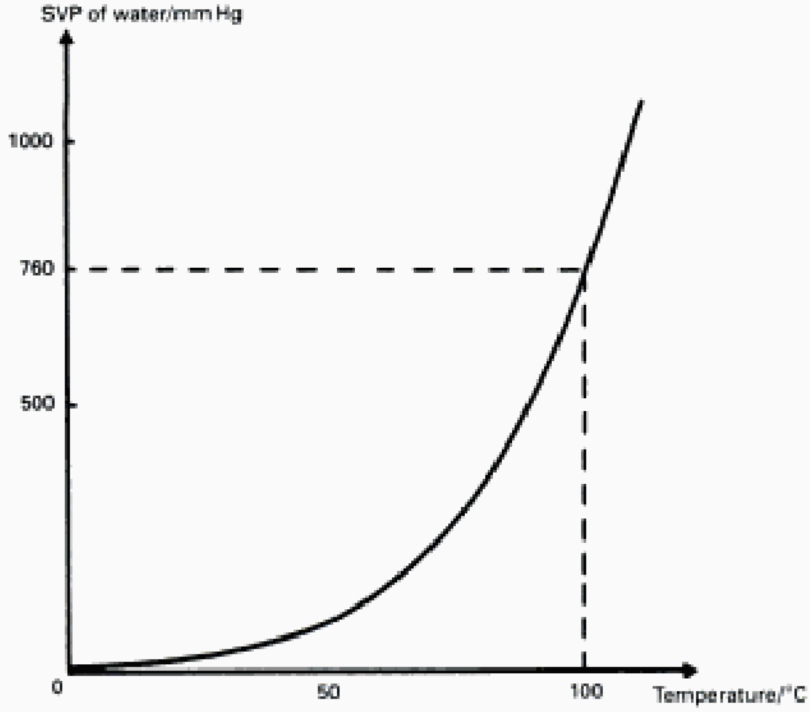


Fig. 15.3 Variation of saturated vapour pressure of water with temperature

Boiling differs from evaporation in that:

(i) boiling occurs throughout the volume of a liquid, whereas evaporation occurs only at the surface, and

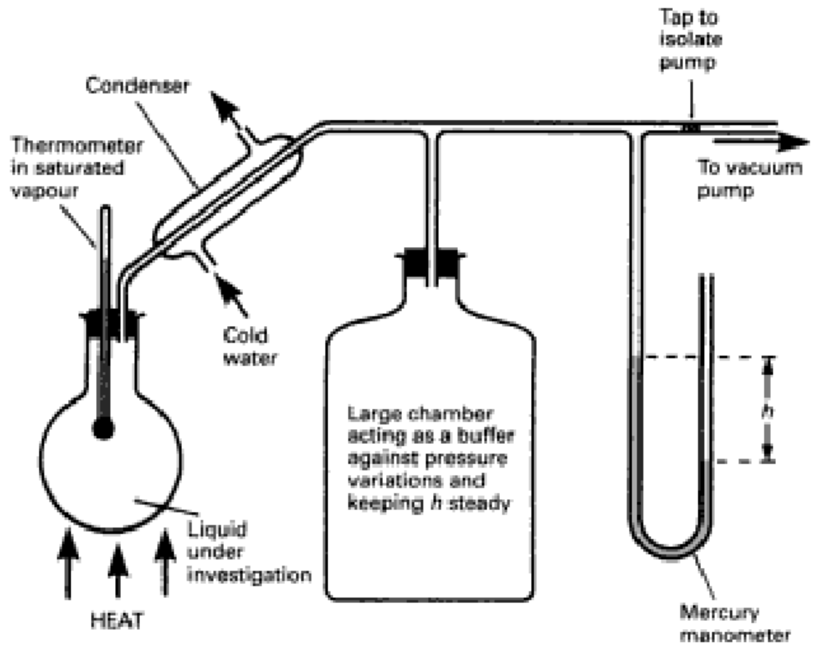
(ii) for any given external pressure a liquid boils at a single temperature only, whereas evaporation takes place at all temperatures.

15.6 EXPERIMENTAL DETERMINATION OF SVP BY THE DYNAMIC METHOD

The apparatus is shown in Fig. 15.4. The pressure above the liquid is reduced to some desired value below atmospheric pressure by means of the vacuum pump. The liquid is then heated gently and it starts to boil at a temperature which is determined by the pressure inside the apparatus. The vapour is condensed and returned to the round-bottomed flask, thereby preventing a pressure build-up within the apparatus. The thermometer registers the temperature of the saturated vapour. The pressure p above the liquid is given by $p = (p_a - h\rho g)$ where p_a is atmospheric pressure and ρ is the density of the mercury. Since a liquid boils when its temperature is such that its SVP is equal to the external pressure, p is the SVP of the liquid at the temperature registered by the thermometer.

Replacing the vacuum pump by a bicyle pump allows SVPs above atmospheric pressure to be determined.

Fig. 15.4
Apparatus for the determination of SVP by the dynamic method



EXAMPLE 15.1

A closed vessel contains air saturated with water vapour at 77 °C. The total pressure in the vessel is 1000 mmHg. Calculate the new pressure in the vessel if the temperature is reduced to 27 °C. (The SVP of water at 77 °C = 314 mmHg; SVP of water at 27 °C = 27 mmHg.)

Solution

By Dalton's law of partial pressures, the pressure of the air at 77 °C (350 K) = 1000 − 314 = 686 mmHg. Treating the air as an ideal gas and assuming that its volume is V and is constant, we see that its pressure, p, at 27 °C (300 K) is (by equation [14.22]) given by

$$\frac{686 \times V}{350} = \frac{pV}{300}$$

i.e. $p = 588$ mmHg

The pressure of the water at 27 °C = 27 mmHg, and therefore the total pressure at 27 °C = 588 + 27 = 615 mmHg.

16

THERMODYNAMICS

The first law of thermodynamics is dealt with in Chapter 14.

16.1 THERMAL EQUILIBRIUM AND THE ZEROTH LAW OF THERMODYNAMICS

If two bodies are in thermal contact and there is no net flow of heat energy between them, the bodies are said to be in **thermal equilibrium** with each other. The bodies must possess some property which determines whether they are in thermal equilibrium – we call this property temperature. It follows that **heat can flow from one body to another only if they are at different temperatures.**

Experiment shows that two bodies which are separately in thermal equilibrium with a third body are also in thermal equilibrium with each other. This is known as the **zeroth law of thermodynamics.** It is called the zeroth law because although the other laws of thermodynamics inherently assume its validity (and therefore logically come after it) they had been established for many years before the first formal statement of it. The reader may feel that the zeroth law is merely a statement of the obvious – maybe it is, but the principle it embodies is fundamental to the whole of thermodynamics and therefore needs to be stated formally.

In order to see how we make use of the zeroth law, suppose we wish to discover whether two bodies, A and B, are at the same temperature, i.e. whether they are in thermal equilibrium with each other. We do this by first bringing A into thermal equilibrium with a third body – a thermometer – and then bringing B into thermal equilibrium with the same thermometer. If the thermometer gives the same reading in each case, by using the zeroth law we can say that A and B are at the same temperature. If, under these circumstances, A and B were not at the same temperature, i.e. if the zeroth law were not true, there would be no point in taking readings with thermometers.

16.2 ENTHALPY

The function $U + pV$ is involved in many applications of thermodynamics; it has therefore been found useful to give it a name – **enthalpy**. Thus the enthalpy, H, of a substance is defined by

$$H = U + pV \qquad [16.1]$$

where U is the internal energy of the substance (see section 14.15) when it is at a pressure p and has a volume V.

Many processes which produce changes in enthalpy take place at constant pressure (chemical reactions for example), and we shall be concerned only with enthalpy changes of this type. Suppose that in some constant pressure process U increases to $(U + \Delta U)$ and V increases to $(V + \Delta V)$. If, as a result of this, the enthalpy increases by ΔH to $(H + \Delta H)$, then from equation [16.1]

$$H + \Delta H = U + \Delta U + p(V + \Delta V) \qquad [16.2]$$

Subtracting equation [16.1] from equation [16.2] gives

$$\Delta H = \Delta U + p\Delta V \quad \text{(at constant pressure)} \qquad [16.3]$$

Note Though p is always measured in N m^{-2}, the units in which ΔH, ΔU and ΔV are measured depend on the amount of substance involved. For unit mass of substance, ΔH and ΔU are respectively called the specific enthalpy change and the specific internal energy change, and are measured in J kg^{-1}. The corresponding value of ΔV is called the specific volume change and is measured in m^3 kg^{-1}. For 1 mole of substance ΔH and ΔU are measured in J mol^{-1} and ΔV is measured in m^3 mol^{-1}. When the amount of substance is neither 1 kg nor 1 mol, or is unspecified, ΔH and ΔU are measured in J and ΔV is measured in m^3.

The term $p\,\Delta V$ in equation [16.3] is the work done by the substance as it expands against the constant pressure p. It therefore follows from the first law of thermodynamics (equation [14.15]) that

$$\Delta Q = \Delta U + p\Delta V \qquad [16.4]$$

where ΔQ is the heat supplied to the substance. Comparing equations [16.3] and [16.4] gives

$$\Delta H = \Delta Q \quad \text{(at constant pressure)} \qquad [16.5]$$

i.e. **in a constant pressure process the enthalpy change is equal to the heat supplied.**

Chemical reactions in which heat is absorbed are called **endothermic reactions**; those in which heat is given out are called **exothermic reactions**. It follows from equation [16.5] that:

$\Delta H > 0$ for an **endothermic** reaction at constant pressure,

$\Delta H = 0$ for an **adiabatic** process at constant pressure,

$\Delta H < 0$ for an **exothermic** reaction at constant pressure.

Equation [16.5] provides two other useful relationships.

$$L_V = \Delta H_{LV} \quad \text{and} \quad L_F = \Delta H_{SL}$$

where

L_V = the specific latent heat of vaporization at some pressure and temperature

ΔH_{LV} = the specific enthalpy change when the substance goes from the liquid to the vapour phase at that pressure and temperature

L_F = the specific latent heat of fusion at some pressure and temperature

ΔH_{SL} = the specific enthalpy change when the substance goes from solid to the liquid phase at that pressure and temperature.

Thus heat has been converted to work. The process stops, however, as soon as the pressure of the gas becomes equal to the pressure outside the cylinder. Before there can be any further conversion of heat into work, the gas has to be returned to its initial (compressed) state. That is, if there is to be a continual conversion of heat into work, the gas has to undergo a cycle. Furthermore, the gas can be returned to its initial state only if some of the heat it initially absorbed is given up to a sink which is at a lower temperature than the source which provided the heat in the first place.

Thus, in practice we find that all heat engines operate by taking some working substance around a cycle, and:

(i) take in heat at a high termperature,

(ii) do work,

(iii) reject some of the heat at a lower temperature.

This is illustrated in Fig. 16.1. Since the engine rejects some of the heat it initially takes in, it has converted only part of it into work. This should not be taken to mean that the engine has violated the first law of thermodynamics. There has been no loss of energy; it is just that some of it is still in the form of heat.

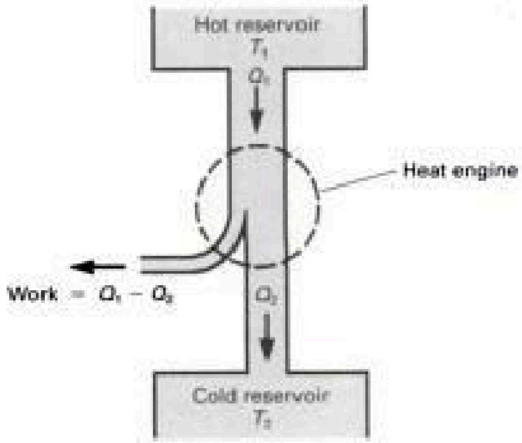


Fig. 16.1 The principle of a heat engine

16.4 THERMAL EFFICIENCY OF HEAT ENGINES

The thermal efficiency η of a heat engine is defined by

$$\eta = \frac{\text{Work done in one cycle}}{\text{Heat taken in at the higher temperature}} \qquad [16.6]$$

At the completion of the cycle the engine's working substance is in the same state as it was initially, and therefore there can have been no change in its internal energy. It follows from the first law of thermodynamics, therefore, that the work done is equal to the net quantity of heat absorbed, i.e.

$$\text{Work done in one cycle} = Q_1 - Q_2$$

where

Q_1 = the heat taken in at the higher temperature

Q_2 = the heat rejected at the lower temperature.

Therefore by equation [16.6]

$$\eta = \frac{Q_1 - Q_2}{Q_1} \quad \text{(for both reversible and irreversible engines)} \qquad [16.7]$$

It can be shown that if the cycle is carried out reversibly (see section 14.13), then

$$\eta = \frac{T_1 - T_2}{T_1} \quad \text{(for all reversible heat engines)} \qquad [16.8]$$

where T_1 and T_2 are the Kelvin temperatures (as measured on the ideal gas scale*) at which the heat is respectively absorbed and rejected.

Notes

(i) Equation [16.8] is valid for all reversible engines, regardless of the particular cycle and the particular working substance, as long as the heat is taken in entirely at the single temperature T_1 and is rejected entirely at the single temperature T_2.

(ii) It can be shown that no heat engine is more efficient than a reversible one working between the same two temperatures, and therefore no heat engine can possibly have an efficiency greater than that given by equation [16.8].

(iii) It follows from equation [16.8] that the efficiency of a heat engine can never be 100% (i.e. η cannot be equal to 1) because the reservoir to which the engine rejects heat would have to be at a temperature of zero kelvin (i.e. $T_2 = 0\,\text{K}$) and this, of course, is impossible.

(iv) Equation [16.8] can be rewritten as $\eta = 1 - T_2/T_1$, and therefore the efficiency is increased by decreasing T_2/T_1, i.e. **the efficiency is increased by taking in heat at as high a temperature as possible and rejecting heat at as low a temperature as possible.**

(v) The efficiency of a real heat engine is less than that given by equation [16.8] because of losses due to frictional effects, turbulence, etc., and because the heat is usually taken in over a range of temperatures and rejected over a range of temperatures.

16.5 THE SECOND LAW OF THERMODYNAMICS

Though there is nothing in the first law of thermodynamics to prevent it being otherwise, it is a matter of common experience that:

(i) no heat engine that works in a cycle completely converts heat into work, and

(ii) when a cold body and a hot body are brought into contact with each other, heat always flows from the hot body to the cold body – never from the cold body to the hot body.

The second law of thermodynamics is a formal statement of these observations. It can be stated in a number of different (but equivalent) ways. One such statement is:

> It is not possible to convert heat continuously into work without at the same time transferring some heat from a warmer body to a colder body.

Thus, whereas the first law tells us of the equivalence of heat and work, the second law is concerned with the circumstances in which heat can be converted into work.

If the second law were not true, it would be possible to run ships on heat extracted from the sea. It is not possible to do so, though, because the second law requires there to be a reservoir at a lower temperature than the sea into which some of the

*The significance of specifying the ideal gas scale will become apparent on reading section 16.6.

rejected heat can be discharged. There is no such reservoir, except perhaps the ship's cold-store, and this is cold only because refrigeration units are consuming energy to keep it so!

The experience that heat cannot be completely converted into work is associated with the fact that heat is fundamentally different from other forms of energy. The heat energy possessed by a body is the energy of the random motions of its molecules. This is quite distinct from, say, the kinetic energy the body has when it is moving. The kinetic energy of a moving body represents the ordered motion which its molecules have superimposed on their random motion. When we try to convert heat into work we are trying to change the random molecular motion into ordered motion. The reason we cannot accomplish this with 100% efficiency is that we cannot control the individual motions of the molecules.

16.6 THE THERMODYNAMIC SCALE OF TEMPERATURE

The efficiency of a reversible heat engine depends only on the temperatures of the source and the sink between which it is operating. Kelvin realized that if a temperature scale were defined in terms of the efficiency of such an engine, it would be independent of the properties of any particular substance – it would therefore be an absolute scale.

Kelvin suggested that the scale (now called the **thermodynamic scale**) should be such that the ratio of any two temperatures on it should be equal to the ratio of the quantities of heat taken in and rejected by a reversible heat engine operating between the same two temperatures. Thus if we represent temperatures on the thermodyanic scale by τ, then for a reversible engine taking in heat Q_1 at temperature τ_1 and rejecting heat Q_2 at a lower temperature τ_2,

$$\frac{\tau_2}{\tau_1} = \frac{Q_2}{Q_1} \qquad [16.9]$$

The efficiency of such an engine is given by equation [16.7] as

$$\eta = \frac{Q_1 - Q_2}{Q_1} \qquad \text{i.e.} \quad \eta = 1 - \frac{Q_2}{Q_1}$$

Therefore, by equation [16.9]

$$\eta = 1 - \frac{\tau_2}{\tau_1} \qquad [16.10]$$

If the temperatures between which the engine is operating had been measured on the **ideal gas scale** (see section 13.2) and had been found to be T_1 and T_2, then from what has been said in section 16.4, the efficiency η would be given by

$$\eta = \frac{T_1 - T_2}{T_1} \qquad \text{i.e.} \quad \eta = 1 - \frac{T_2}{T_1} \qquad [16.11]$$

Comparing equations [16.10] and [16.11] we see that

$$\frac{\tau_2}{\tau_1} = \frac{T_2}{T_1} \qquad [16.12]$$

That is to say, any two temperatures on the thermodynamic scale are in the same ratio as the same two temperatures measured on the ideal gas scale. Finally, by making the temperature of the triple point of water the fixed point of both the thermodynamic scale and the ideal gas scale, and assigning to it the same numerical value (273.16 K) in each case, the two scales become identical. For example, if the temperature of the sink to which the engine is discharging heat is taken to be the temperature of the triple point of water then $\tau_2 = T_2 = 273.16\,\text{K}$ and therefore by equation [16.12]

$$\frac{273.16}{\tau_1} = \frac{273.16}{T_1}$$

i.e. $\tau_1 = T_1$

Now we have established that the two scales are identical, there is no longer any need to distinguish between them. Therefore from now on the single symbol T should be taken to refer to either scale.

16.7 ENTROPY

The first law of thermodynamics is concerned with energy; the second law is concerned with a quantity called **entropy**. It can be defined by*

$$\delta S = \delta Q/T \quad \text{(for a reversible process only)} \qquad [16.13]$$

where

δS = the increase in entropy of some system when it undergoes a reversible change (J K^{-1})

δQ = the heat absorbed by the system, and where δQ is so small that the process can be considered to take place at a constant temperature T measured in kelvins.

For the more general case of a reversible process in which the temperature is not necessarily constant and where a system changes from an initial state (1) to some other state (2)

$$\Delta S = \int_1^2 \frac{\mathrm{d}Q}{T} \quad \text{(for a reversible process only)} \qquad [16.14]$$

where ΔS = the increase in entropy when the system changes reversibly from state 1 to state 2 (J K^{-1})

$\int_1^2 \frac{\mathrm{d}Q}{T}$ = the sum of the ratios of the quantities of heat absorbed at each point on the path from state 1 to state 2 to the temperatures at those points, i.e. the sum of the terms $\delta Q/T$ of equation [16.13].

Notes (i) Equations [16.13] and [16.14] are valid for reversible processes only.

(ii) The entropy of a system depends only on the state of the system. **When the entropy of a system changes the change depends only on the initial and final states of the system, not on the particular process by which it was accomplished, nor on whether it was reversible or irreversible.** At first this statement may seem to contradict note (i), but it does not. Though the changes in entropy are the same, $\int \mathrm{d}Q/T$ for the reversible process is not equal to $\int \mathrm{d}Q/T$ of the irreversible process.

*An alternative definition is given later.

(iii) Since T cannot be negative, it follows from equation [16.14] that **the entropy of a system increases when it absorbs heat and decreases when it rejects heat.**

(iv) For an adiabatic process there is no change in heat content and equation [16.14] reduces to

$$\Delta S = 0 \quad \text{(for any reversible adiabatic process)}$$

Processes which occur without change in entropy are called **isentropic** processes and therefore **reversible adiabatic processes are isentropic**.

(v) For a reversible isothermal process equation [16.4] reduces to

$$\Delta S = \frac{Q}{T} \quad \text{(for any reversible isothermal process)} \qquad [16.15]$$

where Q is the heat absorbed at the constant temperature T.

(vi) It follows from note (ii) that when a substance is taken through a complete cycle the net change in entropy is zero, i.e.

$\Delta S = 0$ for a working substance undergoing a complete cycle (for both reversible and irreversible processes)

When the cycle is carried out reversibly the entropy lost by the source is equal to that gained by the sink, and therefore the entropy change for the whole system (sink, source and working substance) is zero. For an irreversible cycle, though, the entropy lost by the source is less than that gained by the sink. Therefore even though there is no change in the entropy of the working substance, there is an increase in the entropy of the system as a whole.

EXAMPLE 16.2

Calculate the change in entropy of 3.00 kg of water at 100 °C when it is converted to steam at 100 °C. (Specific latent heat of vaporization of water = $2.26 \times 10^6\,\mathrm{J\,kg^{-1}}$ at 100 °C.)

Solution

The process is both isothermal and reversible, in which case the change in entropy, ΔS, is given by equation [16.5] as

$$\Delta S = Q/T$$

Here $Q = 3.00 \times 2.26 \times 10^6 = 6.78 \times 10^6\,\mathrm{J}$

$T = 373\,\mathrm{K}\ (= 100\,^\circ\mathrm{C})$

$$\therefore \quad \Delta S = \frac{6.78 \times 10^6}{373} = 1.82 \times 10^4\,\mathrm{J\,K^{-1}}$$

EXAMPLE 16.3

Calculate the change in entropy of 5.00 kg of water when it is heated reversibly from 0 °C to 100 °C. (Specific heat capacity of water in the range 0 °C to 100 °C = $4.20 \times 10^3\,\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$.)

Solution

The change in entropy, ΔS, is given by equation [16.4] as

$$\Delta S = \int_{T=273\,\mathrm{K}}^{T=373\,\mathrm{K}} \frac{\mathrm{d}Q}{T}$$

It follows from equation [13.7] that $\mathrm{d}Q = mc\,\mathrm{d}T$.

$$\Delta S = \int_{273}^{373} \frac{(5.00)\,(4.20 \times 10^3)}{T}\,\mathrm{d}T$$

$$= 21.0 \times 10^3 [\log_e T]_{273}^{373}$$

$$= 21.0 \times 10^3 \log_e \left(\tfrac{373}{273}\right)$$

$$= 6.55 \times 10^3\,\mathrm{J\,K^{-1}}$$

EXAMPLE 16.4

5.00 kg of water are heated from 0 °C to 100 °C by being placed in contact with a body which has a large heat capacity and which is itself at 100 °C. Calculate the changes in entropy of: (a) the water; (b) the body; (c) the Universe. (Specific heat capacity of water in the range 0 °C to 100 °C = $4.20 \times 10^3\,\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$.)

Solution

(a) This is identical to Example 16.3, and therefore

$$\Delta S_{\text{water}} = +6.55 \times 10^3\,\mathrm{J\,K^{-1}}$$

(b) The body is of very large heat capacity and therefore, to a good approximation, we may assume that its temperature is constant at 100 °C (373 K). The entropy change of the body is therefore given (by equation [16.15]) as

$$\Delta S_{\text{body}} = -Q/373$$

where Q is the heat lost by the body. This is equal to the heat gained by the water, and therefore

$$Q = 5.00 \times 4.20 \times 10^3 \times 100 = 2.10 \times 10^6\,\mathrm{J}$$

$$\therefore \quad \Delta S_{\text{body}} = -\frac{2.10 \times 10^6}{373}$$

i.e. $\Delta S_{\text{body}} = -5.63 \times 10^3\,\mathrm{J\,K^{-1}}$

(c) The change in entropy of the Universe is equal to that of the whole system (i.e. the water and the body), and therefore

$$\Delta S_{\text{Universe}} = \Delta S_{\text{water}} + \Delta S_{\text{body}}$$
$$= 6.55 \times 10^3 - 5.63 \times 10^3$$

i.e. $$\Delta S_{\text{Universe}} = 920\,\text{J K}^{-1}$$

Note The transfer of heat from the body (at 100°C) to the water (initially at 0°C) is an irreversible process, for at no stage could it go in the opposite direction. In performing the calculations, though, we have used equations which govern reversible processes. We are justified in doing this because entropy changes depend only on the initial and final states of the system concerned, and not on the manner in which the changes occur. (See Note (ii) on p. 289.) The change in entropy of the water is therefore the same as it would be if its temperature were increased reversibly to 100°C. Likewise, the entropy lost by the body as a result of losing heat is the same as it would be if the heat had been lost reversibly. Note, though, that there is an overall increase in entropy (of 920 J K^{-1}) as, of course, there must be for an irreversible process.

16.8 TEMPERATURE – ENTROPY DIAGRAMS (*T–S* DIAGRAMS)

These are plots of temperature against entropy and are a useful alternative to *p–V* diagrams. Suppose that during some reversible process the temperature and entropy of a substance vary in the arbitrary manner shown in Fig. 16.2.

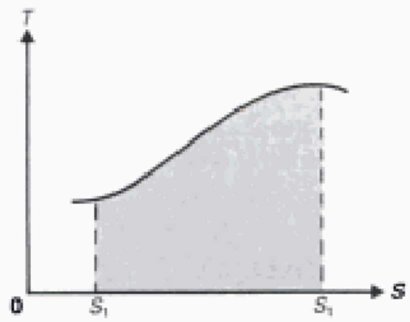


Fig. 16.2
To illustrate the significance of the area under a ***T–S*** curve

It follows from equation [16.13] that for a reversible process

$$\delta Q = T\delta S$$

The heat absorbed by the substance when its entropy changes from S_1 to S_2 is therefore given by Q, where

$$Q = \int_{S_1}^{S_2} T\,\mathrm{d}S$$

i.e. **Heat absorbed = Area of shaded region** (for a reversible process only) [16.16]

When a substance is taken through a complete cycle it ends up in the same state as the one it started in; in particular it has the same temperature and the same entropy, and is therefore represented by a closed loop on a *T–S* diagram. Suppose a

Cooling of Food in a Refrigerator

As the food in a refrigerator cools, its entropy decreases because heat is being removed from it. But this heat goes into the surrounding air, and so the entropy of the air increases. In addition, electrical energy is being consumed, and this will probably have involved the burning of some fuel (coal or oil, for example). The entropy of the combustion products (hot gases, smoke, etc.) is greater than that of the original fuel. Calculations show that there is a net increase in entropy.

Irreversible Heat Engines

The efficiency of a heat engine, reversible or irreversible, which takes in heat Q_1 at a temperature T_1 and rejects heat Q_2 at a temperature T_2 is $(Q_1 - Q_2)/Q_1$ (see section 16.4). In the case of a reversible engine

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

The efficiency of an irreversible engine is less than that of a reversible one, and therefore for an irreversible engine

$$\frac{Q_1 - Q_2}{Q_1} < \frac{T_1 - T_2}{T_1}$$

$$\therefore \quad 1 - \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

$$\therefore \quad \frac{T_2}{T_1} < \frac{Q_2}{Q_1}$$

$$\therefore \quad \frac{Q_1}{T_1} < \frac{Q_2}{T_2}$$

Thus the entropy lost by the source (Q_1/T_1) is less than that gained by the sink (Q_2/T_2). Since the only other component of the system, the working substance, undergoes no entropy change in a complete cycle, there has been an overall increase in entropy.

16.10 PRINCIPLE OF INCREASE OF ENTROPY (ENTROPY VERSION OF THE SECOND LAW)

We have seen that when a system undergoes a reversible process there is no change in the entropy of the system, and that in an irreversible process there is always an increase in entropy. Reversible processes are an ideal that cannot be realized in practice, i.e. all real processes are irreversible. It follows that **all real processes occur in such a way that there is a net increase in entropy.** This is called **the principle of increase of entropy.** It is a consequence of the second law of thermodynamics, and in fact is one of the many ways in which the second law can be stated.

Every time entropy increases the opportunity to convert some heat into work is lost for ever. For example, there is an increase in entropy when hot and cold water are mixed. The warm water which results will never separate itself into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. We can envisage a (distant) future in which the temperature of the Universe is the same throughout. The entropy of the Universe will then have reached its maximum value and all processes will cease – the so-called **'heat death' of the Universe**.

16.11 THE STATISTICAL SIGNIFICANCE OF ENTROPY

Imagine a glass container in which there are a thousand grains of salt, and then imagine that a thousand grains of black pepper are carefully placed on top of them. If the container is shaken, the mixture will become uniformly grey. Continued shaking will keep redistributing the grains at random, but we would not expect that the original distribution would ever return. Thus the system has gone from a highly organized state with salt at the bottom and pepper on top, into a highly disorganized state where there is complete uniformity. The reader should realize that if we were to label the grains in some way (by numbering them, say), then the chance that any particular distribution would occur (all the odd numbers being on top for example) would be just as unlikely as that with all the pepper at the top – no matter how we numbered the grains. The point is, of course, that the grains are not labelled. The system has gone from a statistically unlikely state (salt and pepper separate) to one of a very large number of indistinguishable (uniformly grey) states in which there are approximately five hundred grains of salt and five hundred grains of pepper in each half of the mixture.

This has been just one example of the common experience that in all natural processes (involving large, and therefore statistically meaningful, numbers) the amount of disorder tends to increase up to some maximum value. We saw in section 16.10 that whenever some natural process takes place there is an increase in entropy. Thus natural processes increase both disorder and entropy. This is no coincidence; entropy and disorder are related, and it can be shown that entropy is in fact a measure of disorder. This is not too surprising, for we stated in section 16.5 that when work is converted into heat, ordered motion is being changed into disordered motion, and later saw that increases in heat content are brought about by increases in entropy.

16.12 HEAT PUMPS AND REFRIGERATORS

Both heat pumps and refrigerators (we shall explain the difference in the next paragraph) act like heat engines working in reverse, i.e. they take in heat at a low temperature and reject heat at a higher temperature. In order that they can do this, some external agency (an electric motor for example) has to do work on the working substance of the device. Fig. 16.5 compares the action of a heat engine operating between temperatures T_1 and T_2 with that of a heat pump or refrigerator operating between the same two temperatures.

The purpose of a refrigerator is to cool whatever is inside it, i.e. to remove heat from the low temperature reservoir. The purpose of a heat pump, on the other hand, is to supply heat to the high temperature reservoir. For example, a heat pump might be used to heat a house in winter by taking heat from a (cold) river nearby. The effectiveness of refrigerators and heat pumps is measured by a quantity called the **coefficient of performance**. It is respectively the ratio of the heat extracted or supplied, to the work done by the external agency. Thus

$$\begin{array}{c}\text{Coefficient of}\\ \text{performance of}\\ \text{a refrigerator}\end{array} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2} \quad \left(\begin{array}{c}\text{for a reversible}\\ \text{refrigerator}\end{array}\right)$$

$$\begin{array}{c}\text{Coefficient of}\\ \text{performance of}\\ \text{a heat pump}\end{array} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2} \quad \left(\begin{array}{c}\text{for a reversible}\\ \text{heat pump}\end{array}\right)$$

Heat pumps provide a cheap form of heating, because the heat supplied (Q_1) is greater than the work done by the external agency ($Q_1 - Q_2$). Suppose that a heat pump working reversibly extracts heat from a river at 7 °C (280 K) and delivers it to a room at 21 °C (294 K). The pump is reversible, and therefore

$$\frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

$$= \frac{294}{294 - 280} = 21$$

Thus the coefficient of performance is 21, i.e. 21 joules of heat would be provided with the consumption of only one joule of work! Compare this with a conventional electric fire where one joule of electrical energy can (at best) supply one joule of heat.

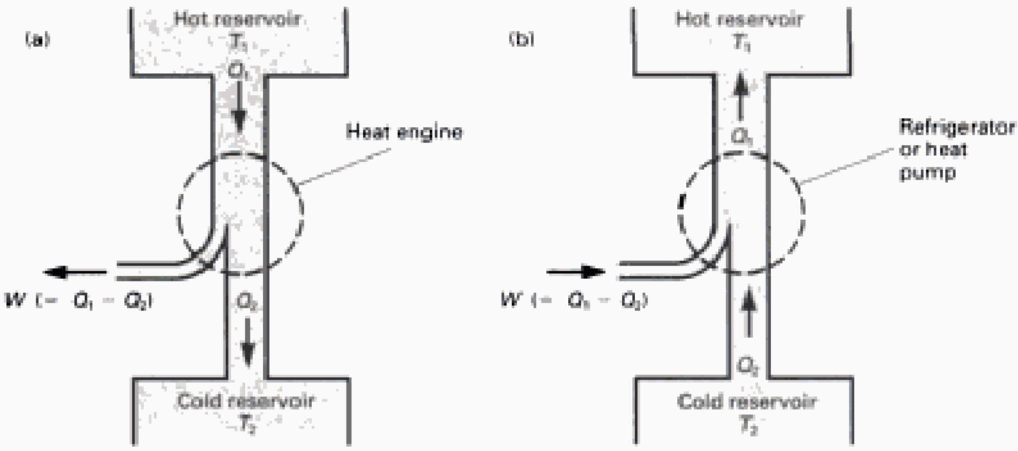


Fig. 16.5
(a) The action of a heat engine, compared with (b) that of a heat pump or refrigerator

16.13 THE PETROL ENGINE CYCLE (OTTO CYCLE)

Fig. 16.6(a) shows the p–V curve for the cycle of operations known as an Otto cycle. **The Otto cycle** is an idealized form of the cycle that occurs in a petrol engine. Refer also to Fig. 16.6(b).

A→A′ The inlet valve opens and the exhaust valve closes.

A′→A **Induction stroke**. A mixture of typically 7% petrol vapour and 93% air (by weight) at about 50 °C is drawn into the cylinder (through the inlet valve) as the piston moves down.

B→C **First part of power stroke**. The fuel (diesel oil) is sprayed into the cylinder and is ignited by the hot air. The fuel enters at such a rate that as it burns (supplying heat Q_1) it forces the piston down at constant pressure.

C→D **Second part of power stroke.** The fuel supply is cut off at C and the burnt gas expands adiabatically and pushes the piston down. The temperature falls.

D→A The exhaust valve opens at D and most of the burnt gas rushes out of the cylinder, removing an amount of heat Q_2. The pressure and temperature of the gas remaining in the cylinder decrease.

A→A **Exhaust stroke.** The rest of the burnt gas is expelled from the cylinder as the piston moves up.

At A′ The exhaust valve closes and the inlet valve opens. The cycle starts again.

Notes

(i) Each cycle consists of four strokes of the piston: A′ to A (down), A to B (up), B to D (down) and A to A′ (up). It is therefore a four-stroke cycle.

(ii) The fuel is burnt inside the cylinder; it is therefore an **internal combustion engine**.

(iii) There is no fuel in the cylinder during compression (A to B) and therefore (unlike the case of the petrol engine) very high compression ratios (typically 16:1) can be utilized without any risk of pre-ignitiion. This makes Diesel engines more efficient than petrol engines.

Diesel engines have the added advantage of using a cheaper fuel. On the other hand, the higher working pressures of Diesel engines makes them more expensive to produce and they have lower power/weight ratios than petrol engines. The theoretical efficiency is typically 65%, but the efficiency of an actual engine is less than this (typically 36%) because of frictional effects, etc. (see note (v) of section 16.13).

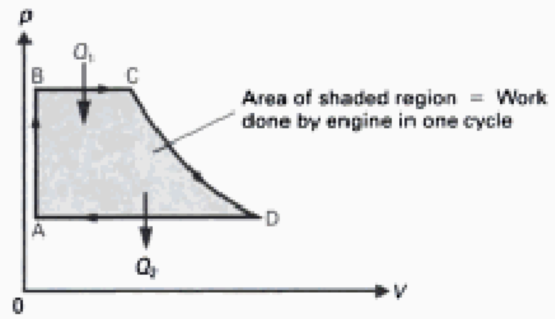
(iv) Note (vi) of section 16.13 applies here too.

16.15 THE STEAM ENGINE CYCLE (RANKINE CYCLE)

Fig. 16.8 shows the p–V curve for the cycle of operations known as a Rankine cycle. **The Rankine cycle** is an idealized form of the cycle which occurs in a steam engine.

A→B Water is compressed adiabatically. There is very little change in volume and only a slight increase in pressure.

Fig. 16.8
Idealized steam engine cycle



B→C The water is heated (in the boiler) at constant pressure to its boiling point at the pressure of the boiler. As the heating continues the water vaporizes, at the same constant pressure, to form steam, which expands into the cylinder.

C→D The steam is now receiving no heat. It expands adiabatically and cools.

D→A The steam is condensed to water at constant pressure and temperature.

Notes (i) Steam engines are known as **external combustion engines** because the fuel is burned outside the cylinder.

(ii) The theoretical efficiency is typically 30%; the actual efficiency is much less, typically 10%. A major cause of this large difference is the drop in pressure that occurs as the stream passes along the pipes leading from the boiler to the cylinder.

(iii) The theoretical efficiency is much less than that of both the Otto cycle and the Diesel cycle. This reflects the fact that the heat is supplied at a much lower temperature (about 250 °C) in the case of the steam engine.

Suppose also that the rate of flow of heat from the hotter face to the colder face is $\delta Q/\delta t$. It can be shown by experiment that **if there are no heat losses from the sides and steady state conditions prevail,** then

$$\frac{\delta Q}{\delta t} \propto A\frac{\delta \theta}{\delta x}$$

With the introduction of a constant of proportionality k this can be written as

$$\frac{\delta Q}{\delta t} = -kA\frac{\delta \theta}{\delta x}$$

which in the limit as $\delta x \to 0$ becomes

$$\frac{dQ}{dt} = -kA\frac{d\theta}{dx} \qquad [17.1]$$

where

dQ/dt is the rate of flow of heat from the hotter face to the colder face and is at right angles to the faces (unit = W)

$d\theta/dx$ is called the **temperature gradient** across the section concerned (unit = $K\,m^{-1}$)

k is a constant whose value depends on the material of the disc. It is called the **coefficient of thermal conductivity** of the material (unit = $W\,m^{-1}\,K^{-1}$). Values of k for some common materials are given in Table 17.1.

Notes

(i) When heat is flowing in the positive direction of x (as in Fig. 17.1) the temperature gradient is negative, and therefore the presence of the minus sign in equation [17.1] makes k a positive constant.

(ii) It is the existence of the temperature gradient which causes the heat to flow. If it were not for the fact that the two faces are being maintained at their respective temperatures, the effect of the heat flow would be to destroy the temperature gradient by warming the cooler regions.

(iii) Equation [17.1] is used to define k. Thus:

The coefficient of thermal conductivity of a material is the rate of flow of heat per unit area per unit temperature gradient when the heat flow is at right angles to the faces of a thin parallel-sided slab of the material under steady state conditions.

Table 17.1
Values of k for some common substances at room temperature

Substance	$k/W\,m^{-1}\,K^{-1}$
Silver	418
Copper	385
Aluminium	238
Iron	80
Lead	38
Mercury	8
Glass (Pyrex)	1.1
Brick	~ 1
Rubber	0.2
Air	0.03

Therefore, from equations [17.2] and [17.3] and since $k_X = k_Y$

$$\left(\frac{d\theta}{dx}\right)_X > \left(\frac{d\theta}{dx}\right)_Y$$

and it follows that:

Temperature gradient decreases with distance from the hot end of an unlagged uniform bar.

Fig. 17.3 is based on these results and shows the steady state temperature distribution of a perfectly lagged uniform bar of length L, together with that of an identical unlagged bar. Each bar has its hot end maintained at a temperature θ_2 and its cold end at θ_1. The situations of the two bars are illustrated in terms of heat flow lines in Fig. 17.4.

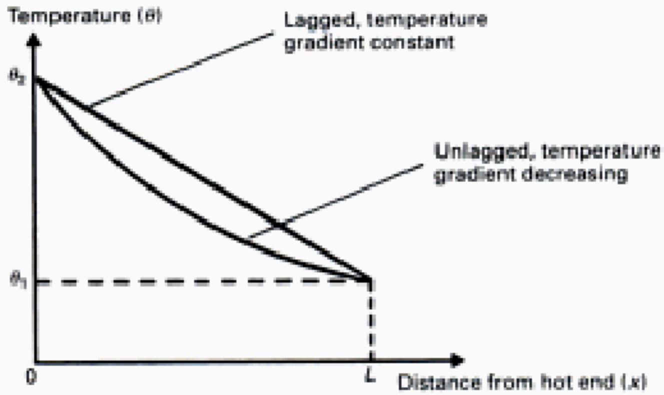


Fig. 17.3
Temperature distribution along a uniform bar

Fig. 17.4
Heat flow lines in a lagged and an unlagged bar

Since the temperature gradient of the lagged bar is constant

$$\frac{d\theta}{dx} = -\frac{(\theta_2 - \theta_1)}{L}$$

Therefore from equation [17.1]

$$\frac{dQ}{dt} = kA\frac{(\theta_2 - \theta_1)}{L} \quad \left(\begin{array}{c}\text{for a perfectly}\\ \text{lagged bar}\end{array}\right) \qquad [17.5]$$

This is a particularly useful form of equation [17.1], but it is relevant only to the case of a perfectly lagged bar.

17.4 ANALOGY BETWEEN THERMAL AND ELECTRICAL CONDUCTION

The flow of heat is anlogous to the flow of electrical charge, and it is possible to think in terms of a **heat current** dQ/dt in the same way as we think in terms of electrical current I. A heat current flows through a perfectly lagged uniform bar whenever there is a temperature difference $(\theta_2 - \theta_1)$ across it. This is equivalent to the way in which an electrical current is caused to flow by a potential difference V. Equation [17.5] can be rearranged as

$$\frac{dQ}{dt} = \frac{(\theta_2 - \theta_1)}{(L/kA)}$$

and for an electrical conductor of resistance R

$$I = \frac{V}{R}$$

Comparing these two equations we see that (L/kA) is equivalent to R, and therefore may be thought of as the **thermal resistance** of the bar. Electrical conductivity σ and resistivity ρ are related by $\sigma = 1/\rho$ (section 36.1), and therefore (from equation [36.2]) electrical resistance R is given by $R = L/(\sigma A)$. Since thermal resistance is equal to $L/(kA)$, it follows that k is equivalent to σ. Table 17.2 summarizes these results. Note also that the temperature gradient is equivalent to potential gradient.

Table 17.2
Electric quantities and their thermal analogues

Electrical quantity		*Analogous thermal quantity*	
Electric current	I	Heat current	$\frac{dQ}{dt}$
Potential difference	V	Temperature difference	$\theta_2 - \theta_1$
Electrical resistance	$R = \frac{L}{\sigma A}$	Thermal resistance	$\frac{L}{kA}$
Electrical conductivity	σ	Thermal conductivity	k
Potential gradient	$\frac{dV}{dx}$	Temperature gradient	$\frac{\theta_2 - \theta_1}{L}$

The analogy is made use of in Example 17.1

EXAMPLE 17.1

Two perfectly lagged metal bars, X and Y, are arranged (a) in series, (b) in parallel. When the bars are in series the 'hot' end of X is maintained at 90 °C and the 'cold' end of Y is mantained at 30 °C. When the bars are in parallel the 'hot' end of each is maintained at 90 °C and the 'cold' end of each is maintained at 30 °C. Calculate the ratio of the total rate of flow of heat in the parallel arrangement to that in the series arrangement. The length of each bar is L and the cross-sectional area of each is A. The thermal conductivity of X is $400\ \text{W m}^{-1}\ \text{K}^{-1}$ and that of Y is $200\ \text{W m}^{-1}\ \text{K}^{-1}$.

mass and specific heat capacity of Y, then the rate at which it is losing heat to the surroundings when its temperature is θ_1 is given by $mc(a/b)$, where a/b is the gradient of the graph (i.e. the rate of fall of temperature at θ_1). The conditions under which Y is losing heat are the same as those at steady state, and therefore

$$kA\frac{(\theta_2 - \theta_1)}{x} = mc\frac{a}{b}$$

from which k can be determined.

Notes (i) The upper and lower surfaces of the sample should be smeared with petroleum jelly (Vaseline) to give good thermal contact with X and Y.

(ii) The thermometers actually register the temperatures of X and Y, but since these are good conductors, the temperature gradients across them are small and therefore $(\theta_2 - \theta_1)$ is, to a good approximation, the temperature difference across the sample.

17.7 THERMAL RADIATION

We shall describe **thermal radiation** as being electromagnetic radiation emitted by a body solely on account of its temperature. The radiation spans a continuous range of wavelengths and the distribution of energy amongst these wavelengths depends on the temperature of the emitter. At temperatures below about 1000 °C the energy is associated almost entirely with infrared wavelengths; at higher temperatures visible and ultraviolet wavelengths are also involved. (These aspects are discussed more fully in section 17.10). Thermal radiation has all the general properties of electromagnetic waves. It can be reflected; its speed in a vacuum is $3 \times 10^8\ \text{m s}^{-1}$; it cannot be deflected by electric and magnetic fields; the intensity of the radiation produced by a point source falls off as the inverse square of the distance from the source; etc.

When thermal radiation is incident on a body some of the radiation may be reflected, some transmitted, and some may be absorbed and produce a heating effect. A substance which transmits the thermal radiation incident on it is said to be **diathermanous**, one which absorbs the radiation is said to be **adiathermanous**. (Equivalent respectively to substances which are transparent and substances whch are opaque to visible light.) The absorption of electromagnetic radiation of any wavelength may produce a heating effect. Thus, though X-radiation, for example, is not normally thought of as thermal radiation, heat is produced when X-rays are absorbed.

17.8 PRÉVOST'S THEORY OF EXCHANGES

According to this theory a body emits radiation at a rate which is determined only by the nature of its surface and its temperature, and absorbs radiation at a rate which is determined by the nature of its surface and the temperature of its surroundings.

Suppose that a body is suspended by a non-conductng thread inside an evacuated enclosure whose walls are maintained at a constant temperature T. Since the enclosure is evacuated, there is no possibility of conduction and convection and events are controlled only by radiative processes, i.e. Prévosts's theory applies. If the temperature of the body is greater than that of the surroundings, the body emits radiation at a greater rate than it absorbs it and its temperature falls, eventually becoming equal to T. Conversely, if the initial temperature of the body is less than that of the enclosure, the temperature of the body increases until it becomes equal to T. It is important to note that emission and absorption do not cease at this stage; instead there is a dynamic equilibrium in which the rate of emission is equal to the rate of absorption.

It follows that **if the surface of a body is such that the body is a good absorber of radiation, it must be an equally good emitter**, otherwise its temperature would rise above that of its surroundings. It also follows that a good emitter is a good absorber. These conclusions are confirmed by simple experiments (e.g. Leslie's cube). In particular, **matt black surfaces are the best absorbers and the best emitters of radiation; highly polished silver surfaces are both poor emitters and poor absorbers.**

17.9 THE BLACK BODY

A black body is a body which absorbs all the radiation which is incident on it.

The concept is an idealized one, but it can be very nearly realized in practice – Fig. 17.7 illustrates how. The inner wall of the enclosure is matt black so that most of any radiation which enters through the small hole is absorbed on reaching the wall. The small amount of radiation which is reflected has very little chance of escaping through the hole before it too is absorbed in a subsequent encounter with the wall.

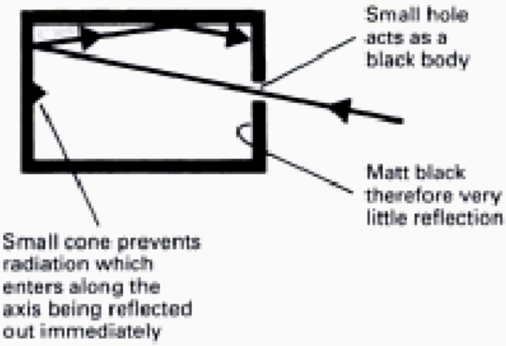


Fig. 17.7 Approximate realization of a black body

A black body radiator (or cavity radiator) is one which emits radiation which is characteristic of its temperature and, in particular, which does not depend on the nature of its surfaces.

A black body radiator can be made by surrounding the enclosure of Fig. 17.7 with a heating coil. The radiation which is emitted by any section of the wall is involved in many reflections before it eventually emerges from the hole. Any section which is a poor emitter absorbs very little of the radiation which is incident on it, and those sections which are good emitters absorb most of the radiation incident on them. This has the effect of mixing the radiations before they emerge, and of making the temperature the same at all points on the inner surface of the enclosure.

increasing temperatures cause the overall colour to change from red through yellow to white. The intensity distribution of the wavelengths emitted by the Sun is the same as that of a black body at about 6000 K, i.e. the temperature of the Sun's surface is about 6000 K. Some stars are much hotter than the Sun and appear blue.

Stefan's Law

The total energy radiated per unit time per unit surface area of a black body is proportional to the fourth power of the temperature of the body expressed in kelvins.

Thus

$$E = \sigma T^4 \qquad [17.7]$$

where

σ = a constant of proportionality known as **Stefan's constant**. Its value is $5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$.

Note that the value of E at any temperature T is equal to the area under the corresponding curve, i.e. $E = \int_0^\infty E_\lambda \, d\lambda$.

If a black body whose temperature is T is in an enclosure at a temperature T_0, the rate at which unit surface area of the black body is receiving radiation from the enclosure is σT_0^4. The net rate of loss of energy by the black body is therefore given by E_{net} where

$$E_{net} = \sigma(T^4 - T_0^4) \qquad [17.8]$$

In the case of a non-black body equations [17.7] and [17.8] are replaced by

$$E = \varepsilon \sigma T^4$$

and

$$E_{net} = \varepsilon \sigma (T^4 - T_0^4)$$

where ε is called the **total emissivity** of the body. Its value depends on the nature of the surface of the body and lies between 0 and 1.

EXAMPLE 17.2

A 100 W electric light bulb has a filament which is 0.60 m long and has a diameter of 8.0×10^{-5} m. Estimate the working temperature of the filament if its total emissivity is 0.70. (Stefan's constant = $5.7 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$.)

Solution

The surface area of the filament is that of a cylinder of diameter 8.0×10^{-5} m and length 0.60 m and is therefore $\pi \times 8.0 \times 10^{-5} \times 0.60 = 1.51 \times 10^{-4}\ \text{m}^2$.

The bulb is rated at 100 W and therefore E, the energy radiated per unit time per unit surface area of the filament, is given by

$$E = \frac{100}{1.51 \times 10^{-4}} = 6.62 \times 10^5\ \text{W m}^{-2}$$

It is possible to estimate the temperatures on the surfaces of the window pane in our example (see later in this section). Such an estimate gives 6.8 °C for the inner surface and 5.8 °C for the outer surface. There is therefore a temperature difference of 1 °C across the glass. The reader should not be surprised by this value – it is one twentieth of the value we used in our original calculation, and that gave an estimate of the heat flow rate which was twenty times too high!

Thermal Resistance Coefficient

The thermal resistance coefficient of a material is the thermal resistance of unit area of the material and is defined by

$$X = \frac{L}{k} \qquad [17.10]$$

where

X = thermal resistance coefficient ($m^2\,K\,W^{-1}$)

L = thickness of material (m)

k = coefficient of thermal conductivity ($W\,m^{-1}\,K^{-1}$).

The thermal resistance coefficient of a structure which consists of a number of different components in series is the sum of the thermal resistance coefficients of the individual components. In Fig. 17.11, for example, the thermal resistance coefficient, X_S, of the structure (for heat transfer between the outer surfaces of A and C is given by

$$X_S = X_A + X_B + X_C$$

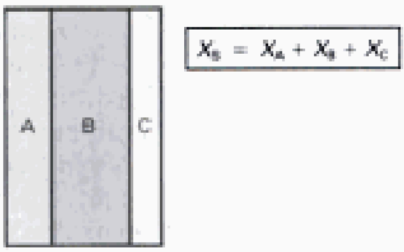


Fig. 17.11
To calculate thermal resistance coefficient of a composite structure

where X_A, X_B and X_C are respectively the thermal resistance coefficients of A, B and C. The thermal resistance coefficient, X_W, of the window in Fig. 17.10 is given by

$$X_W = X_i + X_g + X_o$$

where X_g is the thermal resistance coefficient of the glass and is calculated on the basis of equation [17.10], and X_i and X_o are respectively the effective thermal resistance coefficients of the 'layers' of air on the inner and outer surfaces of the glass. Equation [17.10] cannot be used to calculate these, but it is found by experiment that $X_i = 0.120\,m^2\,K\,W^{-1}$ and $X_o = 0.053\,m^2\,K\,W^{-1}$. The U-value of the window is the reciprocal of its thermal resistance coefficient, i.e.

$$U = \frac{1}{X_W}$$

We are now in a position to show how we estimated the temperatures on the surfaces of the glass in the window. Equation [17.5] can be rewritten as

$$\frac{dQ}{dt} = \frac{A}{X}(\theta_2 - \theta_1)$$

the room temperature on the scale of the resistance thermometer and on the scale of the constant volume gas thermometer.

Why do these values differ slightly? [L]

C8 The value of the property X of a certain substance is given by

$$X_t = X_0 + 0.50t + (2.0 \times 10^{-4})t^2,$$

where t is the temperature in degrees Celsius measured on a gas thermometer scale. What would be the Celsius temperature defined by the property X which corresponds to a temperature of 50 °C on this gas thermometer scale? [L]

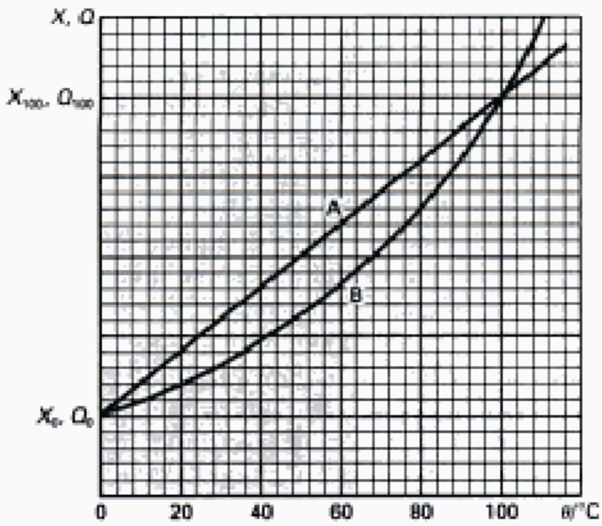
C9 (a) What is meant by a thermometric property? What qualities make a particular property suitable for use in a practical thermometer?

A Celsius temperature scale may be defined in terms of a thermometric property X by the following equation:

$$\theta = \frac{X - X_0}{X_{100} - X_0} \times 100\,^\circ\mathrm{C} \qquad (1)$$

where X_0 is the value of the property at the ice point, X_{100} at the steam point, and X at some intermediate temperature. If X is plotted against θ a straight line always results no matter what thermometric property is chosen. Explain this.

(b) On the graph, line A shows how X varies with θ (following equation (1) above), line B shows how a second thermometric property Q varies with θ, the temperature measured on the X scale.



(i) Describe, in principle, how you would conduct an experiment to obtain line B.

(ii) If $\theta = 40\,^\circ$C recorded by an X-scale thermometer, what temperature would be recorded by a Q-scale thermometer?

(iii) At what two temperatures will the X and Q scales coincide?

(c) The ideal gas scale of temperature is one based on the properties of an ideal gas. What is the particular virtue of this scale? Describe very briefly how readings on such a scale can be obtained using a thermometer containing a real gas. [L]

C10 A temperature T can be defined by $T = T_f(X/X_f)$, where T_f is the assigned temperature of a fixed point and X and X_f are the values of a thermometric property of a substance at T and T_f respectively. On the ideal-gas scale, the fixed point is the triple point of water and $T_f = 273.16$ K.

(a) List *four* thermometric properties which are used in thermometry. Explain why certain thermometric properties of a gas are taken as standard.

(b) Explain what is meant by a fixed point and by the triple point of water.

(c) Sketch and label the simple form of constant-volume gas thermometer found in school laboratories, and describe how it is used to determine the boiling point of a liquid on the ideal-gas scale.

(d) For a thermometer which is not based on the properties of gases, explain how you would calibrate it in terms of the ideal-gas scale.

(e) Compare the advantages and disadvantages of the constant-volume gas thermometer with those of any *two* other types of thermometers.

(f) The pressures recorded in a certain constant-volume gas thermometer at the triple point of water and at the boiling point of a liquid were 600 mm of Hg and 800 mm of Hg respectively. What is the apparent temperature of the boiling point? However, it was found that the volume of the thermometer increased by 1% between the two temperatures. Obtain a more accurate value of the boiling point. [W] source of the energy required to evaporate the water? Estimate the proportion of the water originally in the clothing which remains as ice. State any assumptions you make.

(Specific latent heat of fusion of ice at $273\,K = 333\,kJ\,kg^{-1}$; specific latent heat of vaporization of water at $273\,K = 2500\,kJ\,kg^{-1}$.) [S]

C24 Describe with the aid of a labelled diagram a method of measuring the latent heat of vaporization of a liquid.

In a factory heating system water enters the radiators at 60 °C and leaves at 38 °C. The system is replaced by one in which steam at 100 °C is condensed in the radiators, the condensed steam leaving at 82 °C. What mass of steam will supply the same heat as 1.00 kg of hot water in the first instance?

(The latent heat of vaporization of water is $2.260 \times 10^6\,J\,kg^{-1}$ at 100 °C. The specific heat of water is $4.2 \times 10^3\,J\,kg^{-1}\,°C^{-1}$.) [J]

C25 Describe how you would determine the specific latent heat of vaporization of a liquid by the continuous flow method.

What becomes of the energy used to change a liquid into a vapour at the same temperature?

A beaker containing ether at a temperature of 13 °C is placed in a large vessel in which the pressure can be reduced so that the ether boils; this results in a cooling of the remaining ether. What proportion of the ether has evaporated when the temperature of the remainder has been reduced to 0 °C? (Assume no interchange of heat between the ether and its surroundings.)

(Mean specific heat capacity of ether over the temperature range 0–13 °C = $2.4 \times 10^3\,J\,kg^{-1}\,K^{-1}$.

Mean specific latent heat of vaporization of ether in temperature range 0–13 °C = $3.9 \times 10^5\,J\,kg^{-1}$.) [S]

C26 A domestic kettle is marked 250 V, 2.3 kW and the manufacturer claims that it will heat a pint of water to boiling point in 94 s.

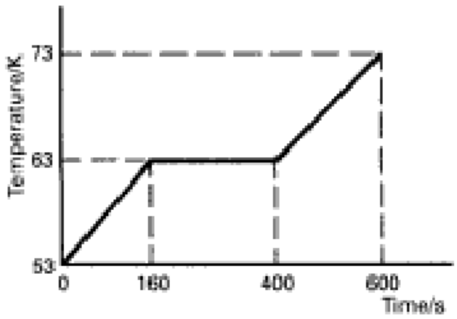
(a) Test this claim by calculation and state any simplifying assumptions you make.

(b) If the kettle is left switched on after it boils, how long will it take to boil away half a pint of water measured from when it first boils?

(c) Estimate the work done against an atmospheric pressure of 100 kPa when $1\,cm^3$ of water evaporates at 100 °C, producing $1600\,cm^3$ of steam. Express this as a percentage of the total energy required to evaporate $1\,cm^3$ of water at 100 °C.

(Specific heat capacity of water = $4.2 \times 10^3\,J\,kg^{-1}\,K^{-1}$, specific latent heat of vaporisation of water = $2.3 \times 10^6\,J\,kg^{-1}$, density of water = $1.0\,g\,cm^{-3}$, 1 pint = $570\,cm^3$.) [J, '92]

C27 The graph refers to an experiment in which an initially solid specimen of nitrogen absorbs heat at a constant rate. Nitrogen melts at 63 K, and the specific heat capacity of solid nitrogen is $1.6 \times 10^3\,J\,kg^{-1}\,K^{-1}$.



Calculate the specific latent heat of fusion of nitrogen.

Calculate the specific heat capacity of liquid nitrogen. [S]

C28 **(a)** In an espresso coffee machine, steam at 100 °C is passed into milk to heat it. Calculate

(i) the energy required to heat 150 g of milk from room temperature (20 °C) to 80 °C,

(ii) the mass of steam condensed.

(b) A student measures the temperature of the hot coffee as it cools. The results are given below:

Time/min	0	2	4	6	8
Temp/°C	78	66	56	48	41

A friend suggests that the rate of cooling is exponential.

(i) Show quantitatively whether this suggestion is valid.

period of three years the pressure has fallen to 2.0×10^6 Pa at the same temperature because of leakage.

(Assume molar mass of nitrogen = $0.028\,\text{kg mol}^{-1}$, $R = 8.3\,\text{J mol}^{-1}\,\text{K}^{-1}$, Avogadro constant = $6.0 \times 10^{23}\,\text{mol}^{-1}$.)

Calculate:

(i) the mass of gas originally present in the cylinder,

(ii) the mass of gas which escaped from the cylinder in three years,

(iii) the average number of nitrogen molecules which escaped from the cylinder per second.
(Take one year to be equal to 3.2×10^7 s.) [O, '92]

C37 A cylinder containing 19 kg of compressed air at a pressure 9.5 times that of the atmosphere is kept in a store at 7 °C. When it is moved to a workshop where the temperature is 27 °C a safety valve on the cylinder operates, releasing some of the air. If the valve allows air to escape when its pressure exceeds 10 times that of the atmosphere, calculate the mass of air that escapes. [L]

C38 A mole of an ideal gas at 300 K is subjected to a pressure of 10^5 Pa and its volume is $0.025\,\text{m}^3$. Calculate:

(a) the molar gas constant R,

(b) the Boltzmann constant k,

(c) the average translational kinetic energy of a molecule of the gas.
($N_A = 6.0 \times 10^{23}\,\text{mole}^{-1}$.) [W, '90]

C39 A vessel of volume $1.0 \times 10^{-3}\,\text{m}^3$ contains helium gas at a pressure of 2.0×10^5 Pa when the temperature is 300 K.

(a) What is the mass of helium in the vessel?

(b) How many helium atoms are there in the vessel?

(c) Calculate the r.m.s. speed of the helium atoms.

(Relative atomic mass of helium = 4, the Avogadro constant = $6.0 \times 10^{23}\,\text{mol}^{-1}$, the molar gas constant $R = 8.3\,\text{J mol}^{-1}\,\text{K}^{-1}$.) [W, '92]

C40 Use a simple treatment of the kinetic theory of gases, stating any assumptions made, to derive the expression $\overline{c^2} = 3p/\rho$ for the mean square speed of the molecules in terms of the density and pressure of the gas.

What would be the total kinetic energy of the atoms of 1 kg of neon gas at a pressure of 10^5 Pa and temperature 293 K, given that the density of neon under these conditions is $828\,\text{g m}^{-3}$. What would be the total kinetic energy of the atoms of 1 kg of neon gas at 300 K? Hence determine the specific heat capacity of neon at constant volume. [S]

C41 **(a)** State Avogadro's law.

(b) The pressure p of an ideal gas is given by:

$$p = \tfrac{1}{3}\,nm\langle c^2\rangle$$

where n is the number of molecules per unit volume, m is the mass of one molecule and $\langle c^2\rangle$ is the mean square speed of the gas molecules.
Use the above equation to deduce Avogadro's law. [W, '91]

C42 **(a)** State the assumptions made in the kinetic theory of gases and prove $p = \frac{1}{3}\rho\overline{c^2}$, in the usual notation. Hence derive **(i)** Boyle's law, and **(ii)** the perfect gas law, assuming that the average kinetic energy of a molecule is proportional to the absolute temperature.

(b) Consider whether the assumptions of the kinetic theory are likely to be true for real gases.

(c) At room temperature, $\sqrt{\overline{c^2}}$ of a gas molecule is typically about $10^2\,\text{m s}^{-1}$. Explain why, if a gas is released at one side of a room, it may be several minutes before it can be detected on the other side of the room.

(d) At a certain instant of time, ten molecules have the following speeds: 100, 300, 400, 400, 500, 600, 600, 600, 700, 900 m s^{-1} respectively. Calculate $\sqrt{\overline{c^2}}$. [W]

C43 **(a)** One mole of an ideal gas at pressure p and Celsius temperature θ occupies a volume V. Sketch a graph showing how the product pV varies with θ. What information can you obtain from the gradient of the graph and the intercept on the temperature axis?

(b) Some helium (molar mass of which = $0.004\,\text{kg mol}^{-1}$) is contained in a vessel of volume $8.0 \times 10^{-4}\,\text{m}^3$ at a temperature of 300 K. The pressure of the gas is 200 kPa. Calculate

Air may be taken to consist of 80% nitrogen molecules and 20% oxygen molecules of relative molecular masses 28 and 32 respectively. Calculate:

(a) the ratio of the root mean square speed of nitrogen molecules to that of oxygen molecules in air,
(b) the ratio of the partial pressures of nitrogen and oxygen molecules in air, and
(c) the ratio of the root mean square speed of nitrogen molecules in air at 10 °C to that at 100 °C. [O & C]

C49 Show that, for an ideal gas, the coefficient of pressure increase at constant volume and the coefficient of cubic expansivity at constant pressure are equal in value. [AEB, '79]

C50 **(a)** A flask is filled with water vapour at 30 °C and sealed. The velocity of any particular water vapour molecule in the flask may vary randomly in two different ways. What are these two ways?

Describe, with the aid of a diagram, how the motion of one of the water vapour molecules could change during a time interval in which it has six collisions with other molecules.

(b) Explain why a small increase in pressure will do more work on a gas than on a liquid. [L, '91]

C51 **(a)** **(i)** Write down the equation which defines a temperature on the Kelvin scale in terms of the properties of an ideal gas. Explain the symbols you use.

(ii) A simple form of gas thermometer consists of a capillary tube sealed at one end and containing a thread of mercury which traps a mass of dry air. Describe how you would calibrate it on the gas scale and use it to determine the boiling point of a liquid known to be about 350 K. Explain how the temperature is calculated from the readings and state any assumptions you make.

(b) A cylinder fitted with a piston which can move without friction contains 0.050 mol of a monatomic ideal gas at a temperature of 27 °C and a pressure of 1.0×10^5 Pa. Calculate:

(i) the volume,
(ii) the internal energy of the gas.

(c) The temperature of the gas in **(b)** is raised to 77 °C, the pressure remaining constant. Calculate:

(i) the change in internal energy,
(ii) the external work done,
(iii) the total heat energy supplied.
(Molar gas constant = 8.3 J mol^{-1} K^{-1}.) [J]

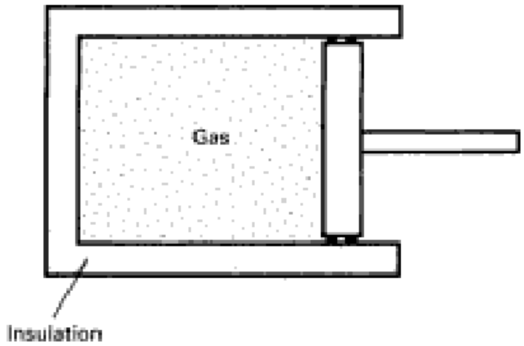
C52 At a temperature of 100 °C and a pressure of 1.01×10^5 Pa, 1.00 kg of steam occupies 1.67 m^3 but the same mass of water occupies only 1.04×10^{-3} m^3. The specific latent heat of vaporization of water at 100 °C is 2.26×10^6 J kg^{-1}. For a system consisting of 1.00 kg of water changing to steam at 100 °C and 1.01×10^5 Pa, find:

(a) the heat supplied to the system,
(b) the work done by the system,
(c) the increase in internal energy of the system. [C]

C53 **(a)** State the *first law of thermodynamics*.
(b) Give *one* practical example of each of the following:

(i) a process in which heat is supplied to a system without causing an increase in temperature,
(ii) a process in which no heat enters or leaves a system but the temperature changes. [C]

C54 Some gas, assumed to behave ideally, is contained within a cylinder which is surrounded by insulation to prevent loss of heat, as shown below.



Initially the volume of gas is 2.9×10^{-4} m^3, its pressure is 1.04×10^5 Pa and its temperature is 314 K.

(a) Use the equation of state for an ideal gas to find the amount, in moles, of gas in the cylinder.

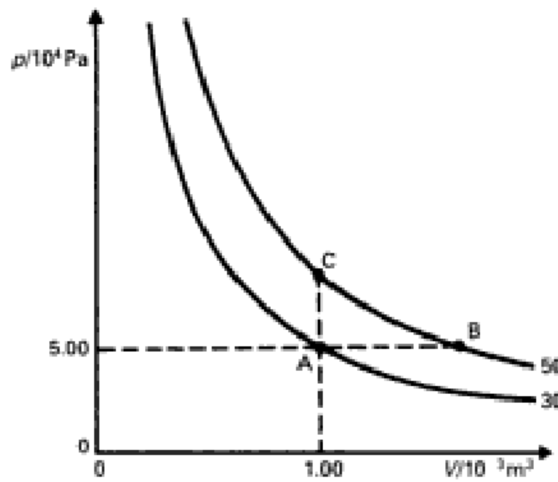
(b) The gas is then compressed to a volume of $2.9 \times 10^{-5}\,m^3$ and its temperature rises to 790 K. Calculate the pressure of the gas after this compression.

(c) The work done on the gas during the compression is 91 J. Use the first law of thermodynamics to find the increase in the internal energy of the gas during the compression.

(d) Explain the meaning of *internal energy*, as applied to this system, and use your result in **(c)** to explain why a rise in the temperature of the gas takes place during the compression.

(Molar gas constant = $8.3\,J\,K^{-1}\,mol^{-1}$.)

C55 The diagram shows curves (not to scale) relating pressure, p, and volume, V, for a fixed mass of an ideal monatomic gas at 300 K and 500 K. The gas is in a container fitted with a piston which can move with negligible friction.



(a) Give the equation of state for n moles of an ideal gas, defining the symbols used. Show by calculation that:

(i) the number of moles of gas in the container is 2.01×10^{-2},

(ii) the volume of the gas at B on the graph is $1.67 \times 10^{-3}\,m^3$.

Molar gas constant, $R = 8.31\,J\,mol^{-1}\,K^{-1}$

(b) The kinetic theory gives the equation $p = \frac{1}{3}\rho\overline{c^2}$ where ρ is the density of the gas.

(i) Explain what is meant by $\overline{c^2}$.

(ii) Use the equation to derive an expression for the total internal energy of one mole of an ideal monatomic gas at kelvin temperature T.

Calculate the total internal energy of the gas in the container at point A on the graph.

(c) State the first law of thermodynamics as applied to a fixed mass of an ideal gas when heat energy is supplied to it so that its temperature rises and it is allowed to expand. Define any symbols used.

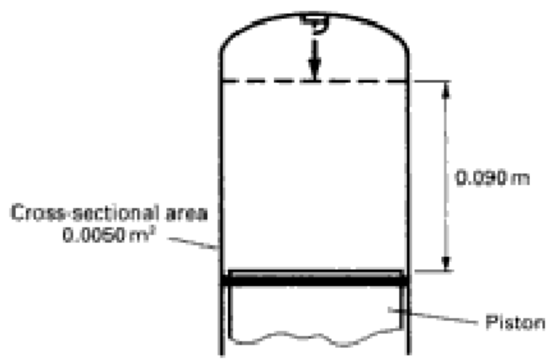
(d) Explain how the first law of thermodynamics applies to the changes represented on the graph by **(i)** A to C and **(ii)** A to B. Calculate the heat energy absorbed in each case. [J, '89]

C56 **(a)** The first law of thermodynamics is represented by the equation

$$Q = \Delta U + W$$

Explain each term in this equation.

(b) An engine (shown below) burns a mixture of petrol vapour and air. When the engine is running it makes 25 power strokes per second and develops a mean power of 18 kW.



Neglecting losses in the engine due to friction and other causes, calculate the work done in each power stroke.

(c) The burning starts when the piston is at the top of its stroke and the resulting high pressure drives the piston downwards through a distance of 0.090 m. The cylinder has a cross-sectional area of $0.0050\,m^2$.

Calculate:

(i) the mean force on the piston head during the power stroke;

(ii) the mean pressure of the hot gas. [O, '92] is now expelled into the atmosphere. Calculate the pressure in A after the second inlet stroke. Calculate the number of strokes of the pump to reduce the pressure in A to 1.0×10^3 Pa. The whole system is maintained at 10 °C throughout the process [O & C]

C76 The specific latent heat of vaporization of a particular liquid at 130 °C and a pressure of 2.60×10^5 Pa is $1.84 \times 10^6\,J\,kg^{-1}$. The specific volume of the liquid under these conditions is $2.00 \times 10^{-3}\,m^3\,kg^{-1}$, and that of the vapour is $5.66 \times 10^{-1}\,m^3\,kg^{-1}$. Calculate:
(a) the work done, and
(b) the increase in internal energy when 1.00 kg of the vapour is formed from the liquid under these conditions.

C77 **(a)** Explain what is meant by a *reversible* change.
(b) State the *first law of thermodynamics*, and discuss the experimental observations on which it is based.
(c) A mass of 0.35 kg of ethanol is vaporized at its boiling point of 78 °C and a pressure of 1.0×10^5 Pa. At this temperature, the specific latent heat of vaporization of ethanol is $0.95 \times 10^6\,J\,kg^{-1}$, and the densities of the liquid and vapour are $790\,kg\,m^{-3}$ and $1.6\,kg\,m^{-3}$ respectively. Calculate:
(i) the work done by the system;
(ii) the change in internal energy of the system.
Explain in molecular terms what happens to the heat supplied to the system. [O]

C78 **(a)** State *four* of the basic assumptions made in developing the simple kinetic theory for an ideal gas.
(b) The theory derives the formula $p = \frac{1}{3}\rho\overline{c^2}$ where p is the pressure of the gas, ρ is the density of the gas and $\overline{c^2}$ is the mean square speed of the molecules. Explain more fully what is meant by $\overline{c^2}$ and explain its significance in relation to the temperature of a gas.
(c) Describe briefly the experiments which Andrews performed on carbon dioxide. (A detailed description of the apparatus is *not* required.)
(i) Draw graphs to show the pressure–volume relationship which Andrews obtained for various temperatures. Indicate on your diagram the various states of the carbon dioxide.
(ii) Use your graphs to explain the meaning of critical temperature. What is its significance in connection with the liquefaction of gases? [AEB, '79]

C79 The model of a gas as a large number of elastic bodies moving about in a random manner is the basic idea of the kinetic theory of gases. In terms of this model, explain:
(a) what is meant by an ideal gas,
(b) how a gas exerts a pressure when enclosed in a container,
(c) why the atmospheric pressure decreases with height,
(d) how the atmosphere, which is not in a container, exerts a pressure at all.
Which of the assumptions made to develop a quantitative expression for the pressure of an ideal gas require to be modified to explain the behaviour of a real gas? Illustrate your answer by considering a p–V isothermal for an ideal gas and a p–V isothermal for a real gas at a temperature below its critical value.

A series of experiments was performed by Andrews to obtain p–V isothermals for carbon dioxide. Sketch a set of p–V isothermals for water, noting particularly any dissimilarities between the curves for water and carbon dioxide. [L]

C80 Explain what is meant by the *critical temperature* of a real gas (such as carbon dioxide), and describe, with the aid of pressure–volume diagrams, the behaviour of a real gas at a temperature **(a)** above the critical temperature, **(b)** equal to the critical temperature, **(c)** below the critical temperature.

Either: Describe, with the aid of a diagram, an experiment by which the departure of a real gas from ideal gas behaviour may be studied.

Or: Explain how van der Waals attempted to produce an equation which would describe the behaviour of a real gas. [S]

C81 What are the conditions under which the equation $pV = RT$ gives a reasonable description of the relationship between the pressure p, the volume V and the temperature T of a real gas?

Sketch p–V isothermals for the gas–liquid states and indicate the region in which $pV = RT$ applies. Indicate the state of the substance in the various regions of the p–V diagram. Mark and explain the significance of the critical isothermal.

Discuss a way in which the equation $pV = RT$ may be modified so that it can be applied more generally. Explain and justify on a molecular basis the additional terms introduced. Discuss the success of this modification. [L]

C82 (a) State the conditions under which the behaviour of a *real* gas will deviate significantly from that expected of an *ideal* gas.

(b) (i) On a pV against p diagram sketch an isotherm for a real gas at the Boyle temperature. On the same set of axes sketch isotherms for temperatures just above and just below the Boyle temperature, labelling the isotherms clearly.

(ii) Explain how the properties of the atoms or molecules of a gas give rise to the shape of the isotherm you have drawn *below* the Boyle temperature.

(c) A quantity of oxygen gas occupies $0.20\,\mathrm{m^3}$ at a temperature of 27 °C and pressure of 10 atmospheres. If it were to be liquefied, what volume of liquid oxygen, density $1.1 \times 10^3\,\mathrm{kg\,m^{-3}}$, would be produced? The oxygen gas in its initial state may be considered to behave as an ideal gas.

What condition must be met before the gas can be liquefied by the increase of pressure alone?

(1 atmosphere $= 1.0 \times 10^5\,\mathrm{Pa}$, relative molecular mass of oxygen = 32, molar gas constant $= 8.3\,\mathrm{J\,mol^{-1}\,K^{-1}}$.) [J, '89]

C83 The equation of state for one mole of a real gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where p is the pressure of the gas, V is the volume and T is the absolute temperature of the gas. Determine the dimensions of **(i)** a, **(ii)** b, **(iii)** R. [W, '90]

C84 (a) Write down van der Waals' equation of state for a real gas and explain how the assumptions of the simple kinetic theory of gases are modified in the derivation of the equation.

(b) Carbon dioxide has a density of $344\,\mathrm{kg\,m^{-3}}$ at its critical pressure $7.5 \times 10^6\,\mathrm{Pa}$ and critical temperature 304 K.

(i) By considering the mass of 1 mol of CO_2 show that the value of the critical volume is $1.28 \times 10^{-4}\,\mathrm{m^3}$.

(ii) Hence calculate the van der Waals' constants a and b given that the critical volume $V_c = 3b$.

(Molar mass of $CO_2 = 4.4 \times 10^{-2}\,\mathrm{kg}$, molar mass constant $= 8.3\,\mathrm{J\,mol^{-1}\,K^{-1}}$.)

(c) Sketch, on the same axes, P–V isotherms for a fixed mass of CO_2 at

(i) the critical temperature,

(ii) a temperature below the critical temperature.

Mark on the same axes points corresponding to the critical volume and critical pressure.

(d) For the isotherm you have drawn in (c) (ii) give the state (or states) of the CO_2 when

(i) it has the critical volume,

(ii) it is at the critical pressure. [J, '91]

C85 (a) Which two assumptions of the kinetic theory of ideal gases are unlikely to be valid for real gases at high pressure?

(b) The equation of state for one mole of an ideal gas is $pV = RT$.

(i) Write down Van der Waals' equation for one mole of a real gas.

(ii) Explain the reasons for the modifications made.

(c) The following data refer to nitrogen gas.

Critical pressure $= 3.4 \times 10^6\,\mathrm{Pa}$

Critical volume $= 9.0 \times 10^{-5}\,\mathrm{m^3\,mol^{-1}}$

Van der Waals' constant, $a = 1.4 \times 10^{-1}\,\mathrm{Pa\,m^6\,mol^{-2}}$

Van der Waals' constant, $b = 3.9 \times 10^{-5}\,\mathrm{m^3\,mol^{-1}}$

(i) Use the given data to calculate the critical temperature of nitrogen.

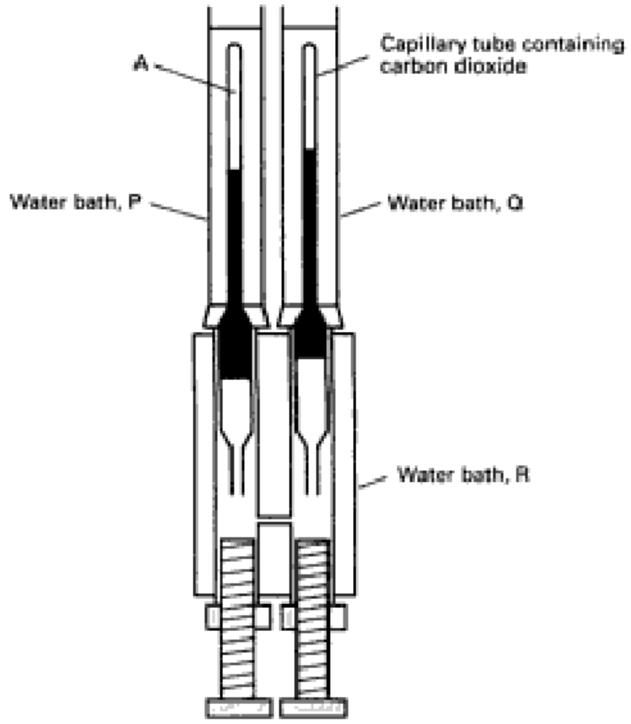
(ii) Calculate the temperature of an ideal gas with the same pressure and volume per mole as given in the data.

(d) Sketch a graph of pV against p for 1 mole of nitrogen at its critical temperature. On the same axes, sketch the graph for 1 mole of an ideal gas at the temperature calculated in (c) (ii).

(c) Explain why some of the assumptions of the kinetic theory of an ideal gas may have to be modified for real gases. Hence explain why a real gas may deviate from Boyle's law. [J]

C90 **(a)** The partly labelled diagram below shows the apparatus used by Andrews in his experiments on carbon dioxide.

State what A is and explain its purpose. Explain why each of the water baths, P, Q and R were used during the experiment.



(b) State the meanings of *critical temperature* and *critical pressure*.

(c) Some carbon dioxide initially at a temperature above its critical temperature is subjected to the following changes.

(i) It is compressed isothermally to a pressure above its critical pressure.

(ii) Then at this pressure it is cooled at constant pressure until the temperature is well below its critical temperature.

(iii) Then at this temperature it is expanded isothermally until all the carbon dioxide is again a gas.

Sketch a graph of pressure against volume to illustrate these changes, and discuss the associated changes of state. [J]

C91 **(a)** In terms of simple kinetic theory, explain qualitatively how a gas exerts a pressure. If the pressure of an ideal gas is given by $p = \frac{1}{3}\rho\overline{c^2}$ where ρ is the density of the gas and $\overline{c^2}$ is the mean square speed of the molecules, explain any change in the pressure that may occur if the gas is:

(i) allowed to expand while the temperature is kept constant,

(ii) heated while the volume is kept constant.

(b) Sketch isothermal curves to show how the pressure of a fixed mass of substance (e.g. carbon dioxide) varies with volume over a wide range of temperature and pressure. Indicate on your sketch the regions where the substance is in *the liquid phase, the saturated vapour phase, the unsaturated vapour phase* and *the gas phase*.

(c) An unsaturated vapour of mass 5×10^{-4} kg and at a temperature of 20 °C is compressed isothermally until, at a volume $V_1 = 9 \times 10^{-5}\,\mathrm{m}^3$ and a pressure 6×10^6 Pa, the vapour first becomes saturated. Further compression of the vapour causes the formation of liquid until, when the volume is V_2, the substance is changed completely to liquid. If V_2 is negligible compared with V_1 and the temperature remains constant throughout the process, calculate:

(i) the work that must be performed during the compression from V_1 to V_2,

(ii) the amount of thermal energy that must be supplied to, or removed from, the substance during the same compression.

(Assume that the specific latent heat of vaporization of the liquid at 20 °C is $1.2 \times 10^5\,\mathrm{J\,kg^{-1}}$.) [AEB, '79]

C92 Sketch a graph to show how the saturated vapour pressure of a liquid varies with temperature. Give a qualitative explanation of the shape of the graph. [C]

C93 In terms of the kinetic theory of matter explain:

(a) what is meant by *saturated vapour* and *saturation vapour pressure*,

(b) how the saturation vapour pressure varies with temperature.

Describe an experiment to measure the saturation vapour pressure of water vapour at 300 K (27 °C). Discuss one practical difficulty

(ii) Calculate the saturation vapour pressure at the temperature of the experiment.

(iii) Calculate the initial pressure of the water vapour. [AEB, '79]

C100 **(a)** **(i)** Explain what is meant by a *saturated vapour*.

(ii) State Dalton's law of partial pressures.

(b) Suggest an experiment to investigate the variation with temperature of the saturated vapour pressure of water vapour over the range of 0 °C to 100 °C. Sketch the apparatus you would use, list the measurements you would make, and describe how the results would be obtained.

(c) The saturated vapour pressure of water at 20 °C is 18 mmHg (= 2.4 kPa). Draw sketch-graphs showing how the pressure, p, of 1 m^3 of water vapour (with a small amount of water present throughout) will vary when:

(i) the vapour is compressed isothermally to a volume of 0.2 m^3,

(ii) the vapour (and the water) are heated at constant volume to the boiling point of water (100 °C).

In each case, show on your graph the final vapour pressure exerted by the water vapour.

(d) In pure atmospheric air it may be assumed that 80% of the molecules present are nitrogen (molar mass = 0.028 kg) and that 20% are oxygen (molar mass = 0.032 kg).

(Take atmospheric pressure as 100 kPa, temperature to be 17 °C, and the molar gas constant to be 8.3 J K^{-1} mol^{-1}.)

Showing all stages in your working, calculate:

(i) the partial pressure exerted by each gas,

(ii) the density of the oxygen present,

(iii) the density of the air. [O]

C101 In an experiment to determine the specific latent heat of vaporization of benzene, it was found that when the electrical power input to the heater was 82 W, 10.0 g of benzene was evaporated in 1 minute; when the power input was reduced to 30 W, the rate of evaporation was 2.0 g per minute. Calculate the specific latent heat of vaporization of benzene. The saturation vapour pressure of benzene is 1.0×10^5 Pa at a temperature of 80 °C; at the same temperature, the saturation vapour pressure of acetone (propanone) is 1.8×10^5 Pa. Which of these two compounds has the higher boiling point, and why?

(Atmospheric pressure = 1.0×10^5 Pa.) [S]

THERMODYNAMICS (Chapter 16)

C102 **(a)** Explain what is meant by the statement that two bodies are in thermal equilibrium.

(b) State the zeroth law of thermodynamics. Explain why it is so called and its relevance in the use of a thermometer to measure temperature.

C103 The specific latent heat of vaporization of a particular liquid at 30 °C and 1.20×10^5 Pa is 3.20×10^5 J kg^{-1}. Under the same conditions of temperature and pressure the specific volume of the liquid is 1.00×10^{-3} m^3 kg^{-1}, and that of its vapour is 4.51×10^{-1} m^3 kg^{-1}. If 5.00 kg of the liquid is vaporized at 30 °C and 1.20×10^5 Pa, what is:

(a) the increase in enthalpy?

(b) the increase in internal energy?

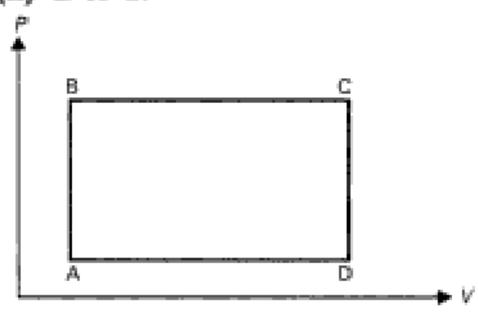
C104 What is the maximum theoretical efficiency of a heat engine which takes in heat at 25.0 °C and rejects it at 10.0 °C?

C105 **(a)** When a system is taken from A to C via B it absorbs 180 J of heat and does 130 J of work. How much heat does the system absorb in going from A to C via D, if it performs 40 J of work in doing so?

(b) The decrease in internal energy in going from D to A is 30 J. Calculate the heat absorbed by the system in going from:

(i) A to D,

(ii) D to C.



Solution

If the initial charge on X is Q_0, then by $Q = VC$ (equation [40.1]),

$$Q_0 = (40) \times (5 \times 10^{-6})$$

i.e. $$Q_0 = 200 \times 10^{-6} \text{ coulombs}$$

The situation after the capacitors have been connected is shown in Fig. 40.9, where Q_X and Q_Y are the final charges on X and Y respectively, and V is the final PD.

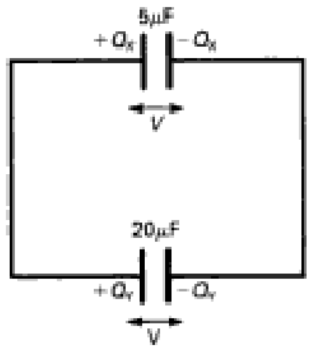


Fig. 40.9 Diagram for Example 40.2

The capacitors are in parallel, and therefore the total capacitance is given (by equation [40.8]) as

$$\text{Total capacitance} = 5 + 20 = 25\mu\text{F}$$

The total charge is unchanged, and therefore

$$\text{Total charge} = 200 \times 10^{-6} \text{ coulombs}$$

Applying $Q = VC$ to the combination gives

$$200 \times 10^{-6} = (V) \times (25 \times 10^{-6})$$

i.e. $$V = 8\text{V}$$

For X:

By $Q = VC$ we have

$$Q_X = (8) \times (5 \times 10^{-6})$$

i.e. $$Q_X = 40 \times 10^{-6} \text{ coulombs}$$

For Y:

Since $Q = VC$ we have

$$Q_Y = (8) \times (20 \times 10^{-6})$$

i.e. $$Q_Y = 160 \times 10^{-6} \text{ coulombs}$$

The energy of a charged capacitor is given by $\frac{1}{2}CV^2$ (equation [40.12]), and therefore

$$\text{Initial energy} = \tfrac{1}{2}(5 \times 10^{-6}) \times (40)^2 = 4 \times 10^{-3}\text{ J}$$

$$\text{Final energy} = \tfrac{1}{2}(5 \times 10^{-6}) \times (8)^2 + \tfrac{1}{2}(20 \times 10^{-6}) \times (8)^2 = 0.8 \times 10^{-3}\text{ J}$$

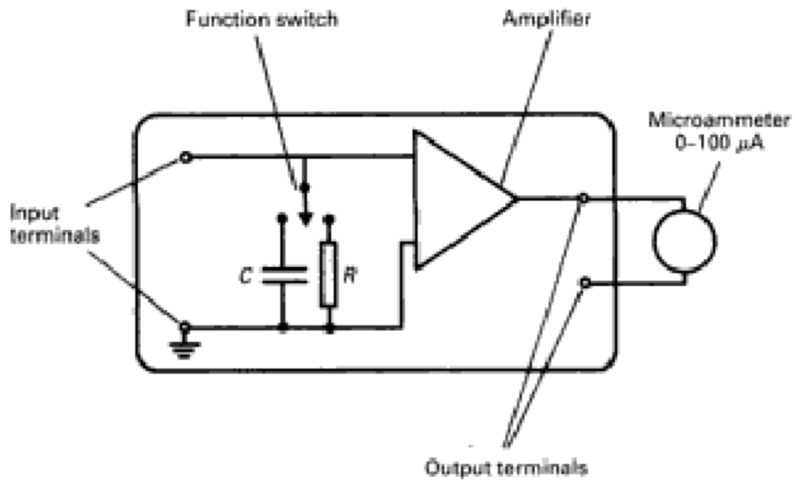
Measurement of the Capacitance of an Isolated Sphere

When the capacitance being measured is that of an isolated sphere M and N in Fig. 40.15(a) are respectively the sphere and an earth.

40.13 THE DC AMPLIFIER

A DC amplifier (Fig. 40.16) is an electronic device which acts as a voltmeter with a very high input resistance (typically $10^{13}\,\Omega$). The instrument is designed to produce an output current which is proportional to the PD across its input terminals. The PD to be measured is applied to the input terminals and produces a reading on a microammeter connected across the output terminals. A typical instrument gives a full scale deflection on a 0–100 μA meter when the PD across the input is 1 V. Before the DC amplifier is used, it must be calibrated by applying a known PD of 1 V to the input and adjusting a sensitivity control (not shown) to give a reading of 100 μA on the microammeter.

Fig. 40.16
DC amplifier



Although it is basically a voltmeter, a DC amplifier can be adapted to measure very small charges ($<10^{-9}$ C) and very small currents ($<10^{-11}$ A).

Measurement of Charge and Capacitance

By use of the function switch a capacitor, C, whose capacitance is known, is connected (internally) across the input. Capacitances of 10^{-7} F, 10^{-8} F and 10^{-9} F are normally available. Suppose that the 10^{-8} F capacitor is selected and that the charge to be measured, Q, is on the plates of a capacitor, C'. (It might equally well be on a proof plane or the dome of a Van de Graaff generator, etc.) In order that it can be measured, Q has to be transferred to the plates of C (the 10^{-8} F capacitor). Accordingly, C' is connected across the input terminals of the DC amplifier so that C and C' are in parallel, in which case, provided the capacitance of C' is much less than that of C, practically the whole of the charge on C' is transferred to C. Suppose that the charge transfer produces a (steady) reading of 20 μA on the microammeter. Since the instrument is calibrated to register 100 μA for an input PD of 1 V, it follows that the PD across C is 0.2 V and that (by $Q = CV$) the charge on C is $10^{-8} \times 0.2 = 2 \times 10^{-9}$ C. As long as the capacitance of C' is

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MAGNETIC EFFECTS OF ELECTRIC CURRENTS

41.1 MAGNETIC FIELDS

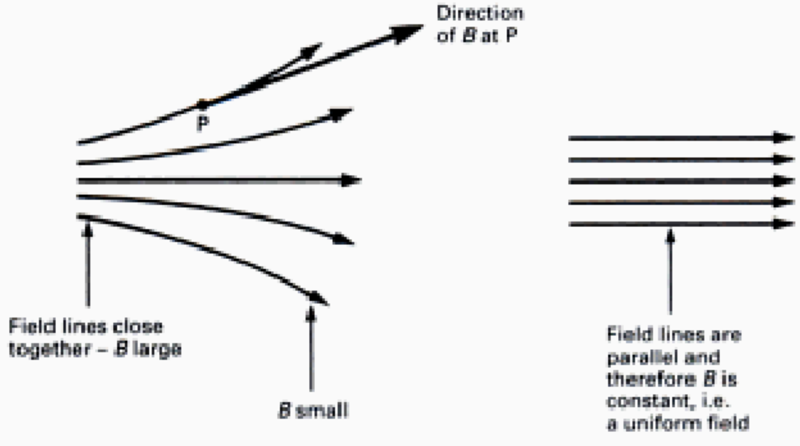
The region around a magnet where magnetic effects can be experienced is called the magnetic field of the magnet. The direction of a field, at a point, is taken to be the direction in which a north magnetic pole would move under the influence of the field if it were placed at that point. The path which such a pole would follow is called a **magnetic field line** (or **line of force**). Field lines are directed away from the north poles of magnets because 'like' poles repel each other.

41.2 MAGNETIC FLUX DENSITY (B)

The magnitude and direction of a magnetic field can be represented by its **magnetic flux density** (B). (This is sometimes called **magnetic induction.**) Flux density is proportional to **magnetic field strength** (H) (see section 41.4). The unit of flux density is the **tesla** (T). (The tesla is defined in section 41.7.)

The direction of the flux density at a point is that of the tangent to the field line at the point. The magnitude of the flux density is high where the number of field lines per unit area is high (see Fig. 41.1).

Fig. 41.1
Field lines and flux density



ANSWERS TO SECTION QUESTIONS

The Examination Boards accept no responsibility whatsoever for the accuracy or method of working in the answers given. These are the sole responsibility of the author.

SECTION A

A1 38 N horizontally, 32 N vertically
A2 36 N at 56° to the horizontal
A3 54.1 N to the right at 20° below the horizontal
A5 33.7°, 3.25×10^{-6} J
A7 **(b)** **(ii)** 11.3° **(c)** 17.0°
A8 $8.75\,\mathrm{m\,s^{-1}}$, **(a)** 385 N **(b)** 770 N
A9 $17.2\,\mathrm{m\,s^{-1}}$
A10 **(a)** $1.92 \times 10^{-2}\,\mathrm{kg\,s^{-1}}$ **(b)** $2.22 \times 10^{-2}\,\mathrm{m\,s^{-2}}$
A11 0.051 kg
A12 5 s, $50\,\mathrm{m\,s^{-1}}$, $11.25\,\mathrm{m\,s^{-1}}$
A13 **(a)** 45 m **(b)** 63.4°
A15 **(a)** 15 s **(b)** 1181 m
A17 80.0 m, 2.90×10^{5} J
A24 $1\,\mathrm{m\,s^{-1}}$, 2.25 J
A25 $0.087\,\mathrm{m\,s^{-1}}$
A26 2/3
A27 **(b)** **(i)** $0.60\,\mathrm{m\,s^{-2}}$ **(ii)** 7508 m **(iii)** 1.2×10^{4} N **(iv)** 50 kg
A28 **(a)** $0.60\,\mathrm{m\,s^{-2}}$ **(b)** 225 J (before), 81.4 J (after)
A30 0.75
A31 $2.57 \times 10^{6}\,\mathrm{m\,s^{-1}}$ at 43.2° to the original direction of the proton
A33 **(a)** $1.6 \times 10^{2}\,\mathrm{m\,s^{-1}}$ **(c)** 0.027°
A34 **(b)** **(i)** 52 N
A36 $\sqrt{2}$ kg (A), 1 kg (C)
A37 23.1 N (CD), 11.6 N (AB)
A38 520 N (X), 380 N (Y)
A39 16 cm
A40 2.0 cm
A41 **(a)** 506 N at 90° to the wall **(b)** 1121 N at 63° to the ground
A43 **(b)** **(i)** 410 N
A44 **(d)** 500 N **(e)** 15 kW
A45 **(b)** **(ii)** 550 N **(iii)** 2.75×10^{6} Pa
A46 9×10^{-15} J
A47 **(a)** $10\,\mathrm{m\,s^{-1}}$ **(b)** 24 J, $12.65\,\mathrm{m\,s^{-1}}$
A48 **(a)** $300\,\mathrm{m\,s^{-1}}$ **(b)** 89.64 J
A49 $319.4\,\mathrm{m\,s^{-1}}$
A50 **(a)** $0.15\,\mathrm{m\,s^{-1}}$ (before), $0.060\,\mathrm{m\,s^{-1}}$ (after) **(c)** 4.6×10^{-3} J (before), 1.8×10^{-4} J (after)
A51 2.5×10^{5} N, 1.54×10^{5} N
A54 **(a)** 4.3×10^{-2} J **(b)** $0.66\,\mathrm{m\,s^{-1}}$ **(c)** $0.44\,\mathrm{m\,s^{-1}}$
A55 $5.0\,\mathrm{m\,s^{-1}}$
A56 $2\,\mathrm{m\,s^{-1}}$ (A), $4\,\mathrm{m\,s^{-1}}$ (B), 0.3 m from A
A57 $2\,\mathrm{m\,s^{-1}}$ (P), $6\,\mathrm{m\,s^{-1}}$ (Q), $15\,\mathrm{m\,s^{-1}}$, 60 750 W
A58 **(a)** 125 m **(b)** $6\frac{2}{3}$ N
A59 **(a)** 45 m **(b)** 400 N
A60 **(a)** **(i)** 250 m **(ii)** 4.0×10^{3} N **(iii)** $2.0 \times 10^{4}\,\mathrm{kg\,m\,s^{-1}}$, 2.0×10^{3} N **(b)** $8.0\,\mathrm{m\,s^{-1}}$, 2.0×10^{5} J, 8.0×10^{4} J
A61 **(a)** $4.0\,\mathrm{m\,s^{-2}}$ **(b)** $7.1\,\mathrm{m\,s^{-1}}$
A63 **(c)** **(i)** 5.0 N **(ii)** 5.0 N **(iii)** $3.6\,\mathrm{m\,s^{-1}}$
A64 **(a)** $3.0\,\mathrm{m\,s^{-2}}$ **(b)** 45 kJ **(c)** 30 kW
A65 **(a)** $1.1 \times 10^{2}\,\mathrm{m\,s^{-1}}$ **(b)** 0.16 m
A67 1.85×10^{3} N, $3.3\,\mathrm{m\,s^{-2}}$
A68 **(a)** $2.0\,\mathrm{m\,s^{-2}}$ **(b)** $1.2 \times 10^{4}\,\mathrm{kg\,m\,s^{-1}}$ **(c)** 2.7×10^{5} J
A69 **(a)** 1.4×10^{4} W **(b)** 7.0×10^{3} W, 3.0×10^{4} W
A70 **(a)** **(i)** 100 N **(ii)** 10 J **(b)** 57.7 N **(c)** 57.7 N
A71 **(a)** $2.5 \times 10^{4}\,\mathrm{kg\,s^{-1}}$ **(b)** $7.1 \times 10^{5}\,\mathrm{J\,s^{-1}}$ **(c)** 2025
A73 90 N
A74 **(a)** $0.13\,\mathrm{rad\,s^{-1}}$ **(b)** 2.7×10^{5} N towards centre of circle
A75 $0.68\,\mathrm{rev\,s^{-1}}$
A77 $31\,\mathrm{rev\,min^{-1}}$
A79 **(a)** **(i)** $3.2\,\mathrm{m\,s^{-1}}$ **(ii)** $2.2\,\mathrm{m\,s^{-1}}$ **(b)** 0.20 N
A80 **(a)** 9.0×10^{-2} J **(b)** **(i)** $2.7\,\mathrm{m\,s^{-1}}$ **(ii)** $1.8\,\mathrm{m\,s^{-1}}$ **(iii)** 0.55 N
A81 **(a)** 8 N **(b)** 8 N
A82 **(a)** 0.01 J **(b)** $2\,\mathrm{m\,s^{-1}}$ **(c)** $20\,\mathrm{m\,s^{-2}}$ **(d)** 0.15 N
A83 **(b)** 18.5 N **(c)** 12.25 m
A85 **(b)** 157 J
A86 $8.9\,\mathrm{rev\,s^{-1}}$
A88 **(b)** **(i)** 2.0×10^{7} J **(ii)** 10 km
A89 58.9 rev, 235.6 s
A90 **(b)** **(i)** 2.9×10^{7} J, 29 km
A91 **(c)** $1.07\,\mathrm{rad\,s^{-1}}$, $0.161\,\mathrm{m\,s^{-1}}$
A92 **(a)** $49\,\mathrm{rad\,s^{-2}}$ **(b)** 5.4×10^{-2} N m
A93 **(c)** **(i)** $2.0 \times 10^{2}\,\mathrm{rad\,s^{-1}}$ **(iii)** 80 N m
A95 **(a)** $0.08\,\mathrm{m\,s^{-1}}$ **(b)** $\pi/2$ s
A96 **(b)** **(i)** $0.10\,\mathrm{m\,s^{-1}}$ **(ii)** $0.25\,\mathrm{m\,s^{-2}}$ **(iii)** 1.5×10^{-3} J
A97 **(a)** $0.026\,\mathrm{m\,s^{-1}}$ **(b)** $0.014\,\mathrm{m\,s^{-2}}$; P = 0.05, Q = $\pi/6$
A98 $168\,\mathrm{m\,s^{-1}}$
A99 $0.42\,\mathrm{m\,s^{-1}}$
A101 0.2 J
A102 **(d)** 9.63×10^{11} Hz
A103 **(a)** $0.25\,\mathrm{m\,s^{-1}}$, 0.016 J
A105 **(b)** 0.12 J
A106 **(a)** **(i)** 0.05 m **(ii)** 1.0 s, $1.8\,\mathrm{m\,s^{-2}}$
A107 $a = 15$ mm, $\omega = 2\pi/3\,\mathrm{s^{-1}}$, $\varepsilon = \pi/2$ rad
A108 **(c)** **(i)** 3.38×10^{-3} J **(ii)** 3.45 N, 2.55 N
A109 **(d)** 139 N
A112 **(a)** 0.2π s **(b)** 1.25×10^{-4} J
A113 **(a)** 1.2 N **(b)** **(i)** 2.9 Hz **(ii)** $3.3\,\mathrm{m\,s^{-2}}$
A115 16 Hz
A116 6.3 cm
A117 **(c)** **(i)** 100 kg **(ii)** 5.03 Hz
A118 **(a)** **(i)** $3.1\,\mathrm{rad\,s^{-1}}$ **(ii)** $0.30\,\mathrm{m\,s^{-2}}$
A119 **(b)** **(i)** 1.0 N **(ii)** 0.79 s, 0.04 J
A120 889 N
A121 14.4 N, 24.5 h
A122 **(d)** 3.91×10^{-4} N **(e)** 2.65×10^{-4} N **(f)** 0.001%
A125 6.0×10^{24} kg
A127 9.0 m
A128 5×10^{24} kg
A130 **(a)** **(ii)** $-5.36 \times 10^{9}\,\mathrm{J\,kg^{-1}}$

(b) (i) 2.68×10^{28} N (ii) 2.59×10^4 m s^{-1}
(iii) 2.43×10^7 s
A131 2×10^{30} kg
A133 3.37×10^5
A134 1.1×10^3 kg m^{-3}
A136 0.35%, 27 days
A138 5.3×10^6 J; 3.9×10^8 m, 2.7×10^{-3} m s^{-2}
A141 (b) 6.1 km
A143 (a) -6.3×10^7 J kg^{-1} (b) -8.9×10^8 J kg^{-1}
A144 (d) 1.07×10^{11} J
A145 (c) (i) 1.26×10^7 J kg^{-1} (ii) 1.26×10^{14} J
A146 (b) 5.4×10^{26} kg (c) 6.9×10^{-11} N m^2 kg^{-2}
A147 (a) (i) -40.0 MJ kg^{-1}, -26.7 MJ kg^{-1}
(ii) 5.33×10^{11} J, 1.07×10^5 N (iii) 11 km s^{-1}
A148 (b) (i) 4.4×10^8 J, 2.1 km s^{-1}
A149 (a) (iv) 7.9×10^3 m s^{-1}, 5.1×10^3 s
(b) (i) 4.2×10^7 m (ii) 0.28 s (iii) 81.3°

SECTION B

B2 (c) 500 N m^{-1} (d) 1.22×10^{14} Hz
B4 2×10^{-21} J, 4×10^4 J kg^{-1}
B5 (a) 4.5×10^{-2} eV
B6 1.3×10^{-20} J
B7 1.25×10^{-28} m^3, 5×10^{-10} m
B8 (a) 3.1×10^{-10} m (b) 2.8×10^{-21} J
B9 (a) 3.7×10^3 kg m^{-3} (b) 3.5×10^3 kg m^{-3}
B10 1.6 kg (X), 1.2 kg (Y)
B11 1.81×10^5 Pa
B12 36.8 cm
B13 3.2
B14 (a) 4.7×10^3 kg m^{-3}
B15 15 N
B16 16 g
B17 9.0×10^2 kg m^{-3}
B18 (a) 600 kg (b) 0.91 m s^{-2}
B19 (a) 4.0 mm
B22 1.6×10^{15}
B24 $\alpha = -\frac{3}{2}, \beta = \frac{1}{2}, \gamma = -\frac{1}{2}$
B25 2.74×10^{-5} J, 0.35 m s^{-1}
B26 5 cm
B27 761.9 mmHg
B28 30 mm
B29 7.5×10^{-2} N m^{-1}. The water would rise to only 4.3 cm
B30 (b) 7 mm
B32 (c) 4.5×10^{-4} J, $10\sqrt{3}$ m s^{-1}
B33 (b) (i) 7.3×10^{-2} N m^{-1}
B34 (a) 1 mm (b) 5.5×10^{-2} J
B35 2.6×10^{-4} m^2
B36 7×10^6 N m^{-2}, 0.14 J
B37 (c) 0.45 m^2
B38 1.5×10^8 Pa
B39 (a) 8:1 (b) $D/9$
B40 (c) (ii) 1.2×10^{-2} J
B41 6.25×10^{11} N m^{-2}
B42 (b) (i) 50 N (ii) 1.8×10^{-3} m, 4.4×10^{-2} J
(iii) 0.85 mm (iv) 0.084 m from B
B43 (b) (i) 2.0×10^{11} Pa (ii) 9.6×10^2 N
(iii) 3.6×10^6 J m^{-3}
B44 5.1×10^6 N m^{-2}, 0.63 s
B45 (a) 50 N (b) 0.5 J
B47 5 mm
B48 (a) 3:2 (Cu:Fe) (b) 6.0 mm, 4.0 mm (c) 780 N
B49 (a) 589 N, 589 N (b) 10.046 m (c) 25.2 J
B51 (a) 3.0×10^{-4} (b) 6.0×10^7 Pa (c) 1.44×10^3 J,
1.56×10^7 J
B52 (a) 1020 °C (b) 1.8×10^8 Pa
B53 (c) (i) 2.5×10^8 Pa (ii) 0.10 J (iii) 0.20 J
(v) 1.9×10^{11} Pa
B55 (b) (i) 1.1 mm (ii) 1.7×10^{-2} J
B56 2.8×10^{-10} m (a) 2.0×10^8 Pa (b) 3.0 J
(c) 1.4×10^{10} Pa, 3.0 mm
B59 (b) 1.9×10^{11} Pa
B61 (a) 0.1 m (A), 0.2 m (B) (b) 2 N m^{-1}
B62 (c) (ii) 5.0 mm
B63 (c) (i) 2.0 m s^{-1} (ii) 3.9×10^{-2} kg s^{-1}
B64 (c) (i) 2.8 m s^{-1} (ii) 5.7×10^{-3} m^3 s^{-1}
B65 2.97×10^4 N
B66 (a) 2.0 m s^{-1} (b) 0.8 m. The second hole must be 20 cm above the base of the tank
B67 (b) (ii) 1.1×10^3 Pa
B68 (a) 133 m s^{-1} (c) 120 m s^{-1}, 1.82×10^5 Pa
B71 (c) 15.8 m s^{-1}
B74 7.6 cm
B75 6000 N m^{-2}
B76 (b) (iii) 386 mm
B80 (a) (ii) 5.0×10^{-7} m (b) (iii) 169 m s^{-1}
B82 2, 2.19
B83 (b) 24 m s^{-1}
B84 $A = 15$ cm^2 g^{-1} s^{-1}, $B = 15$ cm^{-1} s^{-1}, radius = 0.2 cm
B85 (b) (ii) 0.45 N s m^{-2}
B86 (b) (i) 5.0×10^{-4} m^3

SECTION C

C4 290 K
C6 (a) 37 °C (b) 50 °C
C7 16.6 °C, 17.0 °C
C8 49.04 °C
C9 (b) (ii) 23 °C (approx) (iii) 0 °C, 100 °C
C10 (f) 364.21 K; 367.86 K
C11 0.45 W
C12 279 J kg^{-1} K^{-1}
C13 5.7 V
C14 6000 J kg^{-1} K^{-1}, 10%
C15 1700 J kg^{-1} K^{-1}
C16 (d) 2.91×10^3 J kg^{-1} K^{-1}, 12.3 W
C17 4
C18 (b) (iii) 57.6 °C
C19 0.67 kg
C20 (a) 250 W (b) 2.50×10^4 J kg^{-1} (c) 167 J kg^{-1} K^{-1}
C21 0.1225 kg
C22 1.67×10^3 J kg^{-1} K^{-1}
C23 88%
C24 0.0396 kg
C25 7.4%
C26 (c) 1.6×10^2 J, 7.0%
C27 2.4×10^4 J kg^{-1}, 2.0×10^3 J kg^{-1} K^{-1}
C28 (a) (i) 3.6×10^4 J (ii) 15.8 g
C29 1.1×10^5 Pa
C30 30 m
C31 745 mm
C32 (b) 4.6 mol (c) 517 m s^{-1}
C33 (a) 1.33×10^5 Pa (b) 2 g in A, 1 g in B
C34 (a) 2.1×10^3 mol (b) 60 kg
C35 40
C36 (b) (i) 2.3 kg (ii) 1.4 kg (iii) 3.1×10^{17} s^{-1}
C37 0.33 kg
C38 (a) 8.3 J K^{-1} mol^{-1} (b) 1.4×10^{-23} J K^{-1}
(c) 6.3×10^{-21} J
C39 (a) 0.32 g (b) 4.8×10^{22} (c) 1.4×10^3 m s^{-1}
C40 $1.81(2) \times 10^5$ J, $1.85(5) \times 10^5$ J, 6.1×10^2 J kg^{-1} K^{-1}

C42 **(d)** $552\,\mathrm{m\,s^{-1}}$
C43 **(b)** **(i)** $2.57 \times 10^{-4}\,\mathrm{kg}$ **(ii)** 240 J
C44 **(a)** **(i)** 40.2 **(ii)** 14.1 kg **(iii)** $141\,\mathrm{kg\,m^{-3}}$ **(iv)** $146\,\mathrm{m\,s^{-1}}$
C46 **(b)** **(i)** 6.0×10^{-3} **(ii)** 3.6×10^{21} **(iii)** $483\,\mathrm{m\,s^{-1}}$
C47 **(a)** **(i)** 2.4×10^{22} **(ii)** 150 J
C48 **(a)** 1.07:1 **(b)** 4:1 **(c)** 1.15:1
C51 **(b)** **(i)** $1.2 \times 10^{-3}\,\mathrm{m^3}$ **(ii)** $1.9 \times 10^2\,\mathrm{J}$
(c) **(i)** 31 J **(ii)** 21 J **(iii)** 52 J
C52 **(a)** $2.26 \times 10^6\,\mathrm{J}$ **(b)** $1.69 \times 10^5\,\mathrm{J}$ **(c)** $2.09 \times 10^6\,\mathrm{J}$
C54 **(a)** 1.2×10^{-2} **(b)** $2.6 \times 10^6\,\mathrm{Pa}$ **(c)** 91 J
C55 **(b)** **(ii)** 75.0 J **(d)** **(i)** 50.0 J **(ii)** 83.3 J
C56 **(b)** $7.2 \times 10^2\,\mathrm{J}$ **(c)** **(i)** 8.0 kN **(ii)** 1.6 MPa
C59 **(d)** **(i)** $4.75 \times 10^{-3}\,\mathrm{m^3}$ **(ii)** 984 K
(iii) **(1)** $29.1\,\mathrm{J\,K^{-1}\,mol^{-1}}$ **(2)** $4.92 \times 10^{-4}\,\mathrm{m^3}$
(3) $1.76 \times 10^3\,\mathrm{J}$ **(4)** $4.39 \times 10^3\,\mathrm{J}$
C60 $1.44 \times 10^4\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $1.03 \times 10^4\,\mathrm{J\,kg^{-1}\,K^{-1}}$
C61 **(a)** 3000 K **(b)** $600\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $783\,\mathrm{J\,kg^{-1}}$; $268\,\mathrm{m\,s^{-1}}$
C63 $1.07 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$
C67 **(a)** $2.4 \times 10^3\,\mathrm{J}$ **(b)** **(i)** $1.1 \times 10^5\,\mathrm{Pa}$
(ii) $8.8 \times 10^4\,\mathrm{Pa}$
C68 **(c)** **(i)** $4.0 \times 10^2\,\mathrm{N}$ **(ii)** $5.7 \times 10^2\,\mathrm{N}$
(iii) $6.6 \times 10^2\,\mathrm{N}$
C69 **(b)** **(iii)** 150 K
C70 $1.7 \times 10^{-2}\,\mathrm{kg}$, 60 K, $7.3 \times 10^2\,\mathrm{J\,kg^{-1}\,K^{-1}}$
C71 **(b)** $3.76 \times 10^3\,\mathrm{m\,s^{-1}}$
C72 **(a)** $1250\,\mathrm{cm^3}$ **(b)** $1.38 \times 10^5\,\mathrm{Pa}$
C73 **(b)** $7.44 \times 10^{-3}\,\mathrm{m^3}$ **(d)** 1.66
C74 **(b)** **(i)** 301.2 K at A and C, 150.6 K at B **(ii)** 4375 J
(iii) 3125 J **(iv)** 625 J **(v)** 625 J
C75 **(c)** $2.4(5) \times 10^5\,\mathrm{Pa}$ **(d)** $2.5 \times 10^5\,\mathrm{Pa}$, $2.1 \times 10^5\,\mathrm{Pa}$, 31
C76 **(a)** $1.47 \times 10^5\,\mathrm{J}$ **(b)** $1.69 \times 10^6\,\mathrm{J}$
C77 **(c)** **(i)** $2.2 \times 10^4\,\mathrm{J}$ **(ii)** $3.1 \times 10^5\,\mathrm{J}$
C82 **(c)** $2.3 \times 10^{-3}\,\mathrm{m^3}$
C84 **(b)** **(ii)** $a = 0.362\,\mathrm{N\,m^4\,mol^{-2}}$, $b = 4.27 \times 10^{-5}\,\mathrm{m^3}$
C85 **(c)** **(i)** 127 K, 36.9 K **(d)** $1.05 \times 10^3\,\mathrm{J}$, $3.06 \times 10^2\,\mathrm{J}$
C86 **(c)** **(i)** $9.9 \times 10^{-2}\,\mathrm{kg}$ **(ii)** 1.9×10^{24}
(iii) $4.7 \times 10^2\,\mathrm{m\,s^{-1}}$
C87 **(c)** **(i)** 1024 kPa **(ii)** 533 K
C91 **(c)** **(i)** 540 J **(ii)** 60 J
C94 $1.01 \times 10^5\,\mathrm{Pa}$
C96 $9 \times 10^2\,\mathrm{Pa}$
C97 $1.7 \times 10^4\,\mathrm{N\,m^{-2}}$, $3.0 \times 10^3\,\mathrm{N\,m^{-2}}$, 82%
C99 **(c)** **(ii)** 2.0 kPa **(iii)** 1.5 kPa
C100 **(d)** **(i)** 80 kPa (nitrogen), 20 kPa (oxygen)
(ii) $0.27\,\mathrm{kg\,m^{-3}}$ **(iii)** $1.2\,\mathrm{kg\,m^{-3}}$
C101 $3.9 \times 10^5\,\mathrm{J\,kg^{-1}}$
C103 **(a)** $1.60 \times 10^6\,\mathrm{J}$ **(b)** $1.33 \times 10^6\,\mathrm{J}$
C104 5.03%
C105 **(a)** 90 J **(b)** **(i)** 70 J **(ii)** 20 J
C106 $1.22 \times 10^3\,\mathrm{J\,K^{-1}}$
C107 **(a)** 0 **(b)** $40\,\mathrm{J\,K^{-1}}$
C108 **(c)** 19.6%
C110 **(a)** 0 **(b)** $4.2 \times 10^3\,\mathrm{J}$ **(c)** $-3.0 \times 10^3\,\mathrm{J}$
C111 $3.0 \times 10^2\,\mathrm{^\circ C\,m^{-1}}$ (copper), $5.5 \times 10^2\,\mathrm{^\circ C\,m^{-1}}$ (aluminium)
C112 0.2 °C, $2.4 \times 10^6\,\mathrm{J\,kg^{-1}}$
C113 $1.9 \times 10^3\,\mathrm{W}$
C114 240 W, 232 min
C115 48.1 W, 2.16 g
C116 **(d)** 2.6 °C
C117 6400 W, 66:1
C120 102.5 °C
C121 **(a)** 4.0 kW
C122 **(a)** 0.02 **(b)** 0.008
C123 94%
C128 **(a)** **(ii)** $1.5 \times 10^2\,\mathrm{^\circ C\,m^{-1}}$, 29 W
C129 **(b)** 10 °C **(c)** **(ii)** $3.0 \times 10^{-3}\,\mathrm{K\,W^{-1}}$, $1.0 \times 10^3\,\mathrm{W}$, 9.8 W
C130 $6.5 \times 10^2\,\mathrm{J\,s^{-1}\,m^{-2}}$, $2.0 \times 10^{-3}\,\mathrm{mm\,s^{-1}}$
C132 **(e)** 9.0 kW **(f)** 88 W
C133 **(b)** **(ii)** 507 °C
C134 $6.78 \times 10^{-2}\,\mathrm{K\,s^{-1}}$
C137 1105 K
C138 1073 K
C139 2021 K
C141 $19\,\mathrm{W\,m^{-1}}$
C143 **(a)** 0.19 nm **(b)** 1.07 mm
C144 378 K

SECTION D

D1 **(a)** 19.5° **(b)** 10.5°
D2 18.6°
D3 38.5°
D4 26.4°
D5 1.6
D6 40.2°
D8 27.9°
D9 **(b)** **(i)** 60°4′ **(ii)** 48°27′
D10 39.6°
D11 37.2°, 48.6°
D12 32.77°, 41.05°
D13 **(b)** 1.50
D14 3.17°
D15 **(a)** 1.5 cm high, 30 cm from lens on opposite side from object
(b) 1.0 cm high, 20 cm from lens on same side as object
D16 20 cm; 40 cm from lens
D18 100 cm
D19 30 cm above lens, diameter = 6 mm
D20 5 mm, 5 mm, 25.0 mm
D21 100 cm from diverging lens on same side as object
D23 2.6 mm
D24 100 mm, 95 mm
D25 0.2 m (diverging)
D27 **(a)** 70 cm **(b)** 135 cm
D30 **(b)** 100 cm
D31 **(a)** 6 cm behind the mirror, 1.5× **(b)** 3.3 cm behind the mirror, 0.83×
D32 40 cm
D33 **(a)** $41\frac{2}{3}$ mm to 50 mm **(b)** 6 to 5
D36 16.0 cm from the second lens and between the lenses; 27.8 cm away on the side remote from the object
D37 **(a)** 41.7 mm **(b)** 90 **(c)** 10.7 mm
D38 **(b)** 110 mm
D40 **(c)** 22.7 mm, 37.5
D42 **(a)** 550 mm **(d)** 10 **(e)** 55 mm from eyepiece lens
D44 400 mm (objective), 50 mm (eyepiece)
D45 **(b)** 60 mm
D46 4.0 cm
D47 112.4 cm, 15.6
D48 **(a)** 160 **(b)** 25.2 mm **(c)** 0.938 mm
D49 **(c)** **(i)** 1.84 cm
D50 0.18(2) cm; 20 cm from second lens on the same side as the first lens; 0.91 cm
D51 $9.1 \times 10^{-3}\,\mathrm{rad}$
D52 **(a)** 2.1 cm **(b)** 112.5 cm
D53 100

D54 (b) (ii) $104\frac{1}{6}$ cm, 24 (iii) 5.3 cm behind eyepiece (c) (i) $106\frac{2}{3}$ cm

D55 (a) 0.029 cm (b) 0.55 cm (c) 0.11 rad

D58 (a) (i) 50 mm (b) 2.6 mm away from the film

D59 0.065 s

D60 (c) 1.75 cm; 11, 32 ms

D61 $4\frac{16}{21}$ cm, $\frac{5}{21}$ cm, 100 cm (diverging)

D62 $f = 200$ cm diverging

D63 $f = 30$ cm converging

D64 $66\frac{2}{3}$ cm

D65 (a) $f = 400$ cm diverging (b) $44\frac{4}{9}$ cm to infinity

D66 (a) $42\frac{6}{7}$ cm converging (b) 40 cm

D67 (b) (i) $-2.0\ m^{-1}$, 0.28 m

D68 200 cm (diverging), 50 cm (converging)

D69 (b) 3.1×10^4 m, 300 rev s^{-1}

D71 $3.0(2) \times 10^8\ m\ s^{-1}$

SECTION E

E1 6.0×10^4 m

E4 (a) $3.3 \times 10^2\ m\ s^{-1}$ (b) $6.6 \times 10^{-4}\ m\ s^{-1}$

E5 (a) (i) 7.5° (light), 45.6° (sound)

E12 342 $m\ s^{-1}$

E13 (a) 1.5 m

E14 5.0×10^{-7} m

E15 (b) 1.79×10^{-3} rad, 0.064 mm

E17 1.5×10^{-4} rad

E18 (b) 0.28 mm (c) (i) 0.23 mm

E19 6.43×10^{-7} m

E20 (b) 711 nm, 427 nm

E21 (b) 0.021°

E23 1.5×10^{-5} m

E25 (b) 9.82×10^{-8} m

E26 2.7 mm

E28 (b) (iii) 1.26×10^{-5} m

E29 (a) 25 cm from card (b) 20 cm

E31 2.4×10^{-6} m

E32 4.60×10^{-7} m (violet), 6.90×10^{-7} m (red); 66.9°

E33 640 nm, 480 nm; 28.7°

E34 (a) 0.96° (b) 0.34 cm

E36 600 nm, 2.85×10^5, 43.2°

E37 (c) 1.04 cm

E38 4.34×10^{-7} m

E39 (a) $5.5 \times 10^5\ m^{-1}$ (b) (i) 6.2×10^{-7} m (ii) 4

E40 (a) 13.6° (b) 70.5°

E44 53.1°

E45 (a) 1.6 (b) 32°

E55 (a) 1.83 s (b) 19%

E56 (a) $3.2 \times 10^2\ m\ s^{-1}$ (b) $1.6 \times 10^{-3}\ m\ s^{-1}$, 0.13 m

E57 $3\lambda/8$, 300 Hz

E58 $1.4l$

E59 (b) (i) 91.3 Hz (ii) 87.8 Hz

E60 (a) $1.5 \times 10^2\ m\ s^{-1}$ (b) $1.8 \times 10^8\ N\ m^{-2}$ (c) $1.8 \times 10^{11}\ N\ m^{-2}$

E61 (a) $f - 2.0$ (b) $f - 0.7$; 0.76%

E63 (d) 71.5 cm

E64 (a) 300 $m\ s^{-1}$ (b) 9.0 kg

E65 (a) 250 Hz (b) 1.4×10^2 N

E66 1.7×10^2 Hz

E67 (b) (i) 514 Hz (ii) 512 mm

E68 1.0×10^2 Hz

E69 336 $m\ s^{-1}$

E70 (a) 320 $m\ s^{-1}$ (b) 10 cm (c) 40 cm (d) 1200 Hz

E71 5.1 Hz

E73 0.83 m

E74 (b) (ii) 340 $m\ s^{-1}$ (iii) 34.6 cm

E76 336 $m\ s^{-1}$

E77 348 $m\ s^{-1}$, 8.26 mm

E78 (a) (i) 340 $m\ s^{-1}$ (ii) 696 mm below top of tube

E79 (c) (ii) 341 $m\ s^{-1}$, 12 mm (iii) 207 Hz

E82 (a) 1.75 m, 16 Hz

E84 (b) 0.38 m, 4.1 N

E85 0.066 mm

E86 (a) (i) 1700 Hz (ii) 0.050 m

E87 330 $m\ s^{-1}$, 579 mm

E90 362 $m\ s^{-1}$

E94 514 Hz, 545 Hz

E95 425 Hz

E96 $1.11f$, $1.10f$

E97 (b) (i) ±0.003 43 nm (ii) zero

E98 (c) 2.2×10^{-8} m

E99 $1.02 \times 10^5\ m\ s^{-1}$

E100 (b) 37.8 $m\ s^{-1}$, 302 m

E101 (b) 92.1 kHz

E103 (a) (ii) 1.9%

E104 (b) 57.6 Hz (c) 2.7 Hz

E105 (b) $5.0 \times 10^4\ m\ s^{-1}$ (c) 2.8×10^{10} m

SECTION F

F1 (a) 5.0 A (b) 2.0 V (c) 10 V (d) 12 V

F2 $4.8 \times 10^{-11}\ \Omega$ m

F3 11.3 V

F4 0.59 A

F6 (b) $9.5 \times 10^{-3}\ m\ s^{-1}$

F7 (a) 3 Ω (b) 4 Ω

F8 0.25 A

F9 (a) (i) 0.067 A (ii) 0.033 A (iii) 4.0 V (b) (i) zero (ii) 0.054 A (iii) 2.16 V

F10 8.0 V, 50 Ω

F11 0.5 A

F12 60 V, 48 V

F13 40 Ω, 40 Ω

F14 5 V

F15 (a) 9995 Ω in series (b) $2.5 \times 10^{-3}\ \Omega$ in parallel

F16 (a) 0.05 A (b) 0.0125 A (c) 0.5 V

F17 14 950 Ω in series

F18 60.4 V

F19 9.6 V

F20 (a) 360 Ω (b) 0.96 V

F21 (b) (ii) 3.3 V (1.8 kΩ), 8.7 V (4.7 kΩ) (c) (ii) 10.3 kΩ (d) 3.9 V, 8.1 V

F22 2.29 V

F23 (b) 2.0 mA, 2.48 MΩ

F24 (b) (i) 4.0 A (ii) 2.5 Ω (iii) 3.5 Ω

F25 (b) 1.18 Ω (c) 18.3 V (V_1), 13.2 V (V_2)

F26 24.5 Ω

F27 (b) (ii) 7.0 V

F29 2.0 A

F30 (a) 2.0 Ω (b) 1.44 W

F31 (a) 120 Ω (b) 25.3 W

F32 (b) (i) 1 Ω (ii) 3.75 J

F33 (a) 3.75 V (b) 19 mW

F34 0.367 m

F35 (a) (i) 5.0 A (ii) 100 V (b) 2.1 Ω

F36 (a) 76.8 Ω (b) 3.49 m

F37 (b) 6.0 Ω (c) 500%

F39 (a) 0.718 A (b) 4.19 W, 2.095×10^{-3} kW h (c) 8.17 V, 3.59 V (d) 9.15 W, 0.592 W (e) 3.60 Ω

F40 (b) (i) 78 Ω (ii) 8.0×10^2 W (d) (i) 28 W

F42 (a) 0.25 m from one end (b) 2 μA

F43 (a) 17.2 °C (b) $1.21 \times 10^{-7}\,\Omega\,m$
F44 (b) (i) 2.46 A (ii) $0.500\,V\,m^{-1}$, $4.90 \times 10^{6}\,A\,m^{-2}$, $10.2 \times 10^{-8}\,\Omega\,m$
(c) (i) 4.0 V (iii) 6.7 Ω, 14.4 W
F46 (c) 0.390 Ω
F47 2 V
F48 50 Ω
F49 24 V
F50 82.5 cm, 1.29 V
F52 1 Ω, 5.04 V
F53 (c) 50.2 Ω
F54 (a) (iii) 1.36 V (iv) 18.6 mW
F55 (c) 294.7 Ω
F56 (b) 6.330 mV
F58 (b) (i) 0.075 m
F59 6.0×10^{-8} J
F60 −24 mJ
F62 (a) 3.0×10^{-6} N (b) 5.5×10^{-10} C
(c) 1.8×10^{3} V
F64 (b) (i) 2.0×10^{-3} N (ii) 2.0×10^{-3} N
(iii) 6.0×10^{-4} J (iv) 0 (v) 0
F65 (a) 1.45×10^{-10} N, 1.45×10^{-11} J
F68 (i) 36 μA (ii) 1.44×10^{6} V (iii) 51.8 W
F69 (a) 1.1×10^{-3} C (b) 2.8×10^{3} J
(c) $2.5 \times 10^{6}\,V\,m^{-1}$
F70 (a) 1.5 V (b) 0.56 V (c) 1.5 V
F71 (b) 3 μF (c) 36 μC (A_1 and B_1), −36 μC (A_2 and B_2)
F72 13.3 μC
F73 (i) $5 \times 10^{6}\,V\,m^{-1}$ (ii) 4.4×10^{-12} F
(iii) 2.2×10^{-6} J
F74 133 μC on each; 133 μC
F75 1475 J; an increase of 983 J
F76 $1.13 \times 10^{5}\,V\,m^{-1}$; (a) 226 V (b) 2.2×10^{-10} F
(c) 5.65×10^{-6} J
F77 $\frac{4}{3}$ V; $1\frac{1}{3}$ V, $\frac{4}{3} \times 10^{3}\,V\,m^{-1}$
F78 (b) (i) 1.5 μF (ii) 108 μJ (iii) 9.0 V
F79 7
F80 0.2 μF; 0.25 mJ
F81 0.45 J; 0.15 J
F82 (a) 5.18×10^{-9} J (b) 1.56×10^{-8} J, 1.04×10^{-6} N
F83 36 μJ, 24 μJ; (a) 36 μJ, 384 μJ (b) 36 μJ, 54 μJ
F84 40 V
F85 50 J, $33\frac{1}{3}$ J
F86 (a) 1×10^{-7} C (b) 1000 V (c) 4×10^{-5} J
F87 (a) 4 μC on each (b) 2 μC (M), 8 μC (N)
F88 (b) (ii) (I) 16 μJ (II) 32 μJ
F89 (b) (i) 2.0×10^{-4} C (ii) 4.0×10^{-3} J
(iii) 2.0×10^{-3} J
(c) (i) 20 ms (ii) 92 ms
F92 (a) (i) 24 μC, 144 μJ (ii) 4.8 V, 57.6 μJ
(b) (iii) 33 MΩ (approx.)
F95 1.35×10^{-3} C; 3 times; 84.4 V
F96 Initially 3×10^{-9} C on each; finally 1×10^{-9} C and 5×10^{-9} C; 1.2×10^{-8} J
F97 500 μF
F98 4
F100 9×10^{-6} N
F101 21.8°
F102 (a) 1.3×10^{-3} N (b) 1.54; 115 g
F104 (b) (i) 1.6×10^{-4} T (ii) zero
F108 1.08:1
F109 (a) 40 (b) 0.25
F113 (a) $3.0 \times 10^{-3}\,m\,s^{-1}$ (b) $1.0(4) \times 10^{29}\,m^{-3}$
(c) $3.5 \times 10^{-3}\,m^{2}\,s^{-1}\,V^{-1}$ (d) 0.82
F118 0.24 A
F120 (a) (i) 6.0×10^{-5} Wb (ii) 3.0 V
F121 (a) 0.7 A (b) 7.8×10^{-4} J
F122 (a) 0.24 cos θ Wb (b) 15 V
F123 (b) (i) 0.128 Wb, 2.6×10^{-2} V (ii) 1.1 mA, 6.0×10^{-7} T
F124 (b) (i) 140 ms, 2.5 V (c) (i) 75 mA (ii) 0.11 W
F127 16 mV
F129 (a) 4.0 A (b) $2.0\,A\,s^{-1}$
F130 (a) (i) 0 (ii) 2 V (iii) 0.1 A (iv) 1 V (v) 1 s
F134 (a) (i) $0.30\,A\,s^{-1}$ (ii) 0.24 A (c) 1.2 J
F135 (a) (i) 4.0 A (ii) $1.0\,A\,s^{-1}$ (iii) $0.5\,A\,s^{-1}$
(b) 268 divisions
F136 4 mH, $1.25\,A\,s^{-1}$
F137 (a) (i) 50 kW (ii) 340 V (iii) 70.6%
(b) (ii) 500 W
F138 100 μC
F139 (a) 2.5×10^{-3} Wb (b) 25 μC
F143 (b) (i) 0.2 T
F144 0.5 A
F145 (a) 1 A (b) 2 A (c) 2 A
F146 (b) (i) 100 kJ, 300 kJ
F147 (a) 7.1 V (b) zero
F148 7.5 mA, 750 mA
F149 (a) (i) 3.5 V (ii) 8.0×10^{-3} F
F150 (b) 30 Ω (c) 92 mH
F152 80 Hz; 400 μF
F153 (a) 0.10 A, 40 V (b) 3.0 W
F154 (a) (i) 0.50 A (ii) 240 Ω (iii) 240 Ω
(b) (ii) 416 Ω (iii) 7.7 μF
F155 (a) 40 Ω (b) 2.9×10^{-2} H (c) 0.22 rad
F157 (a) $V_R = 3.0$ V RMS, $V_L = 4.0$ V RMS (c) 0.64 rad
F158 (a) 47.2 A (b) 8.91×10^{3} W
F159 (b) (ii) 1.6×10^{4} Hz
F160 1.6×10^{4} Ω
F162 0.337 A; 202.2 V, 214.3 V, 84.6 V
F163 (a) 0.124 A (b) 78.6 V
F164 (i) 10 Ω (ii) 69 mH
F165 (b) (i) 60 Ω, 480 W (ii) 0.33 H, 240 W
(iii) 30.6 μF, 240 W
F166 (a) 149 V (b) 188 V, 2.9 ms
F167 (a) 0.240 A RMS (b) 28.8 W (c) 4.77 μF
F168 0.21 A, 33 mH
F169 (b) 11.5 mH (c) 2.56:1
F170 8 Ω, 50 Hz
F171 (a) 159 Hz (b) 0.03 A
F172 (a) 1.13 pF (b) 5.4 cm (d) 6.9 pF
F173 (b) (i) 100 Hz
F174 (b) (i) X_C = 133 Ω, X_L = 30.0 Ω (ii) 131 Ω
(iii) 1.91 A (iv) 52° (v) 0.61 (c) 101 Hz
F175 (a) 50.0 Ω, 7.96 mH, 3.18 μF
(b) (i) 7.07 V (ii) 22.5 μC (iii) 7.07 V
(iv) $8.9 \times 10^{2}\,A\,s^{-1}$
F176 (a) 141 Ω (b) 0.20 A (c) 4.0 W
F177 227 V, 41.1 Hz
F178 7.8 V, 6.2 V
F179 (a) (ii) 3.79×10^{3} A, 5.7%
(b) (i) 50 Ω, 40 Hz (ii) V_R = 50 V, V_C = 100 V (iv) 90 V, 56°
F182 (a) 20 W
F185 (a) (iii) 0.042 A (RMS)
(c) (ii) 6.25×10^{-3} J (iii) 3.4 V (iv) 0.17 W
F189 (a) 2.4 V (b) 3.6 W (c) 4.0×10^{3} s
F190 $2.1 \times 10^{-4}\,m^{3}$
F191 $3.037 \times 10^{8}\,C\,kg^{-1}$
F192 0.42 A

SECTION G

G1 500 V
G3 1.96×10^{-19} J
G4 1.96×10^{-19} J, $>1.01 \times 10^{-6}$ m
G5 4.45 eV
G6 1.5 V, 2.5×10^{-19} J, 7.3×10^{5} m s^{-1}
G7 6.64×10^{-34} J s
G8 **(c)** **(i)** 2.35×10^{5} m s^{-1} **(ii)** 0.157 V
G9 **(b)** **(i)** 3.6×10^{-19} J **(ii)** 2.8×10^{17} s^{-1} **(iii)** 9.1% **(c)** **(i)** 2.2×10^{-11} m
G10 **(b)** 6.7×10^{-34} J s **(c)** **(i)** no emission **(ii)** electrons emitted with a KE of 3.74×10^{-19} J
G11 **(a)** 1 eV **(b)** **(i)** 1 V **(ii)** 0.75 V
G12 **(c)** **(i)** 3.4×10^{-19} J **(ii)** 6.68×10^{-34} J s
G13 **(b)** **(ii)** **(I)** 2.6×10^{-19} J **(II)** 7.5×10^{-7} m **(c)** 1.24×10^{-6} m
G14 2.5×10^{17} s^{-1}, 3.9 mA, 0.39 V
G15 **(a)** **(i)** 1.6×10^{-9} A **(ii)** 3.2×10^{-9} A **(iii)** zero **(b)** 0.5 V
G16 **(b)** **(i)** 1.0 V **(ii)** 1.6×10^{-19} J **(iii)** 3.8×10^{-19} J
G20 1.2×10^{-11} m
G24 **(a)** 2.19×10^{6} m s^{-1} **(b)** 9.14×10^{-8} m
G25 **(c)** 1.44×10^{-13} m
G26 **(b)** **(i)** 661 nm, 489 nm, 436 nm, 412 nm
G27 **(a)** 3.4×10^{-19} J **(b)** 1.8×10^{20} s^{-1}
G28 1.22×10^{-7} m
G29 **(a)** 2.16×10^{-18} J **(b)** 6.6×10^{-7} m
G30 **(b)** **(ii)** 5.1×10^{-19} J **(iii)** 3.9×10^{-7} m
G31 **(a)** **(i)** 1.9 eV **(ii)** 10.2 eV
G32 **(a)** **(i)** 3.3×10^{15} Hz **(ii)** 2.5×10^{15} Hz
G34 **(b)** **(vi)** 1.1×10^{7} m^{-1}
G35 **(b)** **(ii)** 6.6×10^{-7} m **(c)** **(ii)** 1.2×10^{-10} m
G39 **(c)** **(i)** 10 000 eV **(ii)** 1.23×10^{-11} m **(iii)** 1.60×10^{-15} J, 1.24×10^{-10} m
G40 **(a)** 1.2×10^{19} Hz **(b)** 9.9×10^{2} W
G41 **(b)** **(i)** 12 mA **(ii)** 1.33×10^{8} m s^{-1}
G42 **(a)** 4.1×10^{-11} m **(b)** 30 mA **(c)** 1.9×10^{15} s^{-1}
G44 **(a)** 4.0×10^{2} eV
G45 **(c)** **(i)** 0.1 V **(ii)** 8.4×10^{-24} J
G46 3.3 cm
G47 1.8 cm
G48 **(c)** **(i)** 8.3×10^{-7} s **(ii)** 4.3×10^{-5} T
G49 **(a)** 1.0×10^{8} m s^{-1} **(b)** 1.7×10^{-2} T
G50 1.33×10^{7} m s^{-1}, 1.71°
G51 6.7×10^{7} m s^{-1}
G54 6×10^{-2} mm V^{-1}
G56 **(a)** 4.2×10^{7} m s^{-1} **(b)** 0.12 m
G58 **(b)** 41 kV m^{-1}
G60 6
G61 **(a)** **(iv)** 3.6×10^{-15} N
G62 6.4×10^{-19} C
G63 4.68×10^{-19} C
G64 4.8×10^{-19} C; 1.6×10^{-19} C
G65 **(b)** **(i)** 3.8×10^{-9} s **(ii)** 2.9°
G67 **(b)** 1.14×10^{3} V **(c)** 3.53 V
G69 75% $^{35}_{17}Cl$, 25% $^{37}_{17}Cl$
G70 69% (62.93), 31% (64.93)
G71 **(a)** 3.2×10^{5} m s^{-1} **(b)** 10.6 kV
G72 **(a)** **(iii)** 179 m s^{-1} **(b)** **(i)** 3.2×10^{-15} N **(iii)** 0.11 T **(c)** **(iii)** 0.50 cm
G76 6.0 MeV
G77 **(b)** **(ii)** 0.015 cm **(iii)** ± 0.001 cm
G78 0.11 cm^{-1}
G79 **(c)** **(i)** 7.5×10^{6} min^{-1} **(ii)** 19%
G80 **(c)** 1.0×10^{-6} s^{-1} **(d)** 1.0×10^{18}
G81 8.8×10^{8} Bq
G82 50 s^{-1}
G83 **(b)** **(i)** 6.3×10^{8} **(ii)** 2.0×10^{-6} s^{-1} **(iii)** 1.3×10^{3} Bq
G84 11 μg
G85 4.3 hours
G86 2.8×10^{11} Bq
G87 **(a)** 6.0×10^{11} **(b)** 6.5×10^{11}
G88 1.27×10^{20}
G90 **(c)** **(i)** 1:2 **(ii)** 1:1 **(iii)** 75%
G91 **(b)** **(ii)** 56 years
G92 140 days; $a = 210$, $b = 84$, $c = 4$, $d = 2$, $e = 0$, $f = 0$
G93 6.0 m
G94 **(c)** **(i)** 6.2×10^{11} s
G95 4.0 mg per day
G96 6.0×10^{3} cm^{3}
G97 **(b)** **(i)** 2.0×10^{-6} s^{-1} **(ii)** 8.0×10^{9} **(c)** 40 hours
G98 >57 μCi
G99 **(d)** $\frac{1}{32}$
G100 2.0 m; 24 hours
G105 **(b)** **(i)** 70 s, 10.0 μg **(ii)** 2.5×10^{14} s^{-1}
G106 **(c)** **(ii)** 1.3×10^{-4} s^{-1} **(iii)** 7.7×10^{15} s^{-1} **(iv)** 63 W
G108 4.5×10^{9} years
G109 6.8×10^{8} years
G110 4.2×10^{3} years
G111 8700 years
G112 **(b)** **(ii)** 3.8×10^{3} years
G114 7.16 MeV
G115 2.79×10^{-12} J
G117 **(c)** **(i)** 1.813 u, 1.824 u
G118 **(a)** 28.3 MeV **(b)** 23.8 MeV
G119 **(c)** **(iii)** 2.6×10^{11} **(iv)** 1.7×10^{-15} kg
G120 **(c)** 1.1×10^{11} years
G121 8.2×10^{-14} J
G123 **(b)** **(iii)** 1.06
G124 +0.624 MeV, −4.01 MeV
G125 39.1 MeV
G127 8.53×10^{10} J
G128 **(c)** 2.5×10^{3} mol **(d)** 2.5×10^{7} Pa
G129 **(d)** **(i)** 5.40(4) MeV **(iii)** 5.30(2) MeV
G130 7.1 days
G132 **(c)** **(iii)** 2.1×10^{-13} J
G133 **(b)** 15 hours
G138 60 Ω
G147 2.5×10^{4} Ω (R_1), 5.0×10^{2} Ω (R_2), 15 mA
G148 **(a)** 1.5 V **(b)** 1.5 V
G149 **(b)** **(i)** 250 μA, 12.75 mA, 3.5 V
G150 **(c)** 60 kΩ
G157 **(b)** **(i)** 100 times **(ii)** never
G158 **(a)** 10
G159 **(b)** 8
G160 **(a)** −4.0 V
G162 **(a)** **(i)** 4.7 kΩ, 150 kΩ **(ii)** 33
G164 **(a)** **(i)** 4.5 V **(ii)** 6.0 V **(b)** **(ii)** 5.7 V

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